The *luadraw* package (3.7)

Patrick Fradin

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2D and 3D drawing with Lua and TikZ.

https://github.com/pfradin/luadraw

Abstract

The *luadraw* package defines the environment of the same name, which lets you create mathematical graphs using the Lua language. These graphs are ultimately drawn by TikZ (and automatically saved), so why make them in Lua? Because Lua brings all the power of a simple, efficient programming language with good computational capabilities.

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Chapter 1

2D Drawing

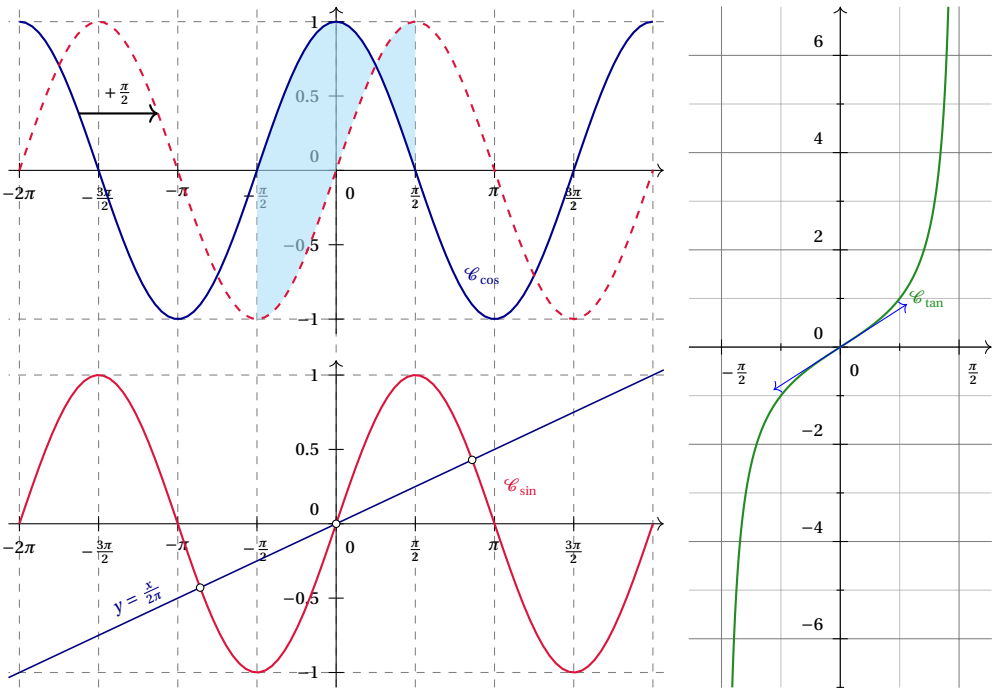


Figure 1: A first example: three sub-figures in the same graph

I Introduction

1) What's New Since Version 3.0

Since version 3.0, **all** data relating to the *luadraw* package is encapsulated in a single namespace, called **luadraw** (it's a table). This means that the data is only accessible using dot notation, specifically:

1. the class *graph* becomes *luadraw.graph*,
2. the class *cpx* (complex numbers) becomes *luadraw.cpx*,
3. the class *graph3d* becomes *luadraw.graph3d*,
4. the class *pt3d* (3D points) becomes *luadraw.pt3d*,
5. all global variables and all non-graphical functions are in the *luadraw* namespace, except for a few exceptions specific to complex numbers which are passed to the *cpx* class, and a few exceptions specific to 3D points which are passed to the *pt3d* class,

6. however, there is no change for the graphical methods since they were already encapsulated in the *graph* and *graph3d* classes.

The Lua language does not allow namespace factorization, so you really have to use dot notation, but fortunately you can create shortcuts (or aliases), for example:

```
local ld = luadraw -- alias on the namespace
local cpx, pt3d = ld.cpx, ld.pt3d -- shortcuts for the cpx and pt3d classes
local Z, M, i = cpx.Z, pt3d.M, cpx.I -- shortcuts to the Z and M functions and the complex number i
local Origin, vecI, vecJ, vecK = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK
local g = ld.graph3d:new{}
...
```

Due to this major change, scripts from previous versions **can no longer be compiled directly**; they must be adapted to this new version. However, with shortcuts, the changes are minimal.

Another new feature: the *luadraw* environment now uses a *luacode* environment (and no longer *luacode**), which allows TeX macros to be inserted into Lua code. Therefore, the most frequently used shortcuts can be put into a macro and used in *luadraw* code, for example:

```
\documentclass{article}
\usepackage[3d]{luadraw}

\newcommand*{\shortcuts}{%
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Z, M, i = cpx.Z, pt3d.M, cpx.I
local Origin, vecI, vecJ, vecK = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK
}%

\begin{document}

\begin{luadraw}{name=test}
\shortcuts
local g = ld.graph3d:new{}
local P = ld.polyreg(0, Z(2,3), 5)
g:Dpolyline(P, true, "red,line width=1.2pt")
local Q = ld.map(pt3d.toPoint3d, P)
P = ld.pyramid(Q, M(0,0,4))
g:Dpoly(P, {color="blue", opacity=0.6})
g:Show()
\end{luadraw}
\end{document}
```

The starred environment *luadraw**, on the other hand, uses a *luacode** environment; in this case, there is no interference from TEX in the Lua code.

NB : Throughout this documentation, we will use the following shortcuts:

```
local ld = luadraw -- alias on the namespace
local cpx = ld.cpx -- shortcut for the cpx class
local pt3d = ld.pt3d -- shortcut for the pt3d class
```

2) Prerequisites

- In the preamble, you must declare the *luadraw* package: `\usepackage[global options]{luadraw}`
- Compilation is done with LuaLateX **exclusively**.
- The colors in the *luadraw* environment are strings that must correspond to colors known to TikZ. It is strongly recommended to use the *xcolor* package with the *svgnames* option.

Regardless of the global options chosen, this package loads the *luadraw_graph2d.lua* module, which defines the *ld.graph* class, and provides the *luadraw* environment for creating graphs in Lua. To create 3D graphics, you must also load the *ld.graph3d* class using the global option 3d:

`\usepackage[3d]{luadraw}`.

Global package options : noexec, 3d, and cachedir=.

- noexec: When this global option is specified, the default value of the *exec* option for the *luadraw* environment will be false (and no longer true).
- 3d: When this global option is specified, the *luadraw_graph3d.lua* module is also loaded. This module also defines the *ld.graph3d* class (which relies on the *ld.graph* class) for 3D drawings.
- cachedir=<folder>: By default, the created files are saved in the *_luadraw* folder, which is a subfolder of the current folder (containing the master document). This folder can be changed with the cachedir option, for example cachedir={tikz}.

NB: In this chapter, we will not discuss the 3d option. This is the subject of the next chapter. We will therefore only discuss the 2D version.

When a graph is finished, it is exported in TikZ format, so this package also loads the TikZ package and the libraries:

- *patterns, patterns.meta*
- *plotmarks*
- *arrows.meta, bending*
- *decorations.markings*
- *backgrounds*

Graphs are created in a *luadraw* environment, which calls *luacode*, so the **lua language** must be used in this environment:

```
\begin{luadraw}{ name=<filename>, exec=true/false, auto=true/false }
local ld = luadraw
-- create a new graph with a local name (this line is a comment)
local g = ld.graph:new{ window={x1,x2,y1,y2 [,xscale,yscale]}, margin={top,right,bottom,left},
size={width,height,ratio}, bg="color", border=true/false }
-- build the g chart
graphic instructions in Lua language ...
-- display the g chart and save it in the <filename>.tkz file
g:Show()
-- or save it only in the <filename>.tkz file
g:Save()
\end{luadraw}
```

Saving the **.tkz* file : the chart is exported in TikZ format to a file (with the *tkz* extension). By default, it is saved in the *_luadraw* folder, which is a subfolder of the current folder (containing the master document), but it is possible to specify a path to another subfolder. with the global option cachedir=....

3) *luadraw* Environment Options

These are:

- name=...: Allows you to name the produced TikZ file. It is given a name without an extension (this will be automatically added; it is *.tkz*). If this option is omitted, then a default name is the name of the master file followed by a number.
- exec=true/false: Allows you to execute or not the Lua code included in the environment. By default, this option is true, **UNLESS** if the global option noexec was mentioned in the preamble with the package declaration. When a complex graph that requires a lot of calculations is ready, it may be useful to add the exec=false option. This will prevent recalculations of the same graph for future compilations.
- auto=true/false: Allows you to automatically include or exclude the TikZ file in place of the *luadraw* environment when the exec option is set to false. By default, the auto option is true.

4) The cpx (complex numbers) class

Reminder: we use the alias `local cpx = ld.cpx`.

It is automatically loaded by the *luadraw_graph2d* module and therefore when the *luadraw* package is loaded. This class allows you to manipulate complex numbers and perform common calculations. We create a complex number with the function **cpx.Z(a, b)** for $a + i \times b$, or with the function **cpx.Zp(r, theta)** for $r \times e^{i\theta}$ in polar coordinates.

- Example: `local z = cpx.Z(a,b)` will create the complex number corresponding to $a + i \times b$ in the variable z. We then access the real and imaginary parts of z like this: `z.re` and `z.im`. Of course, you can create a shortcut to this function with the instruction `local Z = cpx.Z` at the beginning of the code.
- **Warning**: A real number x is not considered complex by Lua. However, the functions provided for graphical constructions perform the verification and conversion from real to complex. However, we can use $Z(x,0)$ instead of x.
- The usual operators have been overloaded, allowing the use of the usual symbols, namely: +, x, -, /, as well as the equality test with = (this equality test is performed to within `ld.epsilon`, where `ld.epsilon` is a global variable that defaults to $1e-16$.). When a calculation fails, the returned result should normally be equal to `nil`. In addition, the following functions are added:
  - modulus: **cpx.abs(z)**,
  - modulus squared: **cpx.abs2(z)**,
  - normalization: **cpx.normalize(z)** (returns *nil* if z is null),
  - norm 1: **cpx.N1(z)**,
  - main argument: **cpx.arg(z)**,
  - conjugate: **cpx.bar(z)**,
  - perfect equality: **cpx.equal(z1,z2)**,
  - complex exponential: **cpx.exp(z)**,
  - the complex hyperbolic cosine and sine: **cpx.cosh(z)** and **cpx.sinh(z)**,
  - the power function a: **cpx.pow(z,a)** (the calculation uses the main argument of z),
  - scalar product: **cpx.dot(z1, z2)**, where the complex numbers represent vector affixes,
  - determinant: **cpx.det(z1, z2)**,
  - the oriented angle (in radians) between two non-zero vectors: **cpx.angle(z1, z2)**
  - rounding: **cpx.round(z, number of decimals)**,
  - the function: **cpx.isNul(z)** tests whether the real and imaginary parts of z are in absolute value less than the global variable `ld.epsilon` which is equal to $1e-16$ by default,
  - the function **cpx.isComplex(u)** tests whether $\langle u \rangle$ is complex or not and returns a boolean (note: if $\langle u \rangle$ is real, the function returns `false`),
  - the function **cpx.toComplex(x)** tests if $\langle x \rangle$ is a real number, if so, it returns the complex number `cpx.Z(x,0)`, if $\langle x \rangle$ is a complex number the function returns $\langle x \rangle$, and in other cases, it returns `nil`,
  - the function **cpx.isobar(L)** where $\langle L \rangle$ is a list of complex numbers, returns the isobarycenter of the points of $\langle L \rangle$.

We also have the constant `cpx.I` which represents the pure imaginary $\imath$.
Example:

```
local cpx = luadraw.cpx
local i = cpx.I
local A = 2+3*i
```

The multiplication symbol is required.

5) Displaying a Variable in the Terminal

The instruction **ld.whatis(variable [, msg])** displays the type of the ⟨*variable*⟩ and its contents in the terminal during compilation. Recognized types include the predefined types plus: *complex number*, *list of (complex) numbers*, and *list of lists of (complex) numbers*. The argument *msg* is an optional string (empty by default) which is displayed with the type to locate the variable in the terminal.

6) Creating a Graph

As seen above, creation is done in a *luadraw* environment. This creation is done by naming the graph:

```
local ld = luadraw
local g = ld.graph:new{ window = {x1,x2,y1,y2 [,xscale,yscale]}, margin = {left,right,top,bottom},
                        size = {width,height,ratio}, bg = "color", border = true/false,
                        bbox = true/false, pictureoptions = "" }
```

The *ld.graph* class is defined in the *luadraw* package. This class is instantiated by invoking its constructor and giving it a name (here it's *g*). This is done locally so that the graph *g* thus created will no longer exist once it leaves the environment (otherwise *g* would remain in memory until the end of the document). Here are the options for the constructor **ld.graph:new**:

- `window={x1,x2,y1,y2 [,xscale,yscale]}`. This option (optional) defines the $\mathbf{R}^2$ tile in which the graph is plotted (it is $[x_1;x_2]\times[y_1;y_2]$), as well as the axis scales, which are the parameters ⟨*xscale*⟩ and ⟨*yscale*⟩. These last two parameters are optional and default to 1; they represent the scale (cm per unit) on the axes. By default, we have `window = {-5,5,-5,5,1,1}`.
- `margin={left,right,top,bottom}`. This option (optional) sets the margins around the graph in cm. By default, we have `margin={0.5,0.5,0.5,0.5}`.
- `size={width,height [,ratio]}`. This option (optional) allows you to impose a size (in cm, including margins) for the graph. The ⟨*ratio*⟩ argument is optional and corresponds to the desired scale ratio (⟨*xscale*⟩/⟨*yscale*⟩). A ratio of 1 will result in an orthonormal coordinate system, and if the ratio is not specified, the default ratio (defined with the `window` option) is retained. A ratio of 0 determines the scales so that the graph is exactly the requested size. Using this parameter will modify the values of ⟨*xscale*⟩ and ⟨*yscale*⟩. The default window size is 11 × 11 (in cm) with the default margins (10 × 10 without margins).
- `bg="color"`. This option (optional) allows you to define a background color for the graph. This color is a string representing a color for TikZ. By default, this string is empty, meaning the background will not be painted.
- `border=true/false`. This option (optional) indicates whether or not a frame should be drawn around the graph (including the margins). By default, this parameter is set to `false`.
- `bbox=true/false`. This option (optional) indicates whether a bounding box should be added to the graph so that it has the desired size. Everything outside of it is clipped by TikZ. By default, this parameter is set to `true`. With the value `false`, no bounding box is added, but everything outside the 2D window, except for the paths, is clipped by *luadraw*. The graph size can be smaller than the requested size.
- `pictureoptions=""`. This option (optional) is a string containing options that will be passed to the *tikzpicture* environment like this:

```
\begin{tikzpicture}[line join=round <,pictureoptions>]
```

Graph construction.

- The instantiated object (*g* in the example) has several methods for drawing (segments, lines, curves, etc.). Drawing instructions are not sent directly to TEX; they are stored as strings in a table that is a property of the *g* object. The **g:Show()** method will send these instructions to TEXwhile saving them in a text file.[1] The **g:Save()** method saves the graph in the file designated by the (environment) option `name` but without sending the instructions to TEX.

[1] This file will contain a *tikzpicture* environment.

- The current graph can be saved to another file using the following method:

g:Savetofile(<filename with extension>)

- You can reset an existing graph—that is, remove all elements already created and restore all attributes to their default values—using the following method:

g:Cleargraph()

- The *luadraw* package also provides a number of mathematical functions, as well as functions for calculating lists (tables) of complex numbers, geometric transformations, etc. These functions are in the namespace *luadraw*.

Coordinate system. Location

- The instantiated object (g in the example) has:

1. An original view: this is the $\mathbf{R}^2$ block defined by the `window` option at creation. This **must not be modified** subsequently.
2. A current view: this is a $\mathbf{R}^2$ block that must be included in the original view; anything outside this block will be clipped. By default, the current view is the original view. To retrieve the current view, you can use the **g:Getview()** method, which returns a table `{x1,x2,y1,y2}`, representing the block $[x_1;x_2]\times[y_1;y_2]$.
3. A transformation matrix: this is initialized to the identity matrix. During a drawing instruction, the points are automatically transformed by this matrix before being sent to TikZ.
4. A coordinate system (Cartesian coordinate system) linked to the current view; this is the user's coordinate system. By default, this is the canonical coordinate system of $\mathbf{R}^2$, but it is possible to change it. Let's say the current view is the $[-5;5]\times[-5;5]$ block. It is possible, for example, to decide that this block represents the $[-1;12]$ interval for the abscissas and the $[0,8]$ interval for the ordinates. The method that makes this change will modify the graph's transformation matrix, so that for the user, everything happens as if they were in the $[-1;12]\times[0;8]$ block. The intervals of the user's coordinate system can be retrieved using the methods:

g:Xinf(), **g:Xsup()**, **g:Yinf()** and **g:Ysup()**.

- Complex numbers are used to represent points or vectors in the user's Cartesian coordinate system.

- In the TikZ export, the coordinates will be different because the lower left corner (excluding margins) will have coordinates $(0,0)$, and the upper right corner (excluding margins) will have coordinates corresponding to the size (excluding margins) of the graph, and with 1 cm per unit on both axes. This means that normally, TikZ should only handle « small » numbers.

- The conversion is done automatically with the **g:strCoord(x, y)** method, which returns a string of the form *(a,b)*, where *a* and *b* are the coordinates for TikZ, or with the **g:Coord(z)** method, which also returns a string of the form *(a,b)* representing the TikZ coordinates of the point with affix z in the user's coordinate system.

7) Saving and restoring settings

The **g:Saveattr([scope_options])** method saves the *luadraw* settings onto a stack and opens a *scope* environment. The ⟨*scope_options*⟩ argument is a string (empty by default) that is passed as an option to the *scope* environment.

The **g:Restoreattr()** method closes the *scope* environment and restores the *luadraw* settings.

8) Can we use TikZ directly in the *luadraw* environment?

Suppose we are creating a graph named g in a *luadraw* environment. It is possible to write a TikZ instruction during this creation, but not using `tex.sprint("<tikz instruction>")`, because then this instruction would not be part of the graph g. To do this, you must use the method **g:Writeln("<tikz instruction>")**, with the constraint that **the backslashes must be doubled**, and without forgetting that the graphic coordinates of a point in g are not the same for TikZ. For example:

```
g:Writeln("\\draw"..g:Coord(Z(1,-1))..." node[red] {Text};")
```

Or to change styles:

```
g:Writeln("\\tikzset{every node/.style={fill=white}}")
```

In a Beamer presentation, this can also be used to insert pauses in a graph:

```
g:Writeln("\\pause")
```

II Graphics Methods

Polygonal lines, curves, paths, points, and labels can be created.

1) Polygonal Lines

Dpolyline

A polygonal line is a list (table) of connected components, and a connected component is a list (table) of complex numbers that represent the affixes of the points. For example, the instruction:

```
local Z = luadraw.cpx.Z
local L = { {Z(-4,0), Z(0,2), Z(1,3)}, {Z(0,0), Z(4,-2), Z(1,-1)} }
```

creates a polygonal line with two components in a variable *L*.

Drawing a polygonal line. This is the

g:Dpolyline(L [, close, draw_options, clip])

method (where *g* denotes the graphic being created), the argument ⟨*L*⟩ is a polygonal line, ⟨*close*⟩ is an optional argument equal to `true` or `false` indicating whether the line should be closed or not (`false` by default), and ⟨*draw_options*⟩ is a string that will be passed directly to the *\draw* instruction in the export (empty by default). The argument ⟨*clip*⟩ must contain either `nil` (default value) or a table of the form `{x1,x2,y1,y2}`, in the first case the line is clipped by the current 2D window **after** its transformation by the 2D matrix of the graph, in the second case the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph matrix.

Options for drawing a polygonal line

Drawing options can be passed directly to the *\draw* instruction in the export, but they will have only a local effect. These options can be modified globally using the method

g:Lineoptions(style [, color, width, arrows])

when one of the arguments is nil, its default value is used.

- ⟨*color*⟩ is a string representing a color known to TikZ (default is black),
- ⟨*style*⟩ is a string representing the type of line to draw. This style can be:
  - `"noline"`: undrawn line,
  - `"solid"`: solid line (default value),
  - `"dotted"`: dotted line,
  - `"dashed"`: dashed line,
  - custom style: the ⟨*style*⟩ argument can be a string of the form (example): `"{2.5pt}{2pt}"` which means: a 2.5pt line followed by a 2pt space, the number of values can be greater than 2, e.g.: `"{2.5pt}{2pt}{1pt}{2pt}"` (succession of on, off).
- ⟨*width*⟩ is a number representing the line thickness expressed in tenths of a point, for example 8 for an actual thickness of 0.8pt (default value of 4),
- ⟨*arrows*⟩ is a string that specifies the type of arrow that will be drawn, this can be:
  - `"-"` which means no arrow (default value),

  - `"->"` which means an arrow at the end,
  - `"<-"` which means an arrow at the beginning,
  - `"<->"` which means an arrow at each end.
    WARNING: The arrow is not drawn when the argument ⟨*close*⟩ is `true`.

The options can be modified individually using the following methods:

- **g:Linecolor(color)**,
- **g:Linestyle(style)**,
- **g:Linewidth(width)**,
- **g:Arrows(arrows)**,
- plus the following methods:
  - **g:Lineopacity(opacity)**, which sets the opacity of the line drawing. The ⟨*opacity*⟩ argument must be a value between 0 (fully transparent) and 1 (fully opaque). The default value is 1.
  - **g:Linecap(style)**, to adjust the line ends, the ⟨*style*⟩ argument is a string that can be:
    * `"butt"` (straight end at the breakpoint, default value),
    * `"round"` (rounded semicircle end),
    * `"square"` (rounded square end).
  - **g:Linejoin(style)**, to adjust the join between segments, the ⟨*style*⟩ argument is a string that can be:
    * `"miter"` (pointed corner, default value),
    * `"round"` (or rounded corner),
    * `"bevel"` (cut corner).

Polygonal line fill options. This is the

g:Filloptions(style [, color, opacity, evenodd])

method (which uses the TikZ *patterns* library, which is loaded with the package). When one of the arguments is `nil`, its default value is used:

- ⟨*color*⟩ is a string representing a color known to TikZ (default is `"black"`).
- ⟨*style*⟩ is a string representing the fill type. This style can be:
  - `"none"`: no fill, this is the default value,
  - `"full"`: full fill,
  - `"bdiag"`: descending hatching from left to right,
  - `"fdiag"`: ascending hatching from left to right,
  - `"horizontal"`: horizontal hatching,
  - `"vertical"`: vertical hatching,
  - `"hvcross"`: horizontal and vertical hatching,
  - `"diagcross"`: descending and ascending diagonals,
  - `"gradient"`: in this case, the fill is done with a gradient defined with the method **g:Gradstyle(string)**, this style is passed as is to the *\draw* instruction. By default, the string defining the gradient style is:

    ```
    "left color=white,right color=red"
    ```

  - Any other style known from the *patterns* library is also possible.
  - ⟨*opacity*⟩: a number between 0 and 1 (1 by default).
  - ⟨*evenodd*⟩: a boolean indicating whether the *even odd rule* option should be used for fill (`false` by default).

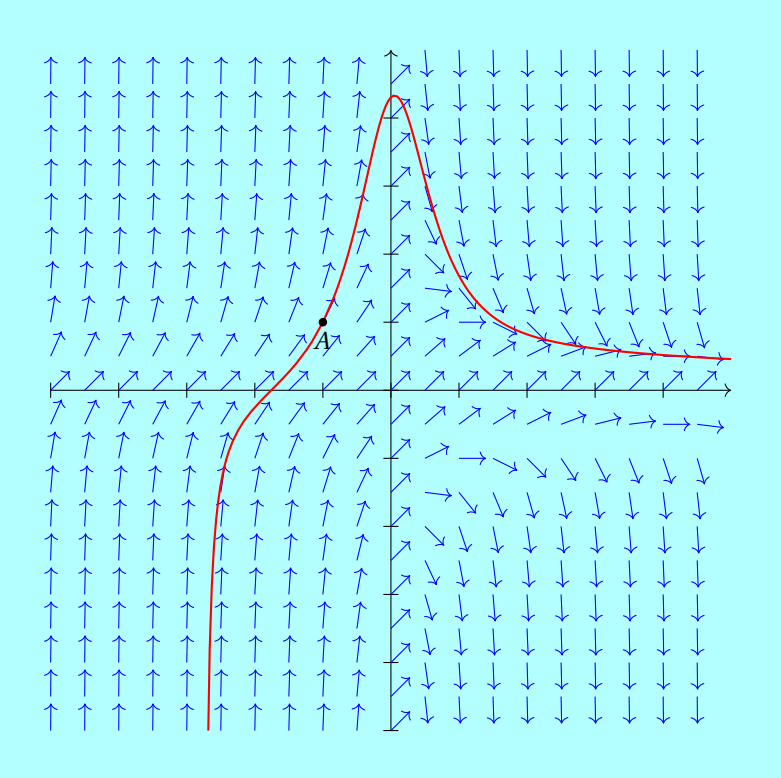
Some options can be modified individually with the methods:

- **g:Fillopacity(opacity)**,
- **g:Filleo(evenodd)**.

```
\begin{luadraw}{name=champ}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z
local g = ld.graph:new{window={-5,5,-5,5},bg="Cyan!30",size={10,10}}
local f = function(x,y) -- ode y'= 1-x*y^2=f(x,y)
    return 1-x*y^2
end
local A = Z(-1,1) -- A = -1+i
local deltaX, deltaY, long = 0.5, 0.5, 0.4
local champ = function(f)
    local vecteurs, v = {}
    for y = g:Yinf(), g:Ysup(), deltaY do
        for x = g:Xinf(), g:Xsup(), deltaX do
            v = Z(1,f(x,y)) -- coordinates 1 and f(x,y)
            v = v/cpx.abs(v)*long -- normalization of v
            table.insert(vecteurs, {Z(x,y), Z(x,y)+v} )
        end
    end
    return vecteurs -- vecteurs is a polygonal line
end

g:Daxes( {0,1,1}, {labelpos={"none","none"}, arrows="->"} )
g:Dpolyline( champ(f), "->,blue")
g:Dodesolve(f, A.re, A.im, {t={-2.75,5},draw_options="red,line width=0.8pt"})
g:Dlabeldot("$A$", A, {pos="S"})
g:Show()
\end{luadraw}
```

Figure 2: Vector field, integral curve of $y' = 1 - xy^2$



Dpolyline2

The **g:Dpolyline2(L [, close, draw_options])** method functions in the same way as the **g:Dpolyline()** method, with two exceptions:

1. In addition to complex numbers, the list ⟨*L*⟩ may contain TikZ nodes expressed as strings—for example: `"(A.south)"`. However, these nodes are only resolved during the compilation of the TikZ code—that is, after the **g:Show()** method has been executed. Consequently, *luadraw* has no knowledge of the numerical coordinates of these nodes, which significantly limits the utility of this method.
2. For the reasons mentioned above, the list ⟨*L*⟩ cannot be clipped by *luadraw*.

2) Segments and Lines

In this section we will use the alias `local ld = luadraw`.

A segment is a list (table) of two complex numbers representing the endpoints. A line is a list (table) of two complex numbers, the first representing a point on the line, and the second a direction vector of the line (this must be non-zero).

Dangle

- The **g:Dangle(B, A, C [, r, draw_options])** method draws the angle *BAC* with a parallelogram (only two sides are drawn). The optional argument ⟨*r*⟩ specifies the length of one side (0.25 by default). The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction.
- The function **ld.angleD(B, A, C, r)** returns the list of points of this angle.

Dbissec

- The method **g:Dbissec(B, A, C, interior, draw_options, scale)** draws a bisector of the geometric angle BAC, interior if the optional argument ⟨*interior*⟩ is `true` (default value), exterior otherwise. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.
- The function **ld.bissec(B, A, C, interior)** returns this bisector as a list $\{A, u\}$ where u is a direction vector of the line.

Dhline

The method **g:Dhline(d [, draw_options, scale])** draws a half-line. The argument ⟨*d*⟩ must be a list of two complex numbers $\{A, B\}$ variables. The half-line $[A, B)$ is drawn.

Variant: **g:Dhline(A, B [, draw_options, scale])**. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.

Dline

The **g:Dline(d [, draw_options, scale])** method draws the line ⟨*d*⟩, which is a list of type $\{A, u\}$ where A represents a point on the line (a complex number) and u a direction vector (a non-zero complex number).

Variant: the **g:Dline(A, B [, draw_options, scale])** method draws the line passing through the points ⟨*A*⟩ and ⟨*B*⟩ (two complex numbers). The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.

DlineEq

- The **g:DlineEq(a, b, c [, draw_options, scale])** method draws the line with the equation $ax + by + c = 0$. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.
- The **ld.lineEq(a, b, c [, inequality])** function returns the line with the equation $ax + by + c = 0$ as a list $\{A, u\}$ where A is a point on the line and u is a direction vector. The optional argument ⟨*inequality*⟩ (`nil` by default) can also take the value `'<'` or `'>'`, the direction vector u is then chosen such that the half-plane satisfying the inequality $ax + by + c < 0$ (or $ax + by + c > 0$ in the other case) is to the left of u (in the trigonometric direction).

Dmarkarc

The method **g:Dmarkarc(b, a, c, r, n [, length, space, draw_options])** marks the arc of a circle with center ⟨*a*⟩, radius ⟨*r*⟩, extending from ⟨*b*⟩ to ⟨*c*⟩, with ⟨*n*⟩ small segments. By default, the ⟨*length*⟩ is 0.25, and the spacing (argument ⟨*space*⟩) is 0.0625. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*.

Dmarkseg

The method **g:Dmarkseg(a, b, n [, length, space, angle, draw_options])** marks the segment defined by ⟨*a*⟩ and ⟨*b*⟩ with ⟨*n*⟩ small segments inclined at ⟨*angle*⟩ degrees (45° by default). By default, the ⟨*length*⟩ is 0.25, and the spacing (argument ⟨*space*⟩) is 0.125. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*.

Dmed

- The **g:Dmed(A, B [, draw_options, scale])** method draws the perpendicular bisector of the segment $[A;B]$.

  Variant: **g:Dmed(seg [, draw_options, scale])** where segment is a list of two points representing the segment. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.
- The function **ld.med(A, B)** (or **ld.med(seg)**) returns the perpendicular bisector of the segment $[A;B]$ as a list $\{C, u\}$ where C is a point on the line and u is a direction vector.

Dparallel

- The method **g:Dparallel(d, A [, draw_options, scale])** draws the parallel to ⟨*d*⟩ passing through ⟨*A*⟩. The argument ⟨*d*⟩ can be either a line (a list consisting of a point and a direction vector) or a non-zero vector. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.
- The function **ld.parallel(d, A)** returns the parallel to ⟨*d*⟩ passing through ⟨*A*⟩ as a list $\{B, u\}$ where B is a point on the line and u is a direction vector.

Dperp

- The method **g:Dperp(d, A [, draw_options, scale])** draws the perpendicular to ⟨*d*⟩ passing through ⟨*A*⟩. The argument ⟨*d*⟩ can be either a line (a list consisting of a point and a direction vector) or a non-zero vector. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint),

or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.

- The function **ld.perp(d, A)** returns the perpendicular to ⟨*d*⟩ passing through ⟨*A*⟩ as a list $\{B, u\}$ where B is a point on the line and u is a direction vector.

Dseg

The **g:Dseg(seg [, scale, draw_options])** method draws the segment defined by the ⟨*seg*⟩ argument, which must be a list of two complex numbers. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.

Dtangent

- The **g:Dtangent(p, t0 [, length, draw_options])** method draws the tangent to the curve parameterized by ⟨*p*⟩: $t \mapsto p(t)$ (with complex values), at the parameter point ⟨*t0*⟩. If the argument ⟨*length*⟩ is `nil` (the default value), then the entire line is drawn. If it is a number, then a segment of length ⟨*length*⟩ is drawn, centered on the point where the calculation is performed (denoted A). If it is a pair of numbers $\{l_1, l_2\}$, then a segment is drawn in which the first part (up to point A) has length l_1 and the second part (starting from point A) has length l_2. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction.

- The function **ld.tangent(p, t0 [, length])** returns either the line or a segment (depending on whether ⟨*length*⟩ is `nil` or not).

DtangentC

- The method **g:DtangentC(f, x0 [, length, draw_options])** draws the tangent to the Cartesian curve with equation $y = f(x)$, at the abscissa point ⟨*x0*⟩. If the argument ⟨*length*⟩ is `nil` (the default value), then the entire line is drawn. If it is a number, then a segment of length ⟨*length*⟩ is drawn, centered on the point where the calculation is performed (denoted A). If it is a pair of numbers $\{l_1, l_2\}$, then a segment is drawn in which the first part (up to point A) has length l_1 and the second part (starting from point A) has length l_2. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*.

- The function **ld.tangentC(f, x0 [, length])** returns either the line or a segment (depending on whether ⟨*length*⟩ is `nil` or not).

DtangentI

- The method **g:DtangentI(f, x0, y0 [, length, draw_options])** draws the tangent to the implicit curve with equation $f(x, y) = 0$, at the assumed point ⟨*(x0,y0)*⟩ on the curve. If the argument ⟨*length*⟩ is `nil` (the default value), then the entire line is drawn. If it is a number, then a segment of length ⟨*length*⟩ is drawn, centered on the point where the calculation is performed (denoted A). If it is a pair of numbers $\{l_1, l_2\}$, then a segment is drawn in which the first part (up to point A) has length l_1 and the second part (starting from point A) has length l_2. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*.

- The function **ld.tangentI(f, x0, y0 [, length])** returns either the line or a segment (depending on whether ⟨*length*⟩ is `nil` or not).

Dtangent_from

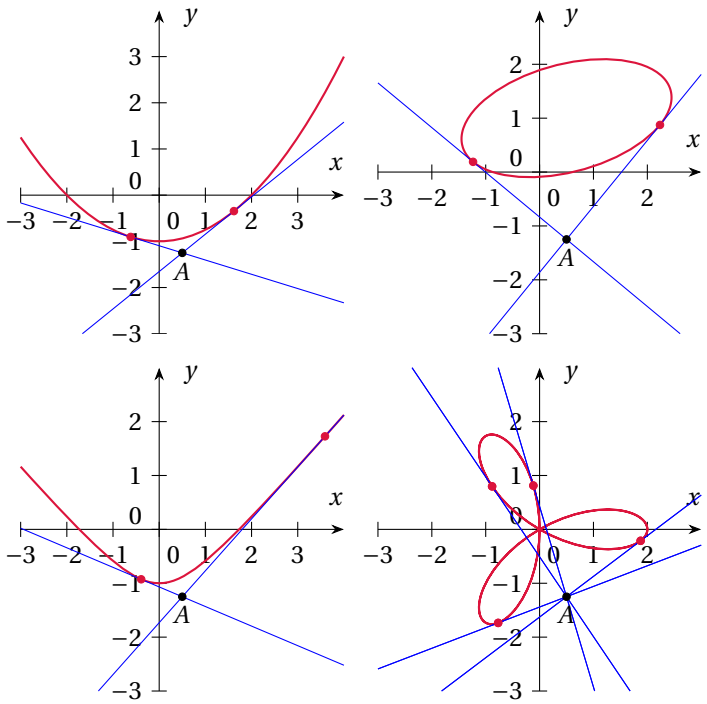
- The method **g:Dtangent_from(A, p, t1, t2 [, dp, draw_options, out, scale])** draws the tangent(s) to the curve parameterized by ⟨*p*⟩: $t \mapsto p(t)$ (with complex values) originating from the point ⟨*A*⟩ (a complex number). The arguments ⟨*t1*⟩ and ⟨*t2*⟩ represent the bounds of the search interval. The optional argument ⟨*dp*⟩ is a function representing the derivative of the function ⟨*p*⟩; by default, this argument is `nil`, and the derivative of ⟨*p*⟩ is replaced

by an approximation. The optional argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction. The optional argument ⟨*out*⟩, if used, must be the name of a variable. This variable must be an array, and at the end of execution, it will contain the points on the curve for which the tangent passes through ⟨*A*⟩. The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.

- The function **ld.tangent_from(A, p, t1, t2 [, dp])** returns the list of points on the curve parameterized by ⟨*p*⟩, on the interval $[t_1; t_2]$, for which the tangent passes through ⟨*A*⟩ (a complex number). If there are no points, the function returns an empty list.

```
\begin{luadraw}{name=tangent_from}
local ld = luadraw
local cpx = ld.cpx
local i, cos, sin, pi = cpx.I, math.cos, math.sin, math.pi
local g = ld.graph:new{window={-5,5,-5,5}, size={10,10}}
local p1 = function(t) return t+i*(t^2/4-1) end -- parabola
local p2 = function(t) return 1/2+i+ld.rotate(2*cos(t)+i*sin(t),15) end -- ellipse
local p3 = function(t) return math.sinh(t)+i*math.cosh(t)-i*2 end -- branch of a hyperbola
local p4 = function(t) return 2*cos(3*t)*cpx.exp(i*t) end -- other
local P = 0.5-1.25*i
local draw = function(p,t1,t2)
    local axis_style = { arrows='-Stealth', legend={'$x$','$y$'} }
    local S = {}
    g:Daxes({0, 1, 1}, axis_style)
    g:Dparametric(p,{t={t1,t2}, draw_options="line width=0.8pt, Crimson"})
    g:Dtangent_from(P,p,t1,t2,"blue",S)
    g:Ddots(S,"Crimson"); g:Ddots(P); g:Dlabel("$A$",P,{pos="S"})
end
g:Saveattr(); g:Viewport(-5,-0.25,0.25,5); g:Coordsystem(-3,4,-3,4); draw(p1,-4,4); g:Restoreattr()
g:Saveattr(); g:Viewport(0.25,5,0.25,5); g:Coordsystem(-3,3,-3,3); draw(p2,-pi,pi); g:Restoreattr()
g:Saveattr(); g:Viewport(-5,-0.25,-5,-0.25); g:Coordsystem(-3,4,-3,3); draw(p3,-4,4); g:Restoreattr()
g:Saveattr(); g:Viewport(0.25,5,-5,-0.25); g:Coordsystem(-3,3,-3,3); draw(p4,-pi,pi); g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 3: Tangents from a point



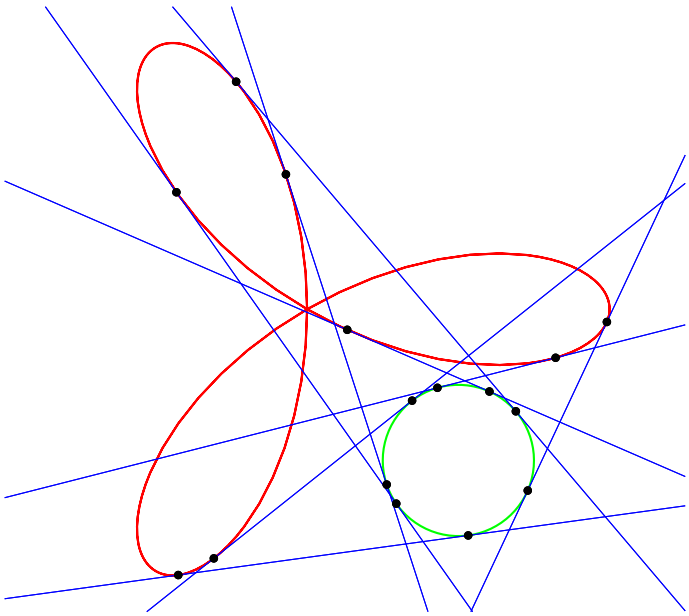
Dcommon_tangent

- The method **g:Dcommon_tangent(p1, p2, options)** draws the common tangent(s) to the curves parameterized by ⟨*p1*⟩: $t \mapsto p(t)$ (complex-valued) and ⟨*p2*⟩: $t \mapsto p(t)$ (complex-valued). The ⟨*options*⟩ argument is a table with the following fields:
  - `t1={-5,5}` : search interval for the parameterization ⟨*p1*⟩.
  - `t2={-5,5}` : search interval for the parameterization ⟨*p2*⟩.
  - `grid={50,50}` : table representing the number of subdivisions for ⟨*p1*⟩ and ⟨*p2*⟩.
  - `draw_options=""` : drawing options for the lines (string passed directly to the *\draw* instruction).
  - `scale=1` : The argument ⟨*scale*⟩ (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.
- The function **ld.common_tangent(p1, u1,u2, p2, v1, v2, grid)** returns a list of values of the form $\{t_1, t_2\}$ such that the line joining points $p_1(t_1)$ and $p_2(t_2)$ is a common tangent to both curves. If none exist, the function returns nothing.

Note: With this method, the calculations can sometimes be somewhat lengthy.

```
\begin{luadraw}{name=common_tangent}
local ld = luadraw
local cpx = ld.cpx
local i, cos, sin, pi = cpx.I, math.cos, math.sin, math.pi
local g = ld.graph:new{window={-4,5,-4,4}, size={10,10}}
local p1 = function(t) return  2-2*i+ cos(t)+i*sin(t) end -- ellipse
local p2 = function(t) return 4*cos(3*t)*cpx.exp(i*t) end -- autre
local dots = {}
g:Dparametric(p1,{t={-pi,pi}, draw_options="green,thick"})
g:Dparametric(p2,{t={-pi,pi}, draw_options="red,thick"})
g:Dcommon_tangent(p1,p2,{t1={-pi,pi}, t2={-pi,pi}, draw_options='blue', out=dots})
g:Ddots(dots)
g:Show()
\end{luadraw}
```

Figure 4: Common tangents



Dparallel_tangent

- The method **g:Dparallel_tangent(p, t0, options)** draws the tangent(s) to the curve parameterized by ⟨*p*⟩: $t \mapsto p(t)$ (with complex values) that are parallel to the tangent at the point with parameter ⟨*t0*⟩ **if the latter is a real number**.

When ⟨*t0*⟩ is a complex number representing a non-zero vector, the method draws the tangents having that vector as their direction. The ⟨*options*⟩ argument is a table with the following fields:

- `t={-5,5}` : search interval.
- `draw_options=""` : drawing options for the lines (string passed directly to the *\draw* instruction).
- `len=nil` : identical to the ⟨*length*⟩ argument of the **g:Dtangent()** method.
- `dp=nil` : allows optionally providing the derivative function of ⟨*p*⟩. By default, the derivative is an approximation.
- `out=nil` : when this option is a variable representing a table, the solutions for the parameter t will be added to that table.

- The function **ld.parallel_tangent(p, t0, t1, t2 [,dp])** returns a list of t values within the interval $[t_1; t_2]$ such that the tangents to the curve parameterized by ⟨*p*⟩ at the points with parameter t are parallel to the tangent at the point with parameter ⟨*t0*⟩, or have the direction of the vector ⟨*t0*⟩. If none exist, the function returns nothing.

Dnormal

- The method **g:Dnormal(p, t0 [, length, draw_options])** draws the normal to the curve parameterized by ⟨*p*⟩: $t \mapsto p(t)$ (with complex values), at the point parameter ⟨*t0*⟩. If the argument ⟨*length*⟩ is `nil` (the default value), then the entire line is drawn. If it is a number, then a segment of length ⟨*length*⟩ is drawn, centered on the point where the calculation is performed (denoted A). If it is a pair of numbers $\{l_1, l_2\}$, then a segment is drawn in which the first part (up to point A) has length l_1 and the second part (starting from point A) has length l_2. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*.

- The function **ld.normal(p, t0 [, long])** returns either the line or a segment (depending on whether ⟨*long*⟩ is `nil` or not).

DnormalC

- The **g:DnormalC(f, x0 [, length, draw_options])** method draws the normal to the Cartesian curve with equation $y = f(x)$, at the abscissa point ⟨*x0*⟩. If the argument ⟨*length*⟩ is `nil` (the default value), then the entire line is drawn. If it is a number, then a segment of length ⟨*length*⟩ is drawn, centered on the point where the calculation is performed (denoted A). If it is a pair of numbers $\{l_1, l_2\}$, then a segment is drawn in which the first part (up to point A) has length l_1 and the second part (starting from point A) has length l_2. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction.

- The function **ld.normalC(f, x0 [, long])** returns either the line or a segment (depending on whether ⟨*long*⟩ is `nil` or not).

DnormalI

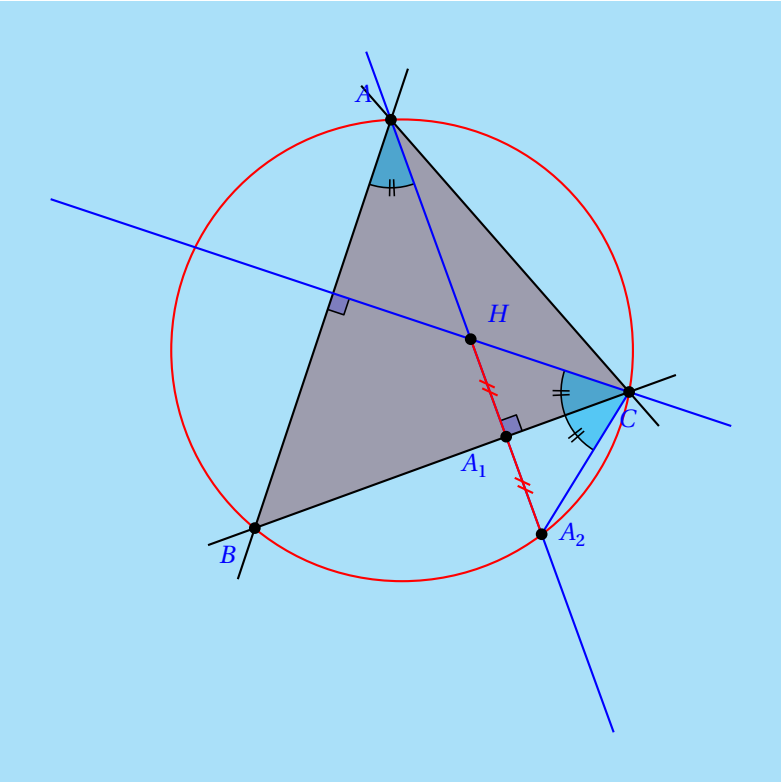
- The **g:DnormalI(f, x0, y0 [, length, draw_options])** method draws the normal to the implicit curve with equation $f(x, y) = 0$, at the assumed point ⟨*(x0,y0)*⟩ on the curve. If the argument ⟨*length*⟩ is `nil` (the default value), then the entire line is drawn. If it is a number, then a segment of length ⟨*length*⟩ is drawn, centered on the point where the calculation is performed (denoted A). If it is a pair of numbers $\{l_1, l_2\}$, then a segment is drawn in which the first part (up to point A) has length l_1 and the second part (starting from point A) has length l_2. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction.

- The function **ld.normalI(f, x0, y0, long)** returns either a line or a segment (depending on whether ⟨*long*⟩ is `nil` or not).

```
\begin{luadraw}{name=orthocentre}
local ld = luadraw
local cpx = ld.cpx
local g = ld.graph:new{window={-5,5,-5,5},bg="cyan!30",size={10,10}}
local i = cpx.I
local A, B, C = 4*i, -2-2*i, 3.5
```

```
local h1, h2 = ld.perp({B,C-B},A), ld.perp({A,B-A},C) -- altitudes of the triangle
local A1, F = ld.proj(A,{B,C-B}), ld.proj(C,{A,B-A}) -- projected
local H = ld.interD(h1,h2) -- orthocenter
local A2 = 2*A1-H -- symmetric to H with respect to BC
g:Dpolyline({A,B,C},true, "draw=none,fill=Maroon,fill opacity=0.3") -- background of the triangle
g:Linewidth(6); g:Filloptions("full", "blue", 0.2)
g:Dangle(C,A1,A,0.25); g:Dangle(B,F,C,0.25) -- right angles
g:Linecolor("black"); g:Filloptions("full","cyan",0.5)
g:Darc(H,C,A2,1); g:Darc(B,A,A1,1) -- arcs
g:Filloptions("none","black",1) -- The opacity is restored to 1
g:Dmarkarc(H,C,A1,1,2); g:Dmarkarc(A1,C,A2,1,2) -- markers
g:Dmarkarc(B,A,H,1,2)
g:Linewidth(8); g:Linecolor("black")
g:Dseg({A,B},1.25); g:Dseg({C,B},1.25); g:Dseg({A,C},1.25) -- sides
g:Linecolor("red"); g:Dcircle(A,B,C) -- circle
g:Linecolor("blue"); g:Dline(h1); g:Dline(h2) -- altitudes of the triangle
g:Dseg({A2,C}); g:Linecolor("red"); g:Dseg({H,A2}) -- segments
g:Dmarkseg(H,A1,2); g:Dmarkseg(A1,A2,2) -- markers
g:Labelcolor("blue") -- for the label
g:Dlabel("$A$",A, {pos="NW",dist=0.1}, "$B$",B, {pos="SW"}, "$A_2$",A2,{pos="E"},
"$C$", C, {pos="S" }, "$H$", H, {pos="NE"}, "$A_1$", A1, {pos="SW"})
g:Linecolor("black"); g:Filloptions("full"); g:Ddots({A,B,C,H,A1,A2})
g:Show()
\end{luadraw}
```

Figure 5: Symmetric of the orthocenter



3) Geometric Figures

Darc

- The **g:Darc(B, A, C, r [, sens, draw_options])** method draws an arc of a circle with center ⟨*A*⟩ (complex number), radius ⟨*r*⟩, going from ⟨*B*⟩ (complex number) to ⟨*C*⟩ (complex) counterclockwise if the argument ⟨*sens*⟩ is 1 (default value), and counterclockwise otherwise. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction.
- The **ld.arc(B, A, C, r, sens)** function returns the list of points on this arc (polygonal line).
- The function **ld.arcb(B, A, C, r, sens)** returns this arc as a path (see *Dpath* page 41) using Bézier curves.

Dcircle

- The method **g:Dcircle(c, r [, d, draw_options])** draws a circle. When the argument ⟨*d*⟩ is `nil`, it is the circle with center ⟨*c*⟩ (complex number) and radius ⟨*r*⟩; when ⟨*d*⟩ is specified (complex number), it is the circle passing through the affix points ⟨*c*⟩, ⟨*r*⟩, and ⟨*d*⟩. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*. Another possible syntax: **g:Dcircle(C [, draw_options])** where ⟨*C*⟩= {*c*, *r*[, *d*]}.
- The **ld.circle(c, r [, d])** function returns the list of points on this circle (polygonal line).
- The function **ld.circle({c, r [, d]}, nbdots)** returns the list of points of this circle (polygonal line) with the ⟨*nbdots*⟩ points.
- The **ld.circleb(c, r [, d])** function returns this circle as a path (see *Dpath* page 41) using Bézier curves.

Dellipse

- The **g:Dellipse(c, rx, ry [, inclin, draw_options])** method draws the ellipse centered at ⟨*c*⟩ (complex number). The arguments ⟨*rx*⟩ and ⟨*ry*⟩ specify the two radii (on *x* and *y*). The optional argument ⟨*inclin*⟩ is an angle in degrees that indicates the inclination of the ellipse relative to the *Ox* axis (zero by default). The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction.
- The **ld.ellipse(c, rx, ry [, inclin])** function returns the list of points on this ellipse (polygonal line).
- The function **ld.ellipse({c, rx, ry [, inclin]}, nbdots)** returns the list of points of this ellipse (polygonal line) with the ⟨*nbdots*⟩ points.
- The function **ld.ellipseb(c, rx, ry [, inclination])** returns this ellipse as a path (see *Dpath* page 41) using Bézier curves.

Dellipticarc

- The method **g:Dellipticarc(B, A, C, rx, ry [, sens, inclination, draw_options])** draws an arc of an ellipse centered at ⟨*A*⟩ (complex number) and radii at ⟨*rx*⟩ and ⟨*ry*⟩, making an angle equal to ⟨*inclination*⟩ with respect to the *Ox* axis (zero by default), going from ⟨*B*⟩ (complex number) to ⟨*C*⟩ (complex number) in the counterclockwise direction if the argument ⟨*sens*⟩ is 1 (default value), and the opposite direction otherwise. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction.
- The **ld.ellipticarc(B, A, C, rx, ry [, sens, inclination])** function returns the list of points of this arc (polygonal line).
- The **ld.ellipticarcb(B, A, C, rx, ry [, sens, inclination])** function returns this arc as a path (see *Dpath* page 41) using Bézier curves.

Dparallelogram

- The method **g:Dparallelogram(a, u, v [, draw_options])** where ⟨*a*⟩, ⟨*u*⟩, ⟨*v*⟩ represent three complex numbers denoting a vertex and two vectors, draws the parallelogram with vertices {*a*, *a*+*u*, *a*+*u*+*v*, *a*+*v*}. The argument ⟨*draw_options*⟩ is a string (empty by default) which will be passed as is to the *\draw* instruction.
- The function **ld.parallelogram(a, u, v)** returns the list of vertices of this parallelogram.

Dpolyreg

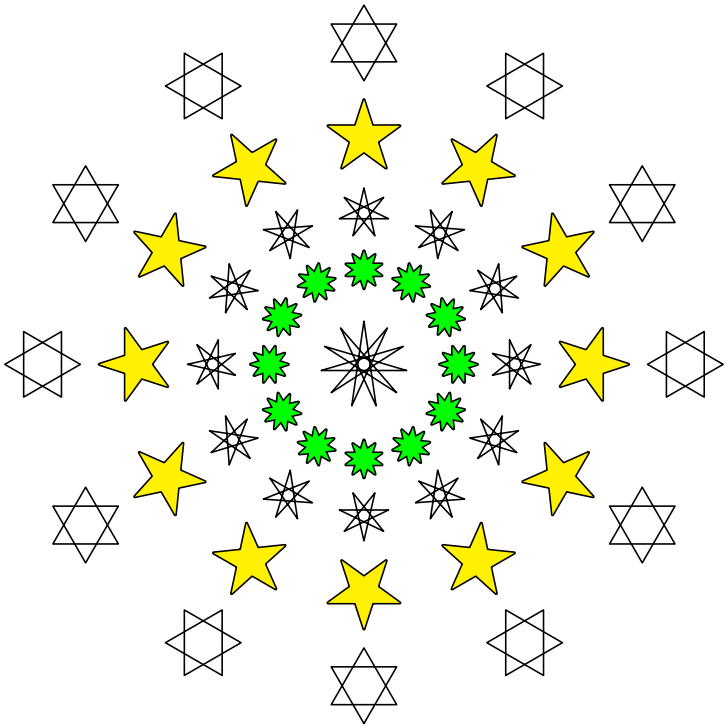
- The **g:Dpolyreg(vertex1, vertex2, nbsides, direction [, draw_options])** or

  g:Dpolyreg(center, vertex, nbsides [, draw_options]) method draws a regular polygon. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction.
- The **ld.polyreg(vertex1, vertex2, nbsides, direction)** function and the **ld.polyreg(center, vertex, nbsides)** function return the list of vertices of this regular polygon.

- {p/q}-regular star polygon (regular star polygon): the function **ld.pq_gonreg(C,S,type)** returns a list of lists of complex numbers representing a regular star polygon with center ⟨*C*⟩, passing through the vertex ⟨*S*⟩, and of type ⟨*type*⟩ where ⟨*type*⟩={*p*, *q*} with *p* and *q* two strictly positive integers, *p* is the number of vertices and *q* is the step (each vertex is connected to the next *q*th), with the condition: $2q < p$.

```
\begin{luadraw}{name=pq_gonreg}
local ld = luadraw
local cpx = ld.cpx
local Z, i = cpx.Z, cpx.I
local g = ld.graph:new{ size={10,10}, margin=0 }
g:Linewidth(6)
local pos1, pos2, pos3, pos4 = Z(0,3), Z(0,2), Z(0,1.25), Z(0,4.25)
local star1 = ld.pq_gonreg(pos1, pos1+i/2, {5,2})
local star2 = ld.pq_gonreg(pos2, pos2+i/3, {7,3})
local star3 = ld.pq_gonreg(pos3, pos3+i/4, {10,4})
local star4 = ld.pq_gonreg(pos4, pos4+i/2, {6,2})
for k = 0, 330, 30 do
    g:Dpolyline(star1, true, "line width=1.2"); g:Dpolyline(star1, "draw=none, fill=yellow")
    g:Dpolyline(star2, true)
    g:Dpolyline(star3, true, "line width=1.2"); g:Dpolyline(star3, "draw=none, fill=green")
    g:Dpolyline(star4, true)
    g:Rotate(30)
end
g:Dpolyline( ld.pq_gonreg(0,i/1.75,{11,5}), true)
g:Show()
\end{luadraw}
```

Figure 6: regular star polygon



Drectangle

- The **g:Drectangle(a, b, c [, draw_options])** method draws a rectangle with consecutive vertices ⟨*a*⟩ and ⟨*b*⟩, and whose opposite side passes through ⟨*c*⟩ (complex numbers). The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction.
- The **ld.rectangle(a, b, c)** function returns the list of vertices of this rectangle.
- The method **g:Drectangle(a, b [, draw_options])** draws the rectangle whose sides are parallel to the axes and whose opposite vertices are ⟨*a*⟩ and ⟨*b*⟩. The argument ⟨*draw_options*⟩ is a string (empty by default) which is passed as is to

the *\draw* instruction.

- The function **ld.rectangle(a, b)** returns the list of vertices of this rectangle.

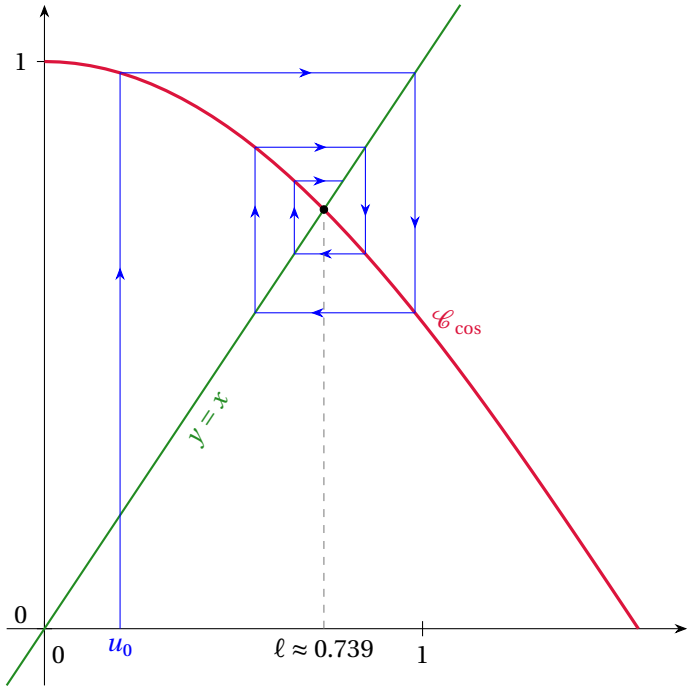
Dsequence

- The **g:Dsequence(f, u0, n, draw_options)** method draws the "staircases" of the recurrent sequence defined by its first term ⟨*u0*⟩ and the relation $u_{k+1} = f(u_k)$. The argument ⟨*f*⟩ must be a function of a real-valued variable, and the argument ⟨*n*⟩ is the number of terms calculated. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction.

- The **ld.sequence(f, u0, n)** function returns the list of points constituting these "staircases".

```
\begin{luadraw}{name=sequence}
local ld = luadraw
local cpx = ld.cpx
local g = ld.graph:new{window={-0.1,1.7,-0.1,1.1},size={10,10,0}}
local i, pi, cos = cpx.I, math.pi, math.cos
local f = function(x) return cos(x)-x end
local ell = ld.solve(f,0,pi/2)[1]
local L = ld.sequence(cos,0.2,5) -- u_{n+1}=cos(u_n), u_0=0.2
local seg, z = {}, L[1]
for k = 2, #L do
    table.insert(seg,{z,L[k]})
    z = L[k]
end -- seg is a list of segments (staircases)
local styleA = "\\tikzset{->-/.style={decoration={markings, mark="
local styleB = "at position #1 with {\\arrow{Stealth}}}, postaction={decorate}}}"
g:Writeln(styleA..styleB)
g:Daxes({0,1,1}, {arrows="-Stealth"})
g:DlineEq(1,-1,0,"line width=0.8pt,ForestGreen")
g:Dcartesian(cos, {x={0,pi/2},draw_options="line width=1.2pt,Crimson"})
g:Dpolyline(seg,false,"->-=0.65,blue")
g:Dlabel("$u_0$",0.2,{pos="S",node_options="blue"})
g:Dseg({ell, ell*(1+i)},1,"dashed,gray")
g:Dlabel("$\\ell\\approx"..ld.round(ell,3).."$", ell,{pos="S"})
g:Ddots(ell*(1+i)); g:Labelcolor("Crimson")
g:Dlabel("${\\mathcal C}_{\\cos}$",1+i*cos(1),{pos="E"})
g:Labelcolor("ForestGreen"); g:Labelangle(g:Arg(1+i)*180/pi)
g:Dlabel("$y=x$",0.4+i*0.4,{pos="S",dist=0.1})
g:Show()
\end{luadraw}
```

Figure 7: Sequence $u_{n+1} = \cos(u_n)$



The **g:Arg(z)** method calculates and returns the *real* argument of the complex z, that is, its argument (in radians) when exported to the TikZ coordinate system (to do this, you must apply the graph's transformation matrix to z, then convert the coordinate system to that of TikZ). If the graph coordinate system is orthonormal and the transformation matrix is the identity matrix, then the result is identical to that of `cpx.arg(z)` (this is not the case in the example above).

Similarly, the **g:Abs(z)** method calculates and returns the *real* modulus of the complex z, that is, its modulus when exported to the TikZ coordinate system; it is therefore a length in centimeters. If the graph coordinate system is orthonormal with 1 cm per unit on each axis, and if the transformation matrix is an isometry, then the result is identical to that of `cpx.abs(z)`.

Dsquare

- The **g:Dsquare(a, b [, sens, draw_options])** method draws the square with consecutive vertices ⟨*a*⟩ and ⟨*b*⟩ (complex numbers), in the counterclockwise direction when ⟨*sens*⟩ is 1 (the default value). The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction.
- The **ld.square(a, b, sens)** function returns the list of vertices of this square.

Dwedge

The method **g:Dwedge(B, A, C, r [, sens, draw_options])** draws an angular sector with center ⟨*A*⟩ (complex number), radius ⟨*r*⟩, going from ⟨*B*⟩ (complex number) to ⟨*C*⟩ (complex number) counterclockwise if the argument ⟨*sens*⟩ is 1, and counterclockwise otherwise. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the instruction *\draw*.

4) Curves

Parametric: Dparametric

- The function **ld.parametric(p, t1, t2 [, nbdots, discont, nbdiv])** calculates the points of the curve and returns a polygonal line (a list of lists of complex numbers, no drawing).
  - The argument ⟨*p*⟩ is the parameterization; it must be a function of a real variable t and complex-valued variables, for example: `local p = function(t) return cpx.exp(t*cpx.I) end`
  - The arguments ⟨*t1*⟩ and ⟨*t2*⟩ are required with $t_1 < t_2$; they form the bounds of the interval for the parameter.

  - The argument ⟨*nbdots*⟩ is optional; it is the (minimum) number of points to calculate; it is 40 by default.
  - The argument ⟨*discont*⟩ is an optional Boolean that indicates whether there are discontinuities or not; it is `false` by default.
  - The argument ⟨*nbdiv*⟩ is a positive integer that is 5 by default and indicates the number of times the interval between two consecutive values of the parameter can be cut in two (dichotomized) when the corresponding points are too far apart.

- The method **g:Dparametric(p, options)** calculates the points and draws the curve parametrized by ⟨*p*⟩. The parameter ⟨*options*⟩ is a table whose fields are the possible options. These are, with their default values:
  - `t= {g:Xinf(),g:Xsup()}`, this is the interval for the parameter t (by default, it's the entire range of the window's x-coordinates),
  - `nbdots=40`,
  - `discont=false`,
  - `nbdiv=5`,
  - `draw_options=""`, string which will be passed as is to the instruction *\draw*,
  - `clip=nil`, this option is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph's matrix.

Polars: Dpolar

- The function **ld.polar(rho, t1, t2 [, nbdots, discont, nbdiv])** calculates the points of the curve and returns a polygonal line (no drawing). The argument ⟨*rho*⟩ is the polar parameterization of the curve; it must be a function of a real variable t and with real values, for example:

  ```
  local rho = function(t) return 4*math.cos(2*t) end
  ```

  The other arguments are identical to those for parameterized curves.

- The method **g:Dpolar(rho, options)** calculates the points and draws the polar curve parameterized by *rho*. The parameter ⟨*options*⟩ is a table whose fields are the possible options. These are, with their default values:
  - `t= {-pi,pi}`, this is the interval for the parameter t,
  - `nbdots=40`,
  - `discont=false`,
  - `nbdiv=5`,
  - `draw_options=""`, string which will be passed as is to the instruction *\draw*,
  - `clip=nil`, this option is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph's matrix.

Cartesian: Dcartesian

- The function **ld.cartesian(f, x1, x2 [, nbdots, discont, nbdiv])** calculates the points and returns a polygonal line (no drawing). The argument ⟨*f*⟩ is the function whose curve we want to obtain. It must be a function of a real variable x and with real values, for example:

  ```
  local f = function(x) return 1+3*math.sin(x)*x end
  ```

  The arguments ⟨*x1*⟩ and ⟨*x2*⟩ are required and form the bounds of the interval for the variable. The other arguments are identical to those for parametric curves.

- The method **g:Dcartesian(f, options)** calculates the points and draws the curve of f. The argument ⟨*options*⟩ is a table whose fields are the possible options. These are, with their default values:

- `x= {g:Xinf(),g:Xsup()}`, this is the interval for the parameter t (by default, it's the entire range of the window's x-coordinates),
- `nbdots=40`,
- `discont=false`,
- `nbdiv=5`,
- `draw_options=""`, string which will be passed as is to the instruction *\draw*,
- `clip=nil`, this option is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph's matrix.

Periodic Functions: Dperiodic

- The function **ld.periodic(f, period, x1, x2 [, nbdots, discont, nbdiv])** calculates the points of the curve and returns a polygonal line (no drawing).
  - The argument $\langle f\rangle$ is the function whose curve we want; it must be a function of a real variable x and with real values.
  - The argument $\langle period\rangle$ is a table of the type $\{a,b\}$, with $a<b$ representing a period of the function $\langle f\rangle$.
  - The arguments $\langle x1\rangle$ and $\langle x2\rangle$ are required and form the bounds of the interval for the variable.
  - The other arguments are identical to those for parametric curves.
- The method **g:Dperiodic(f, period, options)** calculates the points and draws the curve of $\langle f\rangle$. The argument $\langle options\rangle$ is a table whose fields are the possible options. These are, with their default values:
  - `x= {g:Xinf(),g:Xsup()}`, this is the interval for the parameter t (by default, it's the entire range of the window's x-coordinates),
  - `nbdots=40`,
  - `discont=false`,
  - `nbdiv=5`,
  - `draw_options=""`, string which will be passed as is to the instruction *\draw*,
  - `clip=nil`, this option is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph's matrix.

Step Functions: Dstepfunction

- The **ld.stepfunction(def, discont)** function calculates the points of the curve and returns a polygonal line (no drawing).
  - The $\langle def\rangle$ argument defines the step function; it is a table with two elements:

    ```
    { {x1,x2,x3,...,xn}, {c1,c2,...} }
    ```

    The first element `{x1,x2,x3,…,xn}` must be a subdivision of the segment $[x_1;x_n]$.

    The second element `{c1,c2,…}` is the list of constants, with `c1` for the segment $[x_1;x_2]$, `c2` for the segment $[x_2;x_3]$, etc.
  - The argument $\langle discont\rangle$ is a Boolean that defaults to `true`.
- The method **g:Dstepfunction(def, options)** calculates the points and draws the curve of the step function.
  - The argument $\langle def\rangle$ is the same as the one described above (definition of the step function).
  - The argument $\langle options\rangle$ is a table whose fields are the possible options. These are, with their default values:
    * `discont=true`,

* `draw_options=""`, string which will be passed as is to the instruction *\draw*,
* `clip=nil`, this option is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph's matrix.

Piecewise Affine Functions: Daffinebypiece

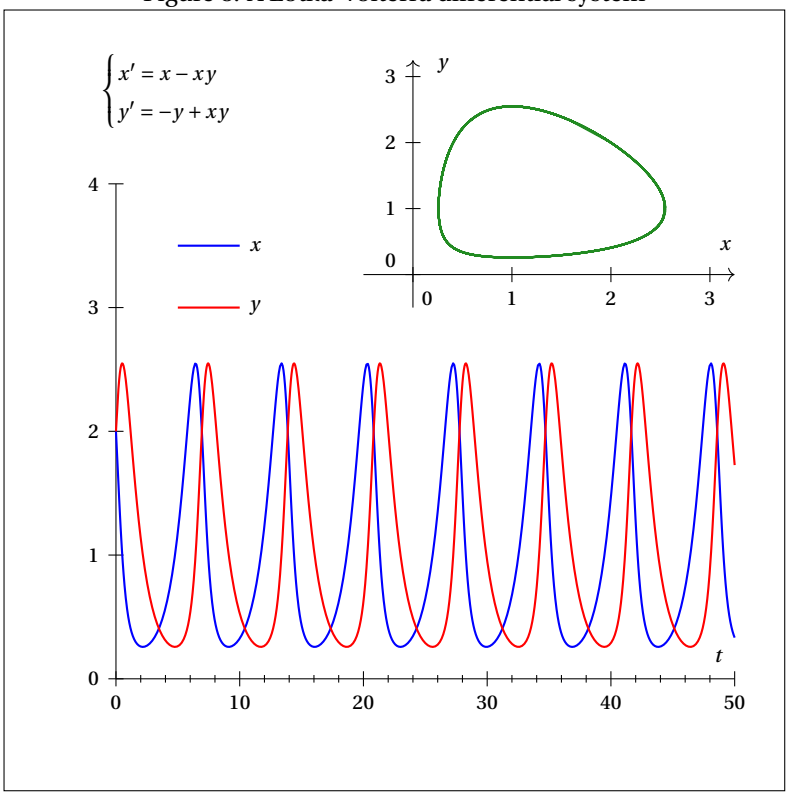
- The function **ld.affinebypiece(def, discont)** calculates the points of the curve and returns a polygonal line (no drawing).
  - The argument ⟨*def*⟩ defines the step function; it is a two-field table:
    ```
    { {x1,x2,x3,...,xn}, { {a1,b1}, {a2,b2},...} }
    ```
    The first element `{x1,x2,x3,…,xn}` must be a subdivision of the segment $[x_1;x_n]$.
    The second element `{ {a1,b1},{a2,b2},…}` means that on $[x_1;x_2]$ the function is $x\mapsto a_1x+b_1$, on $[x_2;x_3]$ the function is $x\mapsto a_2x+b_2$, etc.
  - The argument ⟨*discont*⟩ is a boolean that defaults to `true`.
- The method **g:Daffinebypiece(def, options)** calculates the points and draws the curve of the piecewise affine function.
  - The argument ⟨*def*⟩ is the same as the one described above (definition of the piecewise affine function).
  - The argument ⟨*options*⟩ is a table whose fields are the possible options. These are, with their default values:
    * `discont=true`,
    * `draw_options=""`, string which will be passed as is to the instruction *\draw*,
    * `clip=nil`, this option is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph's matrix.

Differential Equations: Dodesolve

- The function **ld.odesolve(f, t0, Y0, tmin, tmax, nbdots [, method])** allows an approximate solution of the differential equation $Y'(t)=f(t,Y(t))$ in the interval $[t_{\min};t_{\max}]$ which must contain ⟨*t0*⟩, with the initial condition $Y(t_0)=Y_0$.
  - The argument ⟨*f*⟩ is a function $f:(t,Y)->f(t,Y)$ with values in $\mathbf{R}^n$ and where Y is also in $\mathbf{R}^n$: $Y=\{y_1,y_2,\dots,y_n\}$, but when $n=1$, Y is a real number.
  - The arguments ⟨*t0*⟩ and ⟨*Y0*⟩ give the initial conditions with $Y_0=\{y_1(t_0),\dots,y_n(t_0)\}$ (the y_i numbers are real), or $Y_0=y_1(t_0)$ when $n=1$.
  - The arguments ⟨*tmin*⟩ and ⟨*tmax*⟩ define the resolution interval; this must contain ⟨*t0*⟩.
  - The argument ⟨*nbdots*⟩ indicates the number of points calculated on either side of ⟨*t0*⟩.
  - The optional argument *method* is a string that can be `"rkf45"` (default), or `"rk4"`. In the first case, we use the Runge Kutta-Fehlberg method (with variable step size), in the second case, it is the classic Runge-Kutta method of order 4.
  - As output, the function returns the following matrix (list of lists of real numbers):
    ```
    { {tmin,...,tmax}, {y1(tmin),...,y1(tmax)}, {y2(tmin),...,y2(tmax)},...}
    ```
    The first component is the list of values of t (in ascending order), the second is the list of (approximate) values of the component y_1 corresponding to these values of t, etc.
- The method **g:DplotXY(X, Y [, draw_options, clip])**, where the arguments ⟨X⟩ and ⟨Y⟩ are two lists of real numbers of the same length, draws the polygonal line consisting of the points $(X[k],Y[k])$. The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed as is to the *\draw* instruction. The argument ⟨*clip*⟩ is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the graph's matrix.

```
\begin{luadraw}{name=lotka_volterra}
local ld = luadraw
local cpx = ld.cpx
local g = ld.graph:new{window={-5,50,-0.5,5},size={10,10,0}, border=true}
local i = cpx.I
local f = function(t,y) return {y[1]-y[1]*y[2],-y[2]+y[1]*y[2]} end
g:Labelsize("footnotesize")
g:Daxes({0,10,1},{limits={{0,50},{0,4}}, nbsubdiv={4,0}, legendsep={0.1,0},
originpos={"center","center"}, legend={"$t$",""}})
local y0 = {2,2}
local M = ld.odesolve(f,0,y0,0,50,250) -- approximate resolution
-- M est une table à 3 éléments: t, x et y
g:Lineoptions("solid","blue",8)
g:Dseg({5+3.5*i,10+3.5*i}); g:Dlabel("$x$",10+3.5*i,{pos="E"})
g:DplotXY(M[1],M[2]) -- points (t,x(t))
g:Linecolor("red"); g:Dseg({5+3*i,10+3*i}); g:Dlabel("$y$",10+3*i,{pos="E"})
g:DplotXY(M[1],M[3]) -- points (t,y(t))
g:Lineoptions(nil,"black",4)
g:Saveattr(); g:Viewport(20,50,3,5) -- change of view
g:Coordsystem(-0.5,3.25,-0.5,3.25) -- new associated landmark
g:Daxes({0,1,1},{legend={"$x$","$y$"},arrows="->"})
g:Lineoptions(nil,"ForestGreen",8); g:DplotXY(M[2],M[3]) -- points (x(t),y(t))
g:Restoreattr() -- back to the old view
g:Dlabel("$\\begin{cases}x'=x-xy\\\\y'=-y+xy\\end{cases}$", 5+4.75*i,{})
g:Show()
\end{luadraw}
```

Figure 8: A Lotka-Volterra differential system



- The method **g:Dodesolve(f, t0, Y0, options)** allows the drawing of a solution to the equation $Y'(t) = f(t, Y(t))$.
  - The required argument $\langle f \rangle$ is a function $f : (t, Y) -> f(t, Y)$ with values in $\mathbf{R}^n$ and where Y is also in $\mathbf{R}^n$: $Y = \{y_1, y_2, \ldots, y_n\}$, but when $n = 1$, Y is a real number.
  - The arguments $\langle t0 \rangle$ and $\langle Y0 \rangle$ give the initial conditions with $Y_0 = \{y_1(t_0), \ldots, y_n(t_0)\}$ (the y_i are real), or $Y_0 = y_1(t_0)$ when $n = 1$.
  - The argument $\langle options \rangle$ is a table whose fields are the possible options. These are, with their default values:
    * `t={g:Xinf(), g:Xsup()}`, determines the interval for the variable t,

* `out={1,2}`, table of two integers $\{i_1, i_2\}$. If M denotes the matrix returned by the `odesolve` function, the points drawn will have the $M[i_1]$ as abscissas and the $M[i_2]$ as ordinates. By default, we have $i_1 = 1$ and $i_2 = 2$, which corresponds to the y_1 function as a function of t,
* `nbdots=50`, determines the number of points to calculate for the function,
* `method="rkf45"`, determines the method to use; possible values are `"rkf45"` (default value), or `"rk4"`,
* `draw_options=""`, is a string (empty by default) that will be passed as is to the *\draw* instruction,
* `clip=nil`, this option is either `nil` (default value) or a table `{x1,x2,y1,y2}`. In the first case, the line is clipped by the current 2D window **after** its transformation by the graph's 2D matrix. In the second case, the line is clipped by the window $[x_1; x_2] \times [y_1; y_2]$ **before** being transformed by the graph's matrix.

Implicit Curves: Dimplicit

- The function **ld.implicit(f, x1, x2, y1, y2, grid)** calculates and returns a polygonal line constituting the implicit curve with equation $f(x,y) = 0$ in the box $[x_1, x_2] \times [y_1, y_2]$. This box is split according to the argument $\langle grid \rangle$.
  - The argument $\langle f \rangle$ is a function $f : (x,y) -> f(x,y)$ with values in **R**.
  - The arguments $\langle x1 \rangle$, $\langle x2 \rangle$, $\langle y1 \rangle$, $\langle y2 \rangle$ define the plot window, which will be the $[x_1, x_2] \times [y_1, y_2]$ box. We must have $x_1 < x_2$ and $y_1 < y_2$.
  - The argument $\langle grid \rangle$ is a table containing two positive integers: $\{n_1, n_2\}$. The first integer indicates the number of subdivisions following the x-axis, and the second the number of subdivisions following the y-axis.
- The **g:Dimplicit(f, options)** method draws the implicit curve of equations $f(x,y) = 0$.
  - The argument $\langle f \rangle$ is a function $f : (x,y) -> f(x,y)$ with values in **R**.
  - The argument $\langle options \rangle$ is a table whose fields are the possible options. These are, with their default values:
    * `view={g:Xinf(), g:Xsup(), g:Yinf(), g:Ysup()}`, table of the form `{x1,x2,y1,y2}` which determines the drawing area $[x_1, x_2] \times [y_1, y_2]$,
    * `grid={50,50}`, determines the subdivisions,
    * `draw_options=""`, string (empty by default) that will be passed as is to the *\draw* instruction.

Contour Lines: Dcontour

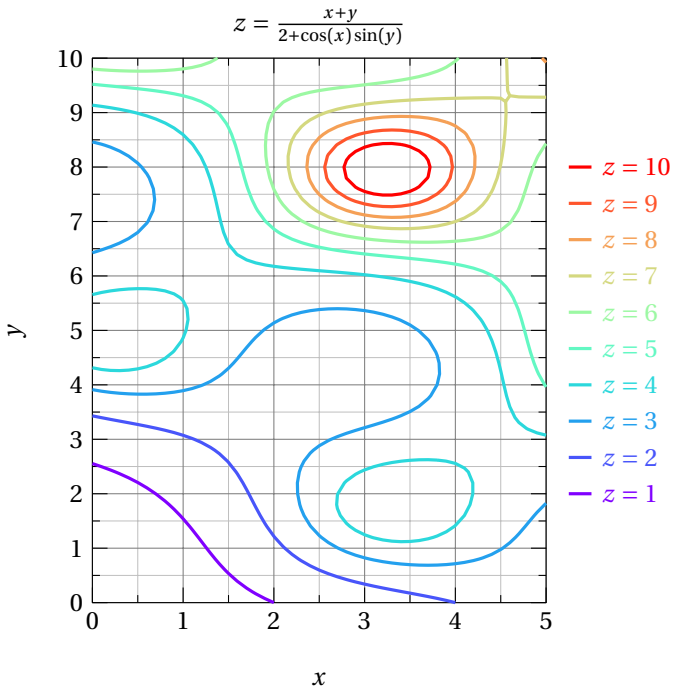
The **g:Dcontour(f, z, options)** method draws **contour lines** of the function $f : (x,y) -> f(x,y)$ with real values.

- The argument $\langle f \rangle$ is the function.
- The argument $\langle z \rangle$ is the list of different levels to plot.
- The argument $\langle options \rangle$ is a table whose fields are the possible options. These are, with their default values:
  - `view={g:Xinf(), g:Xsup(), g:Yinf(), g:Ysup()}`, table of the form `{x1,x2,y1,y2}` which determines the drawing area $[x_1, x_2] \times [y_1, y_2]$,
  - `grid={50,50}`, determines the subdivisions,
  - `colors={}`, list of colors per level. By default, this list is empty and the current drawing color is used.
  - `draw_options=""`, string (empty by default) that will be passed as is to the *\draw* instruction.

```
\begin{luadraw}{name=Dcontour}
local ld = luadraw
local cpx = ld.cpx
local g = ld.graph:new{window={-1,6.5,-1.5,11},size={10,10,0}}
local i, sin, cos = cpx.I, math.sin, math.cos
local f = function(x,y) return (x+y)/(2+cos(x)*sin(y)) end
local Lz = ld.range(1,10) -- levels to be drawn
local Colors = ld.getpalette(ld.palRainbow,10) -- 10 evenly distributed colors in the palRainbow palette
g:Dgradbox({0,5+10*i,1,1},{legend={"$x$","$y$"}, grid=true, title="$z=\\frac{x+y}{2+\\cos(x)\\sin(y)}$"})
```

```
g:Linewidth(12); g:Dcontour(f,Lz,{view={0,5,0,10}, colors=Colors})
for k = 1, 10 do
    local y = (2*k+4)/3*i
    g:Dseg({5.25+y,5.5+y},1,"color="..Colors[k])
    g:Labelcolor(Colors[k])
    g:Dlabel("$z="..k.."$",5.5+y,{pos="E"})
end
g:Show()
\end{luadraw}
```

Figure 9: Example with Dcontour



Parameterization of a Polygonal Line: *curvilinear_param*

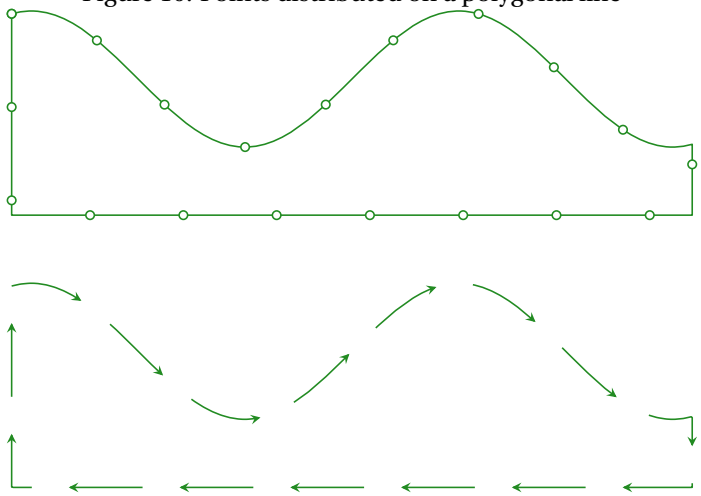
Let L be a list of complex numbers representing a « continuous »line. It is possible to obtain a parameterization of this line based on a parameter t between 0 and 1 (t is the curvilinear abscissa divided by the total length of L).

The function **ld.curvilinear_param(L [, close])** returns a function of one variable $t \in [0;1]$ and values on the line $\langle L \rangle$ (list of complex numbers). The value at $t = 0$ is the first point of $\langle L \rangle$, and the value at $t = 1$ is the last point; this function is followed by a number representing the total length of $\langle L \rangle$. The optional argument $\langle close \rangle$ indicates whether the line $\langle L \rangle$ should be closed (`false` by default).

```
\begin{luadraw}{name=curvilinear_param}
local ld = luadraw
local cpx = ld.cpx
local g = ld.graph:new{bbox=false,size={10,10}}
local i = cpx.I; g:Linewidth(8)
local L = ld.cartesian(math.sin,-5,5)[1]
table.append(L, {5-2*i, -5-2*i})
local f = ld.curvilinear_param(L, true)
local I = ld.map(f,ld.linspace(0,1,20)) -- 20 points distributed over L
g:Shift(4*i)
g:Lineoptions(nil,"ForestGreen",6); g:Dpolyline(L,true)
g:Filloptions("full","white"); g:Ddots(I) -- The first and last points coincide because L is closed
-- Another example of use:
local nb = 16 -- number of arrows
local t = ld.linspace(0,1,3*nb+1)
g:Shift(-4*i)
for k = 0,nb-1 do
g:Dparametric(f,{t={t[3*k+1],t[3*k+3]},nbdots=10,nbdiv=2,draw_options="-stealth"})
```

```
end
g:Show()
\end{luadraw}
```

Figure 10: Points distributed on a polygonal line



5) Domains related to Cartesian curves

Ddomain1

- The function **ld.domain1(f, a, b [, nbdots, discont, nbdiv])** returns a list of complex numbers that represents the contour of the part of the plane bounded by the curve of the function $\langle f \rangle$ on an interval determined by $\langle a \rangle$ and $\langle b \rangle$, the x-axis, and the lines $x = a$, $x = b$.
- The method **g:Ddomain1(f, options)** draws this contour. The argument $\langle options \rangle$ is a table whose fields are the possible options. These are, with their default values:
  - `x= {g:Xinf(),g:Xsup()}`, this is the interval for the parameter t (by default, it's the entire range of the window's x-coordinates),
  - `nbdots=40`,
  - `discont=false`,
  - `nbdiv=5`,
  - `draw_options=""`, string which will be passed as is to the instruction *\draw*.

Ddomain2

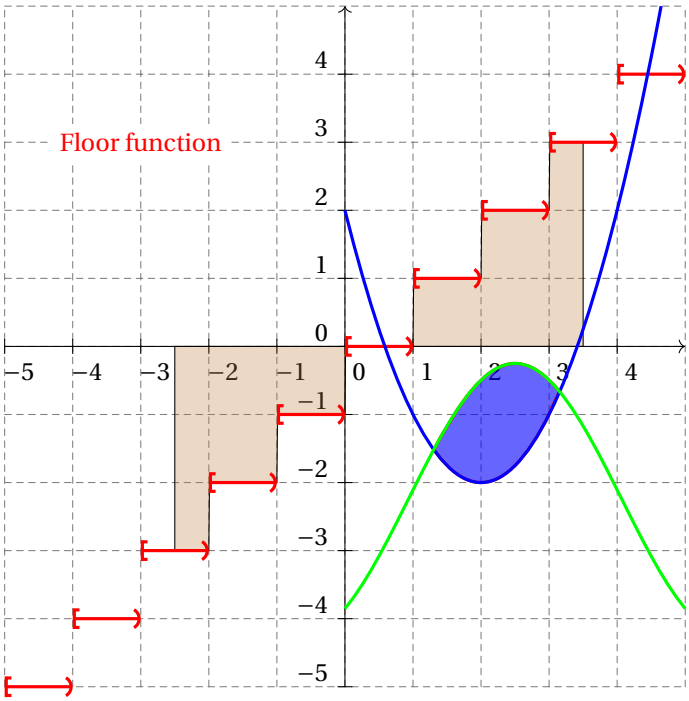
- The **ld.domain2(f, g, a, b [, nbdots, discont, nbdiv])** function returns a list of complex numbers that represents the contour of the part of the plane bounded by the curve of the function $\langle f \rangle$, the curve of the function $\langle g \rangle$, and the lines $x = a$, $x = b$.
- The **g:Ddomain2(f, g, options)** method draws this contour. The argument $\langle options \rangle$ is a table whose fields are the possible options. These are, with their default values:
  - `x= {g:Xinf(),g:Xsup()}`, this is the interval for the parameter t (by default, it's the entire range of the window's x-coordinates),
  - `nbdots=40`,
  - `discont=false`,
  - `nbdiv=5`,
  - `draw_options=""`, string which will be passed as is to the instruction *\draw*.

Ddomain3

- The **ld.domain3(f, g, a, b [, nbdots, discont, nbdiv])** function returns a list of complex numbers that represents the contour of the part of the plane bounded by the curve of the function ⟨*f*⟩ and that of the function ⟨*g*⟩, searching for intersection points in the interval determined by ⟨*a*⟩ and ⟨*b*⟩.
- The **g:Ddomain3(f, g, options)** method draws this contour. The argument ⟨*options*⟩ is a table whose fields are the possible options. These are, with their default values:
  - `x= {g:Xinf(),g:Xsup()}`, this is the interval for the parameter *t* (by default, it's the entire range of the window's x-coordinates),
  - `nbdots=40`,
  - `discont=false`,
  - `nbdiv=5`,
  - `draw_options=""`, string which will be passed as is to the instruction *\draw*.

```
\begin{luadraw}{name=courbe}
local ld = luadraw
local Z = ld.cpx.Z
local g = ld.graph:new{ window={-5,5,-5,5}, bg="", size={10,10} }
local f = function(x) return (x-2)^2-2 end
local h = function(x) return 2*math.cos(x-2.5)-2.25 end
g:Daxes( {0,1,1},{grid=true,gridstyle="dashed", arrows="->"})
g:Filloptions("full","brown",0.3)
g:Ddomain1( math.floor, { x={-2.5,3.5} })
g:Filloptions("none","white",1); g:Lineoptions("solid","red",12)
g:Dstepfunction({ld.range(-5,5), ld.range(-5,4)},{draw_options="arrows={Bracket-Parenthesis}"})
g:Labelcolor("red")
g:Dlabel("Floor function",Z(-3,3),{node_options="fill=white"})
g:Ddomain3(f,h,{draw_options="fill=blue,fill opacity=0.6"})
g:Dcartesian(f, {x={0,5}, draw_options="blue"})
g:Dcartesian(h, {x={0,5}, draw_options="green"})
g:Show()
\end{luadraw}
```

Figure 11: Floor function, Ddomain1 and Ddomain3 functions

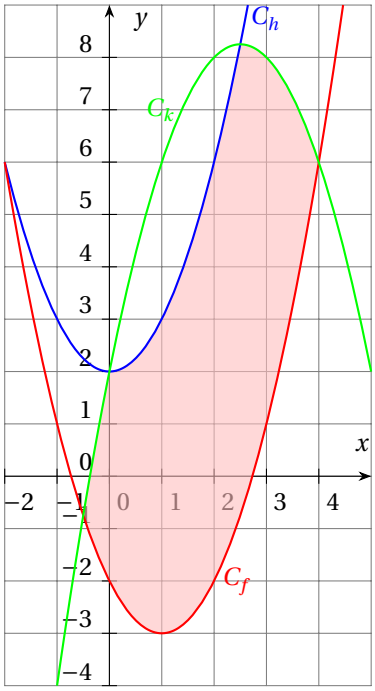


Dinequalities

- The function **ld.inequalities(constraints, x1, x2, y1, y2)** returns a list of complex numbers representing the boundary of the portion of the plane located within the cuboid $[x_1;x_2]\times[y_1;y_2]$ and satisfying the constraints expressed in the argument ⟨*constraints*⟩. This argument is a list of the form `{f1,sg1,f2,sg2,...fn,sgn}` where the ⟨*fi*⟩ are functions ($f_i\colon x\mapsto f_i(x)\in\mathbf{R}$), and the ⟨*sgi*⟩ are either `">"` or `"<"`. The first constraint is given by ⟨*f1*⟩ and ⟨*sg1*⟩, and this constraint is either $y>f_1(x)$, or $y<f_1(x)$ depending on the value of ⟨*sg1*⟩. The same applies to the following constraints.
- The **g:Dinequalities(constraints, options)** method allows you to draw this contour. The argument ⟨*options*⟩ is a table whose fields are the possible options. These are, with their default values:
  - `view={g:Xinf(),g:Xsup(),g:Yinf(),g:Ysup()}`, this is the window in which the resolution will take place. By default, it is the current 2D window.
  - `draw_options=""`, a string that will be passed as is to the *\draw* instruction.
  - `useclip=false`, with the value `false` the method uses the previous function (which calculates the contour of the solution), with the value `true` the method does not calculate the contour but uses clips (one per constraint).

```
\begin{luadraw}{name=Dinequalities}
local ld = luadraw
local Z = ld.cpx.Z
local g = ld.graph:new{window={-2,5,-4,9}, size={10,10}}
g:Daxes({0,1,1}, {grid=true, arrows="-Stealth", legend={"$x$","$y$"}})
local f = function(x) return x*x - 2*x - 2 end
local h = function(x) return x*x + 2 end
local k = function(x) return -x*x+5*x + 2 end
g:Dinequalities({f,'>', h,'<', k,'<'}, {draw_options="draw=none,fill=pink,fill opacity=0.6"})
g:Dcartesian(h, {draw_options="blue, line width=0.8pt"}); g:Dlabel("$C_h$",Z(3,8.75),{node_options="blue"})
g:Dcartesian(f, {draw_options="red, line width=0.8pt"}); g:Dlabel("$C_f$",Z(2,-2),{pos="E",node_options="red"})
g:Dcartesian(k, {draw_options="green, line width=0.8pt"}); g:Dlabel("$C_k$",Z(1,7),{node_options="green"})
g:Show()
\end{luadraw}
```

Figure 12: Dinequalities



Dimplicit_inequalities

- The function **ld.implicit_inequality(f, sg, x1, x2, y1, y2, grid)** returns a **path** representing the boundary of the portion of the plane located within the tile $[x_1;x_2]\times[y_1;y_2]$ and satisfying the condition $f(x,y)>=0$ if ⟨*sg*⟩=">", or $f(x,y)<=0$

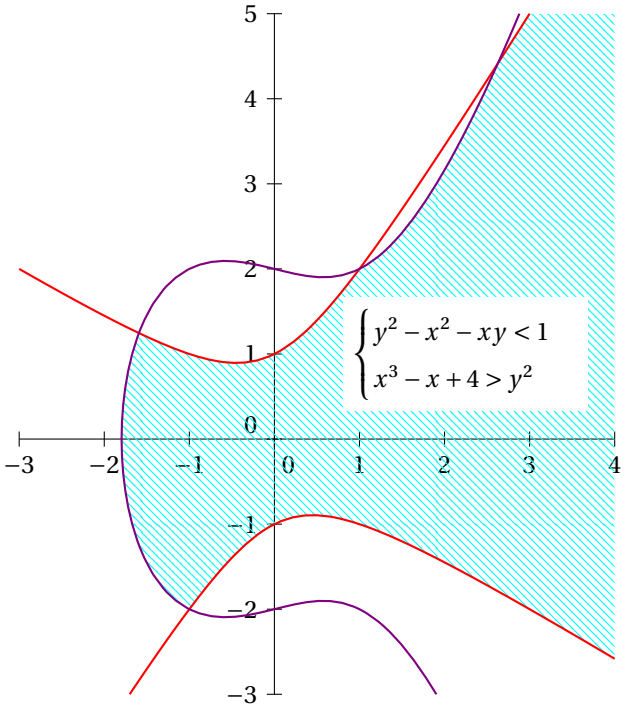
if ⟨*sg*⟩="<". The argument ⟨*f*⟩ is a function of two variables ⟨*f*⟩ : $(x,y) \mapsto f(x,y) \in \mathbf{R}$. The argument ⟨*grid*⟩ is a table of two integers: ⟨*grid*⟩={⟨*n1*⟩,⟨*n2*⟩}, the first indicates the number of subdivisions of the interval $[x_1;x_2]$, and the second, the number of subdivisions of the interval $[y_1;y_2]$.

- The method **g:Dimplicit_inequalities(constraints, options)** allows you to fill in a portion of the design that satisfies the constraints expressed in the argument ⟨*constraints*⟩. This argument is a list of the form `{f1,sg1,f2,sg2,...fn,sgn}` where the ⟨*fi*⟩ are functions ($f_i : (x,y) \mapsto f_i(x,y) \in \mathbf{R}$), and the ⟨*sgi*⟩ are either `">"` or `"<"`. The first constraint is given by ⟨*f1*⟩ and ⟨*sg1*⟩, and this constraint is either $f_1(x,y) > 0$ or $f_1(x,y) < 0$ depending on the value of ⟨*sg1*⟩. The argument ⟨*options*⟩ is a table whose fields are the possible options. These are, with their default values:
  - `view={g:Xinf(),g:Xsup(),g:Yinf(),g:Ysup()}`, this is the window in which the resolution will take place. By default, it is the current 2D window.
  - `draw_options=""`, a string that will be passed as is to the *\draw* instruction.
  - `grid={50,50}`: a table of two integers, the first indicating the number of subdivisions of the interval of x, and the second, the number of subdivisions of the interval of y.

NB: This method does not calculate the contour but uses clipping (one per constraint).

```
\begin{luadraw}{name=implicit_inequalities}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z
local g = ld.graph:new{ window = {-3, 4, -3, 5}, size = {10, 10} }
local f1 = function(x,y) return -x*y+y^2-x^2-1  end
local f2 = function(x,y) return x^3-x-y^2+4 end
local filloptions = "draw=none,pattern= north west lines, pattern color=cyan"
g:Daxes()
g:Dimplicit_inequalities( {f1,'<',f2,">"}, {view={-2,5,-5,5}, draw_options=filloptions})
g:Dimplicit(f1, {draw_options="red,thick"})
g:Dimplicit(f2, {view={-2,5,-5,5}, draw_options="violet,thick"})
local eq = \luastring0{$\begin{cases}y^2-x^2-xy < 1\\ x^3-x+4 > y^2\end{cases}$}
g:Dlabel( eq, Z(2.25,1), {node_options="fill=white"})
g:Show()
\end{luadraw}
```

Figure 13: The **g:Dimplicit_inequalities** method



6) Points (Ddots) and Labels (Dlabel)

- The method for drawing one or more points is: **g:Ddots(dots [, mark_options])**.
  - The argument ⟨*dots*⟩ can be either a single point (i.e., a complex number), a list (a table) of complex numbers, or a list of lists of complex numbers. The points are drawn in the current color of the line plot.
  - The ⟨*mark_options*⟩ argument is an string (empty by default) that will be passed as is to the *\draw* instruction (local modifications), for example:

```
mark_options = "color=green, line width=1.2, scale=0.25"
```

  - Two methods to globally modify the appearance of points:
    - The **g:Dotstyle(style)** method defines the point style. The ⟨*style*⟩ argument is a string that defaults to `"*"`. The possible styles are those of the *plotmarks* library.
    - The **g:Dotscale(scale)** method allows you to adjust the dot size. The ⟨*scale*⟩ argument is a positive integer that defaults to 1. It is used to multiply the default dot size. The current line width also affects the dot size. For "solid" dot styles (e.g., the `"triangle*"` style), the current fill style and color are used by the library.

- The method for placing a label is:

  g:Dlabel(text1, anchor1, options1, text2, anchor2, options2, ...)

  - The arguments ⟨*text1*⟩, ⟨*text2*⟩, ..., are strings; they are the labels.
  - The arguments ⟨*anchor1*⟩, ⟨*anchor2*⟩, ..., are complex numbers representing the anchor points of the labels.
  - The arguments ⟨*options1*⟩, ⟨*options2*⟩, ..., allow you to locally define the label parameters; they are tables whose fields are the possible options. These are, with their default values:
    - `pos="center"`, indicates the position of the label relative to the anchor point. `"center"` (centered), `"N"` (north), `"NE"` (northeast), `"E"` (east), `"SE"` (southeast), `"S"` (south), `"SW"` (southwest), `"W"` (west), `"NW"` (northwest). By default, it is set to `"center"`, and in this case the label is centered on the anchor point,
    - `dist=0`, distance in cm between the label and its anchor point when `pos` is not equal to `"center"`,
    - `dir= {}`, this is a table of the form `{dirX [,dirY,dep]}` which indicates the writing direction. The three values `dirX`, `dirY`, and `dep` are three complex numbers representing three vectors. The first two indicate the writing direction, and the third indicates a translation of the label relative to the anchor point. The vector `dep` is zero by default, and the vector `dirY`, if absent, is equal to the vector `dirX` rotated 90 degrees in the clockwise direction. By default, the `dir` option is equal to the current value of the writing direction,
    - `node_options=""`, string (empty by default) intended to receive options that will be passed directly to TikZ in the *node[]* instruction.
    - The labels are drawn in the current color of the document text, but the color can be changed with the `node_options` argument, for example, by setting: `node_options="color=blue"`.
      Warning: The options chosen for a label also apply to subsequent labels if they are unchanged.

  Global options for labels:

  - The **g:Labelstyle(position)** method allows you to specify the position of the labels relative to the anchor points. The ⟨*position*⟩ argument is a string that can be: `"N"` for north, `"NE"` for northeast, `"NW"` for northwest, or `"S"`, `"SE"`, `"SW"`. By default, it is set to `"center"`, and in this case the label is centered on the anchor point.
  - The **g:Labelcolor(color)** method allows you to set the color of the labels. The ⟨*color*⟩ argument is a string representing a color for TikZ. By default, the argument is an empty string, which represents the current color of the document.
  - The **g:Labelangle(angle)** method allows you to specify an ⟨*angle*⟩ (in degrees) for rotating the labels around the anchor point. This angle is zero by default.
  - The **g:Labelsize(size)** method allows you to manage the size of the labels. The ⟨*size*⟩ argument is a string that can be: `"tiny"`, or `"scriptsize"`, or `"footnotesize"`, etc. By default, the argument is an empty string, which represents the `"normalsize"` size.

  - The **g:Labeldir(dir)** method allows you to manage the writing direction. The argument ⟨*dir*⟩ is an table of the form `{dirX [,dirY,dep]}` which indicates the writing direction. The three values `dirX`, `dirY`, and `dep` are three complex numbers representing three vectors. The first two indicate the writing direction, and the third indicates a translation of the label relative to the anchor point. The vector `dep` is zero by default, and the vector `dirY`, if absent, is equal to the vector `dirX` rotated 90 degrees in the clockwise direction. When ⟨*dir*⟩ is an empty list, this represents the usual writing direction.

- The **g:Dlabeldot(text, anchor, options)** method allows you to place a label and draw the anchor point at the same time.

  - The ⟨*text*⟩ argument is a string; it is the label.
  - The ⟨*anchor*⟩ argument is a complex representing the label's anchor point.
  - The argument ⟨*options*⟩ is a table whose fields are the possible options. These are identical to those of the **Dlabel** method, plus the `mark_options=""` option, which is a string that will be passed as is to the *\draw* instruction when drawing the anchor point.

7) Paths: Dpath, Dspline, and Dtcurve

What is a path

A path is a table of complex numbers and instructions (in the form of strings). This table represents a succession of different "pieces", each piece being a sequence of data (2D points and sometimes numeric values) and ending with a string of characters that represents an instruction. The path is governed by the following rule:

the last point of one piece is the first point of the next piece (it is therefore not repeated)

Example:

```
local Z = cpx.Z
local L = { Z(-3,2),"m",-3,-2,"l", 0,2,2,-1,"ca", 3,Z(3,3),0.5,"la",1,Z(-1,5),Z(-3,2),"b" }
```

The path *L* is composed of five pieces, which are:

1. `{Z(-3,2),"m"}`: there is a data element and the instruction `"m"` which means *moveto*. This instruction does not actually draw the path, but it allows a new component to begin, starting at the first point $Z(-3,2)$ (the last point of the previous piece, if there is one, is not taken into account by this instruction; this is the only exception).

2. `{Z(-3,2),-3,-2,"l"}`: the first point of the second piece is indeed $Z(-3,2)$ and not -3, because $Z(-3,2)$ is the last point of the previous piece. Therefore, there are three data points, and the instruction `"l"` which means *lineto*, it is like executing the instruction `g:Dpolyline(Z(-3,2),-3,-2)`, so these three points are connected by a segment. The last point of this piece is -2.

3. `{-2,0,2,2,-1,"ca"}`: the first point of the third piece is indeed -2 and not 0, because -2 is the last point of the previous piece. There are five data points, and the instruction "ca" stands for *circle arc*. It's as if we were executing the instruction `g:Darc(-2,0,2,2,-1)`, so the center is 0, the arc goes from -2 to 2 with a radius equal to 2 and in a clockwise direction (last value -1). The last point of this piece is 2.

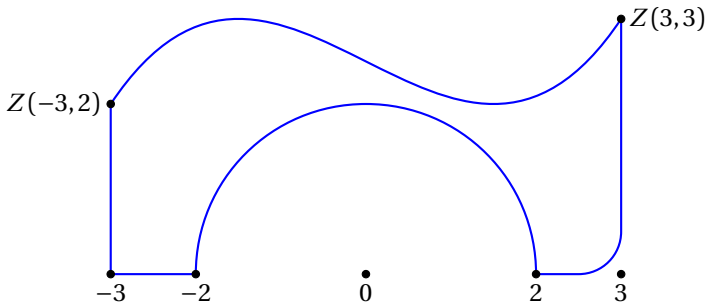
4. `{2,3,Z(3,3),0.5,"la"}`: the first point of the fourth piece is 2 (not 3). There are four data points, and the instruction `"la"` stands for *line arc*. This is a polygonal line with rounded corners and a circular arc of radius 0.5 (the value preceding the instruction). The points of this line are `{2,3,Z(3,3)}`, so there will be a rounding to 3. The last point of this piece is $Z(3,3)$.

5. `{Z(3,3),1,Z(-1,5),Z(-3,2),"b"}`: the first point of the fifth piece is $Z(3,3)$ (not 1). The instruction `"b"` stands for *bezier*, so we draw a Bézier curve from $Z(3,3)$ to $Z(-3,2)$. The other two points, 1 and $Z(-1,5)$, are the first and second control points of the curve.

Here is what this path gives:

```
\begin{luadraw}{name=path_example}
local ld = luadraw
local Z = ld.cpx.Z
local g = ld.graph:new{window={-4,4,-0.5,3}, size={10,10}}
local L = { Z(-3,2),"m",-3,-2,"l",0,2,2,-1,"ca",3,Z(3,3),0.5,"la",1,Z(-1,5),Z(-3,2),"b" }
g:Dpath(L, "line width=0.8pt, blue")
g:Ddots({Z(-3,2),-3,-2,0,2,3,Z(3,3)})
g:Dlabel("$Z(-3,2)$",Z(-3,2),{pos="W"}, "$-3$",-3,{pos="S"}, "$-2$",-2,{},
"$0$",0,{}, "$2$",2,{}, "$3$",3,{}, "$Z(3,3)$",Z(3,3),{pos="E"})
g:Show()
\end{luadraw}
```

Figure 14: Path example



Note: In the example above, you can replace the part: `Z(-3,2),"m",-3,-2,"l"`, with: `Z(-3,2),-3,-2,"l"` because there is no other part before the *moveto*.
Available commands and their syntax, the word *last* denotes the last point of the previous piece:

- `z1,"m"` (moveto) starts a new component of the path at the point with affix z_1.
- `z1,...,zn,"l"` (lineto) draws the polyline `{last,z1,...,zn}`.
- `c1,c2,z2,"b"` (bézier) draws the Bézier curve `{last,c1,c2,z2}`, where c_1 and c_2 are the two control points.
- `z1,...,zn,"s"` (spline) draws the natural cubic spline through the points `{last,z1,...,zn}`.
- `z1,"c"` (circle) draws the circle with center z_1 passing through the point *last*. There is another possible syntax for the circle: `z1,z2,"c"`, in which case the circle passing through the points *last*, z_1, and z_2 is drawn.
- `z1,z2,r,sens,"ca"` (circular arc) draws a circular arc with center z_1 and radius r, going from *last* to z_2, counterclockwise when `sens=1` (and therefore clockwise when `sens=-1`).
- `z1,z2,rx,ry,sens,inclinaison,"ea"` (elliptic arc) draws an elliptic arc with center z_1 going from *last* to z_2; rx and ry are the two radii along the two axes of the ellipse; `inclinaison` is the angle in degrees between the first axis of the ellipse (the one corresponding to rx) and the horizontal. The parameter `inclinaison` is optional and defaults to 0.
- `z1,rx,ry,inclinaison,"e"` (ellipse) draws the ellipse with center z_1 passing through *last*; rx and ry are the two radii along the two axes of the ellipse; `inclinaison` is the angle in degrees between the first axis of the ellipse (the one corresponding to rx) and the horizontal. The parameter `inclinaison` is optional and defaults to 0. When the point *last* is not on this ellipse, a segment is drawn between this point and a point on the ellipse.
- `z1,...,zn,r,"la"` (line arc) draws the polyline `{last,z1,...,zn}` while replacing each "corner" by a circular arc of radius r.
- `z1,...,zn,r,"cla"` (closed line arc) same as the previous command, except that the polyline is closed.
- `"cl"` (closepath) this command is used alone; it closes the current component by drawing a segment joining the last point to the first point of the current component.

Functions on paths

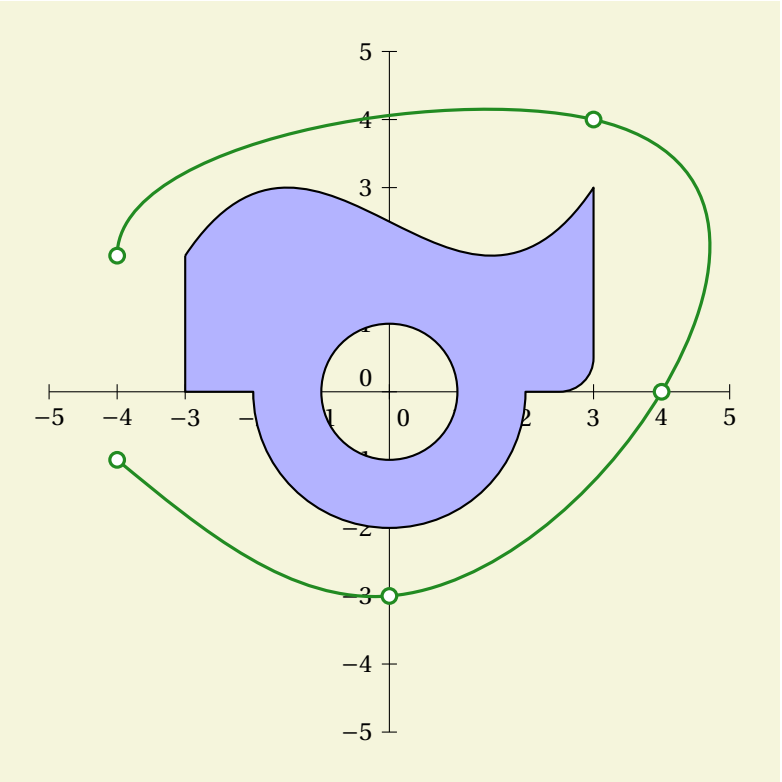
- The function **ld.path(path [, nbdots])** returns a polygonal line containing the points that make up the ⟨*path*⟩. The optional argument ⟨*nbdots*⟩ is the minimum number of points calculated for each Bézier curve within the path; it defaults to the global variable `ld.bezier_nbdots`, which is initialized to 12.
- Conversely, the function **ld.polyline2path(L [, close])**—where ⟨*L*⟩ is a polygonal line (a list of complex numbers or a list of lists of complex numbers)—returns ⟨*L*⟩ in the form of a path. The optional argument ⟨*close*⟩ allows each component of the path to be closed or not (`false` by default).
- The function **ld.convpath(path)** converts the ⟨*path*⟩ into another path using only the "m", "l", and "b" instructions (all other instructions are converted into Bézier curves). The function returns the new path; the advantage of this new path is that a transformation matrix can be applied to it (using the **ld.mtransform()** function).
- The function **ld.parabola(A, B, C)** returns a path that is a Bézier curve representing a parabolic arc from point ⟨*A*⟩ (complex number) to point ⟨*C*⟩ (complex number), passing through point ⟨*B*⟩ (complex number).
- The function **ld.splitpath(P,I)**, where ⟨*P*⟩ is a path and ⟨*I*⟩ is a point belonging to that path, returns a sequence of two paths: the first corresponds to the part of ⟨*P*⟩ preceding point *I*, and the second is the part following point *I*.

Draw a Path

- The **g:Dpath(path [, draw_options])** method draws the ⟨*path*⟩ (which was described above) using Bézier curves as much as possible, including arcs, ellipses, etc. The ⟨*draw_options*⟩ argument is a string that will be passed directly to the *\draw* instruction.
- The function **ld.spline(points, v1, v2)** returns a path (to be drawn with **Dpath**) of the cubic spline passing through the points of the argument ⟨*points*⟩ (which must be a list of complex numbers). The arguments ⟨*v1*⟩ and ⟨*v2*⟩ are tangent vectors imposed at the ends (constraints); when these are equal to `nil` (default value), a natural cubic spline (i.e., unconstrained) is calculated.
- The method **g:Dspline(points [, v1, v2, draw_options])** draws the spline described above. The argument ⟨*draw_options*⟩ is a string that will be passed directly to the instruction *\draw*.

```
\begin{luadraw}{name=path_spline}
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5},size={10,10},bg="Beige"}
local i = ld.cpx.I
local p = {-3+2*i,"m",-3,-2,"l",0,2,2,1,"ca",3,3+3*i,0.5,"la",1,-1+5*i,-3+2*i,"b",-1,"m",0,"c"}
g:Daxes( {0,1,1} )
g:Filloptions("full","blue!30",1,true); g:Dpath(p,"line width=0.8pt")
g:Filloptions("none")
local A,B,C,D,E = -4-i,-3*i,4,3+4*i,-4+2*i
g:Lineoptions(nil,"ForestGreen",12); g:Dspline({A,B,C,D,E},nil,-5*i) -- constraint in E
g:Ddots({A,B,C,D,E},"fill=white,scale=1.25")
g:Show()
\end{luadraw}
```

Figure 15: Path and Spline



- The function **ld.tcurve(L)** returns a curve passing through given points as a path, with tangent vectors (left and right) imposed at each point. The argument ⟨*L*⟩ is a table of the form:

```
L = {point1,{t1,a1,t2,a2}, point2,{t1,a1,t2,a2}, ..., pointN,{t1,a1,t2,a2}}
```

  ⟨*point1*⟩, ..., ⟨*pointN*⟩ are the interpolation points of the curve (complex numbers), and each of them is followed by a table of the form `{t1,a1,t2,a2}` which specifies the tangent vectors to the curve to the left of the point (with ⟨*t1*⟩ and ⟨*a1*⟩) and to the right of the point (with ⟨*t2*⟩ and ⟨*a2*⟩). The left tangent vector is given by the formula $V_g = t_1 \times e^{ia_1\pi/180}$, so ⟨*t1*⟩ represents the modulus and ⟨*a1*⟩ is an argument **in degrees** of this vector. The same is true for ⟨*t2*⟩ and ⟨*a2*⟩ for the right tangent vector, **but these are optional**, and if not specified, they take the same values as ⟨*t1*⟩ and ⟨*a1*⟩.

  Two consecutive points will be connected by a Bézier curve; the function calculates the control points to obtain the desired tangent vectors.

- The method **g:Dtcurve(L, options)** draws the path obtained by ⟨*tcurve*⟩ described above. The argument *options* is a table whose fields are the possible options. These are, with their default values:

  – `showdots=false`, this option allows you to draw or not the given interpolation points as well as the calculated control points, allowing for visualization of the constraints.

  – `draw_options=""`, this is a string that will be passed directly to the *\draw* instruction.

- A path consisting solely of Bézier curves can be drawn using the method:

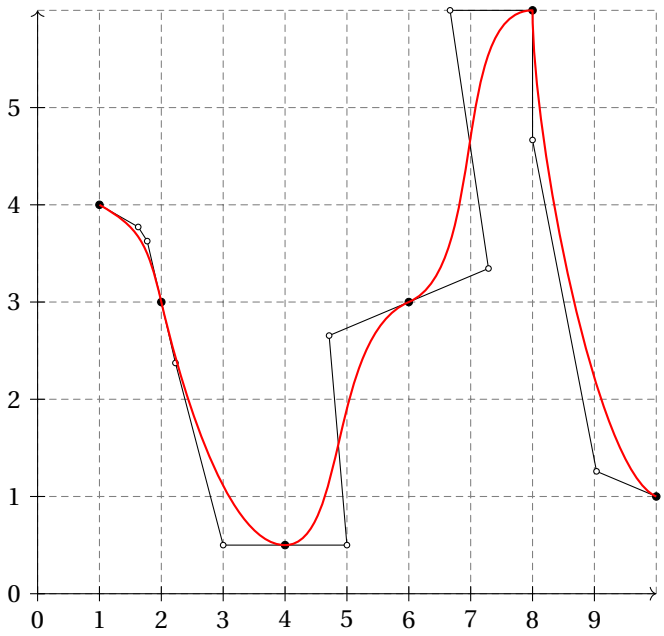
  g:Dbezier(L [, draw_options])

  where ⟨*L*⟩ is a list of the form $\{A_1, c_1, c_2, A_2, c_3, c_4, A_3, \ldots\}$, the complex numbers A_1, A_2, ..., are the points on the curve, and the complex numbers c_1, c_2, c_3, c_4, ..., are the control points.

```
\begin{luadraw}{name=tcurve}
local ld = luadraw
local g = ld.graph:new{window={-0.5,10.5,-0.5,6.5},size={10,10,0}}
local i = ld.cpx.I
local L = {
1+4*i,{2,-20},
2+3*i,{2,-70},
4+i/2,{3,0},
6+3*i,{4,15},
```

```
8+6*i,{4,0,4,-90}, -- angular point
10+i,{3,-15}}
g:Dgrid({0,10+6*i},{gridstyle="dashed"})
g:Daxes(nil,{limits={{0,10},{0,6}},originpos={"center","center"}, arrows="->"})
g:Dtcurve(L,{showdots=true,draw_options="line width=0.8pt,red"})
g:Show()
\end{luadraw}
```

Figure 16: Interpolation curve with imposed tangent vectors

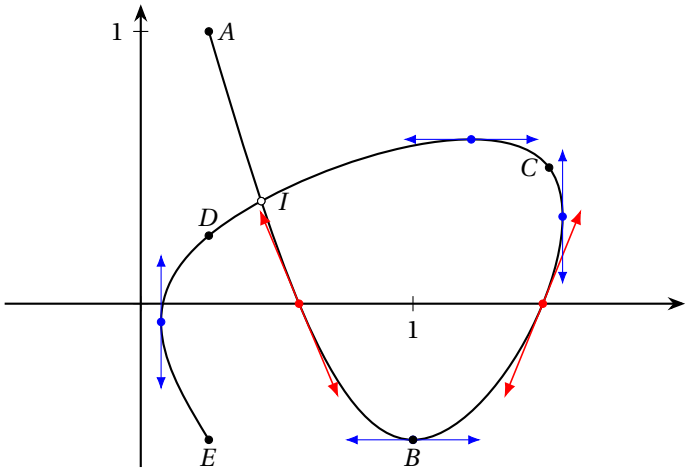


8) Parametrizing a path

The function **ld.param_path(P)**, where $\langle P\rangle$ is a table representing a path, returns a function $p:\ t\mapsto p(t)\in\mathbb{C}$ with $t\in[0;1]$; this function is a parametrization of the path $\langle P\rangle$.

```
\begin{luadraw}{name=param_path}
local ld = luadraw
local Z = ld.cpx.Z
local g = ld.graph:new{ window={-0.5,2,-0.6,1.1}, size={10,10}}
local A, B, C, D, E = Z(0.25,1), Z(1,-0.5), Z(1.5,0.5), Z(0.25,0.25), Z(0.25,-0.5)
P = ld.spline({A,B,C,D,E})
local p = ld.param_path(P)
local T1, T2 = table.unpack(ld.solve(function(t) return p(t).im end, 0, 2))
local L = ld.parametric(p,0,1)
local I = ld.interL(L,L) -- self intersection
g:Daxes({0,1,1},{arrows="thick,-Stealth",originpos={"none","none"}})
g:Dpath(P,"thick")
g:Dtangent(p, T1, 0.75, "red,semithick,Latex-Latex")
g:Dtangent(p, T2, 0.75, "red,semithick,Latex-Latex")
g:Ddots({p(T1), p(T2)}, "red")
local dots = {}
g:Dparallel_tangent(p,Z(1,0),{t={0,1}, len=0.5, draw_options="blue,Latex-Latex", out=dots})
g:Dparallel_tangent(p,Z(0,1),{t={0,1}, len=0.5, draw_options="blue,Latex-Latex", out=dots})
for _,t in ipairs(dots) do g:Ddots(p(t),"blue") end
g:Ddots(I,"circle,fill=white")
g:Ddots({A,B,C,D,E})
g:Dlabel("$A$",A, {pos="E"},"$B$",B, {pos="S"},"$C$",C, {pos="W"},"$D$",D, {pos="N"},
"$E$",E, {pos="S"}, "$I$", I[1],{pos="E",dist=0.1} )
g:Show()
\end{luadraw}
```

Figure 17: Parametrization of a spline



9) Paths and Clipping: Beginclip() and Endclip()

A path can be used for clipping via two methods:

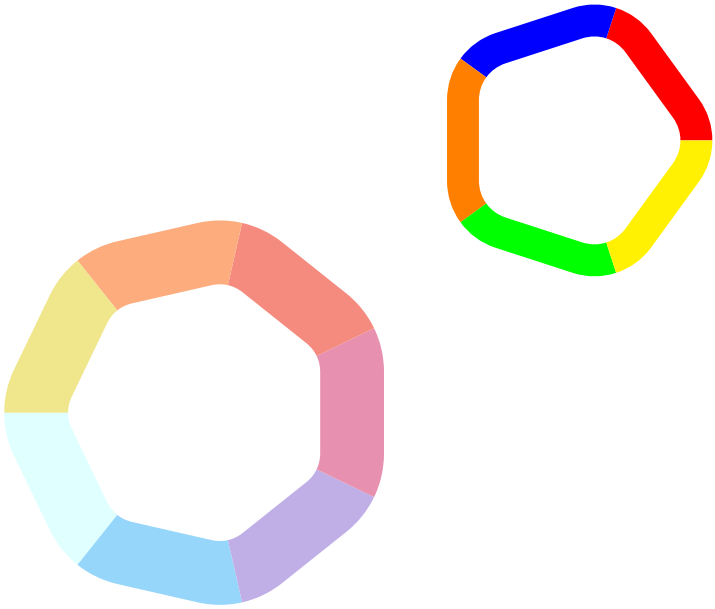
g:Beginclip(2D or 3D path [, inverse]) and **g:Endclip([draw_path, draw_options])**.

The first opens a *scope* group and passes the ⟨*path*⟩ as an argument to the TikZ *\clip* function. The ⟨*inverse*⟩ argument is a boolean that defaults to `false`; when set to `true`, the clipping is inverted—meaning only the area outside the ⟨*path*⟩ is drawn—though this requires the path to be defined in a counter-clockwise direction.

The second method closes the *scope* group; it is essential (otherwise, a compilation error will occur). The optional argument ⟨*draw_path*⟩ is a boolean (defaulting to `false`); when set to `true`, the ⟨*path*⟩ is drawn after the group closes, and in this case, the optional argument ⟨*draw_options*⟩, which must be a character string (empty by default), is passed to the *\draw* command.

```
\begin{luadraw}{name=polygon_with_different_line_color_and_rounded_corners}
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5},size={10,10}}
local i = ld.cpx.I
local Dcolored_polyreg = function(c,a,nb,r,wd,colors)
-- c=center, a=vertex, nb=number of sides, r=radius, wd=width in point, colors=list of colors
    local L = ld.polyreg(c,a,nb)
    table.append(L,{r,"cla"}) --polygon with rounded corners (radius=r)
    local angle = 360/nb
    local b = a
    for k = 1, nb do
        a = b; b = ld.rotate(a,angle,c)
        g:Beginclip({2*a-c,c,2*b-c,"l"}) -- definition of an angular sector for clipping
        g:Dpath(L,"line width="..wd.."pt,color="..colors[k])
        g:Endclip()
    end
end
Dcolored_polyreg(3+2*i,5+2*i,5,0.8,12,{"red","blue","orange","green","yellow"}) -- pentagon
Dcolored_polyreg(-2.5-2*i,-5-2*i,7,1,24,ld.getpalette(ld.palGasFlame,7)) -- heptagon
g:Show()
\end{luadraw}
```

Figure 18: Clipping Example



10) Axes and Grids

Global variables used for axes and grids:

- `ld.maxGrad = 100`: Maximum number of tick marks on an axis.
- `ld.defaultlabelshift = 0.1875`: When a grid is drawn with the axes (option `grid=true`), the labels are automatically shifted along the axis using this variable.
- `ld.defaultxylabelsep = 0`: Sets the default distance between labels and tick marks.
- `ld.defaultlegendsep = 0.2`: Sets the default distance between the legend and the axis.
- `ld.digits = 4`: Default number of decimal places in string conversions; terminal 0s are removed.
- `ld.dollar = true`: to add dollars around the tick labels.
- `ld.siunitx = false`: with the value `true`, the labels are formatted with the macro `\num{..}` of the *siunitx* package, which allows you to use certain options of this package, such as replacing the decimal point with a comma by doing:

```
\usepackage[local=FR]{siunitx}
```

or by doing:

```
\usepackage{siunitx}
\sisetup{output-decimal-marker={,}}
```

For axes, in both 2D and 3D, all labels are formatted as strings with the **ld.num(x)** function. This transforms the number *x* into a string *str* with the number of decimal places set by the global variable `ld.digits`. When the `ld.siunitx` variable has the value `true`, the function returns `"\num{str}"`, otherwise it simply returns *str*. This also applies to 3D axes. Here is the code for this function:

```
function ld.num(x) -- x is a real, returns a string
    local rep = ld.strReal(x) -- conversion to string with digits decimals max
    if ld.siunitx then rep = "\\num{"..rep.."}" end --needs \usepackage{siunitx}
    return rep
end
```

Daxes

The axes are plotted using the method **g:Daxes([{A, xstep, ystep}, options])**.

- The first argument specifies the intersection point of the two axes (this is the complex ⟨*A*⟩), the graduation spacing on the *x*-axis (this is ⟨*xstep*⟩), and the graduation spacing on *y*-axis (this is ⟨*ystep*⟩). By default, the point ⟨*A*⟩ is the origin *Z*(0,0), and both steps are equal to 1.
- The argument ⟨*options*⟩ is a table whose fields are the possible options. Here are these options with their default values:
  - `showaxe=1,1`. This option specifies whether or not the axes should be plotted (1 or 0). The first value is for the *x*-axis and the second for the *y*-axis.
  - `arrows="-"`. This option allows you to add an arrow to the axes (no arrow by default; enter `"->"`for example to add an arrow).
  - `limits="auto","auto"`. This option specifies the extent of the two axes (first value for *x*-axis, second value for *y*-axis). The value `"auto"` means that it is the entire line, but you can specify the extreme abscissas, for example: `limits=-4,4,"auto"`.
  - `gradlimits={"auto","auto"}`. This option allows you to specify the range of the graduations on both axes (first value for *x*-axis, second value for *y*-axis). The value `"auto"` means that it is the entire line, but you can specify the extreme graduations, for example: `gradlimits={{-4.4},{-2.3}}`.
  - `unit={"",""}`. This option allows you to specify the range of the graduations on the axes. The default value means that the step value should be taken (⟨*xstep*⟩ on *x*-axis, or ⟨*ystep*⟩ on *y*-axis), EXCEPT when the option `labeltext` is not the empty string, in which case `unit` takes the value 1.
  - `nbsubdiv={0,0}`. This option specifies the number of subdivisions between two main ticks on the axis.
  - `tickpos={0.5,0.5}`. This option specifies the position of the ticks relative to each axis. These are two numbers between 0 and 1. The default value of 0.5 means they are centered on the axis. (0 and 1 represent the ends).
  - `tickdir={"auto","auto"}`. This option specifies the direction of the ticks on the axis. This direction is a non-zero vector (complex number). The default value `"auto"` means the ticks are orthogonal to the axis.
  - `xyticks={0.2,0.2}`. This option specifies the length of the ticks on the axis.
  - `xylabelsep={0,0}`. This option specifies the distance between the labels and the tick marks on the axis.
  - `originpos={"right","top"}`. This option specifies the position of the label at the origin on the axis. Possible values are: `"none"`, `"center"`, `"left"`, `"right"` for *x*-axis, and `"none"`, `"center"`, `"bottom"`, `"top"` for *y*-axis.
  - `originnum={A.re,A.im}`. This option specifies the value at the origin point of the graduations (graduation number 0).

    The formula that defines the label at tick mark number *n* is:

    ```
    (originnum + unit*n)"labeltext"/labelden.
    ```
  - `originloc=A`. This option specifies the origin point of the graduations.
  - `legend={"",""}`. This option allows you to specify a legend for the axis.
  - `legendpos={0.975,0.975}`. This option specifies the position (between 0 and 1) of the legend relative to each axis.
  - `legendsep={ld.defaultlegendsep,ld.defaultlegendsep}`. This option specifies the distance between the legend and the axis. The legend is on the other side of the axis from the graduations.
  - `legendangle={"auto","auto"}`. This option specifies the angle (in degrees) that the legend should make for the axis. The default value `"auto"` means that the legend must be parallel to the axis if the `labelstyle` option is also set to `"auto"`, otherwise the legend is horizontal.
  - `legendstyle={"auto","auto"}`. Specifies the label style for the legends (position relative to the anchor point); with the value `"auto"`, it is determined automatically; otherwise, the following values can be used: `"center"`, `"N"`, `"NW"`, `"W"`, `"SW"`, `"S"`, `"SE"`, `"E"`, `"NE"`.

- labelpos={"bottom","left"}. This option specifies the position of the labels relative to the axis. For the x-axis, the possible values are: "none", "bottom" or "top", for the y-axis it is: "none", "right" or "left".
- labelden={1,1}. This option specifies the denominator of the labels (integer) for the axis. Remember that the formula that defines the label at graduation number n is:

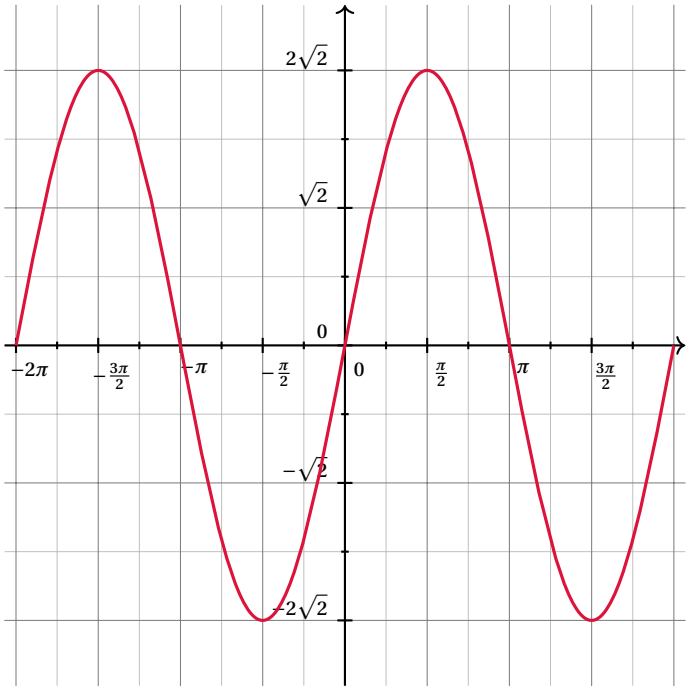
  (originnum + unit*n)"labeltext"/labelden.

- labeltext={"",""}. This option defines the text that will be added to the numerator of the labels for the axis.
- labelstyle={"S","W"}. This option sets the label style for each axis. Possible values are "auto", "N", "NW", "W", "SW", "S", "SE", "E", "NE".
- labelangle={0,0}. This option sets the angle of the labels in degrees from the horizontal for each axis.
- labelcolor={"",""}. This option allows you to choose a color for the labels on each axis. The empty string represents the default color.
- labelshift={0,0}. This option allows you to define a systematic offset for the labels on the axis (offset along the axis). With the option grid=false the default values are 0 and 0, but with the option grid=true the default is labelshift={ld.defaultlabelshift,ld.defaultlabelshift} where ld.defaultlabelshift is a global variable initialized to 0.1875.
- xynode_options="". String of characters that will be passed as is to the instruction \node{} for all labels on both axes (but not for legends).
- xnode_options=xynode_options. String of characters that will be passed as is to the instruction \node{} for all labels on the x axis (but not for the legend).
- ynode_options=xynode_options. String of characters that will be passed as is to the instruction \node{} for all labels on the y axis (but not for the legend).
- nbdeci={2,2}. This option specifies the number of decimal places for numeric values on the axis.
- use_siunitx={ld.siunitx,ld.siunitx}. This option specifies whether numeric values should be formatted using the *siunitx* package; the default value is that of the global variable ld.siunitx, which is false by default.
- myxlabels="". This option allows you to impose custom labels on the x-axis. When any are present, the value passed to the option must be a list of the type: {pos1,"text1",pos2,"text2",…}. The number ⟨*pos1*⟩ represents an abscissa in the (A,xstep) coordinate system, which corresponds to the affix point A+pos1*xstep.
- myylabels="". This option allows you to impose custom labels on the y-axis. When any are present, the value passed to the option must be a list of the type: {pos1,"text1",pos2,"text2",…}. The number ⟨*pos1*⟩ represents an abscissa in the coordinate system (A,i*ystep), which corresponds to the affix point A+pos1*ystep*i.
- grid=false. This option allows you to add a grid or not.
- showlines={true,true }. When grid=true, this option allows you to display or hide the vertical lines of the grid (corresponding to the x axis) as well as the horizontal lines of the grid (corresponding to the y axis).
- drawbox=false. This option draws the axes as a box; in this case, the graduations are on the left and bottom sides.
- gridstyle="solid". This option sets the line style for the primary grid.
- subgridstyle="solid". This option sets the line style for the secondary grid. A secondary grid appears when there are subdivisions on one of the axes.
- gridcolor="gray". This sets the color of the primary grid.
- subgridcolor="lightgray". This sets the color of the secondary grid.
- gridwidth=4. Line thickness of the primary grid (which is 0.4pt).
- subgridwidth=2. Line thickness of the secondary grid (which is 0.2pt).

```
\begin{luadraw}{name=axes_grid}
local ld = luadraw
local g = ld.graph:new{window={-6.5,6.5,-3.5,3.5}, size={10,10,0}}
local i, pi, a = ld.cpx.I, math.pi, math.sqrt(2)
```

```
local f = function(x) return 2*a*math.sin(x) end
g:Labelsize("footnotesize"); g:Linewidth(8)
g:Daxes({0,pi/2,a},{labeltext={"\\pi","\\sqrt{2}"}, labelden={2,1},nbsubdiv={1,1},grid=true,arrows="->"})
g:Lineoptions("solid","Crimson",12); g:Dcartesian(f, {x={-2*pi,2*pi}})
g:Show()
\end{luadraw}
```

Figure 19: Example with axes with grid



DaxeX and DaxeY

The methods **g:DaxeX([{A, xstep}, options])** and **g:DaxeY([{A, ystep}, options])** allow you to plot the axes separately.

- The first argument specifies the point serving as the origin (the complex number $\langle A \rangle$) and the step size of the tick marks on the axis. By default, the point $\langle A \rangle$ is the origin $Z(0,0)$, and the step size is equal to 1.
- The argument $\langle options \rangle$ is a table specifying the possible options. Here are these options with their default values:
  - `showaxe=1`. This option specifies whether or not the axis should be plotted (1 or 0).
  - `arrows="-"`. This option allows you to add an arrow to the axis (no arrow by default; enter `"->"` to add an arrow).
  - `limits="auto"`. This option allows you to specify the range of the two axes. The value "auto" means that it is the entire line, but you can specify the extreme abscissas, for example: `limits={-4.4}`.
  - `gradlimits="auto"`. This option allows you to specify the range of the graduations on both axes. The value `"auto"` means that it is the entire line, but you can specify the extreme graduations, for example: `gradlimits={-2.3}`.
  - `unit=""`. This option allows you to specify the range of the graduations on the axis. The default value ("") means to take the step value, EXCEPT when the `labeltext` option is not the empty string, in which case `unit` takes the value 1.
  - `nbsubdiv=0`. This option specifies the number of subdivisions between two main tick marks.
  - `tickpos=0.5`. This option specifies the position of the tick marks relative to the axis. These are two numbers between 0 and 1. The default value of 0.5 means they are centered on the axis. (0 and 1 represent the ends).
  - `tickdir="auto"`. This option indicates the direction of the tick marks on the axis. This direction is a non-zero (complex) vector. The default value "auto" means the tick marks are orthogonal to the axis.
  - `xyticks=0.2`. This option specifies the length of the tick marks.

- xylabelsep=0. This option specifies the distance between the labels and the tick marks.
- originpos="center". This option specifies the position of the label at the origin on the axis. Possible values are: "none", "center", "left", "right" for x-axis, and "none", "center", "bottom", "top" for y-axis.
- originnum=A.re for x-axis and originnum=A.im for y-axis. This option specifies the value of the tick mark at the origin (tick mark number 0).

  The formula that defines the label at tick mark number n is:

  ```
  (originnum + unit*n)"labeltext"/labelden.
  ```
- legend="". This option allows you to specify a legend for the axis.
- legendpos=0.975. This option specifies the position (between 0 and 1) of the legend relative to the axis.
- legendsep=ld.defaultlegendsep. This option specifies the distance between the legend and the axis. The legend is on the other side of the axis from the graduations.
- legendangle="auto". This option specifies the angle (in degrees) that the legend should make for the axis. The default value "auto" means that the legend must be parallel to the axis if the labelstyle option is also set to "auto", otherwise the legend is horizontal.
- legendstyle="auto". Specifies the label style for the legend (position relative to the anchor point); with the value *"auto"*, the style is determined automatically; otherwise, the following values can be used: "center", "N", "NW", "W", "SW", "S", "SE", "E", "NE".
- labelpos="bottom" for x-axis and labelpos="left" for y-axis. This option specifies the position of the labels relative to the axis. For the x-axis, the possible values are: "none", "bottom" or "top", for the y-axis it is: "none", "right" ou "left".
- labelden=1. This option specifies the denominator of the labels (integer) for the axis. The formula that defines the label at graduation number n is:

  ```
  (originnum + unit*n)"labeltext"/labelden.
  ```
- labeltext="". This option defines the text that will be added to the numerator of the labels.
- labelstyle="S" for x-axis and labelstyle="W" for y-axis. This option defines the style of the labels. The possible values are: "auto", "N", "NW", "W", "SW", "S", "SE", "E", "NE".
- labelangle=0. This option sets the angle of the labels in degrees from the horizontal.
- labelcolor="". This option allows you to choose a color for the labels. The empty string represents the current text color.
- labelshift=0. This option allows you to set a systematic offset for labels on the axis (along-axis offset).
- nbdeci=2. This option specifies the number of decimal places for numeric labels.
- use_siunitx=ld.siunitx. This option specifies whether numeric values should be formatted using the *siunitx* package; the default value is that of the global variable ld.siunitx, which is false by default.
- mylabels="". This option allows you to impose custom labels. When there are any, the value passed to the option must be a list of the type: `{pos1,"text1",pos2,"text2",…}`. The number ⟨*pos1*⟩ represents an abscissa in the coordinate system `(A,xstep)` for x-axis, or `(A,ystep*i)` for y-axis, which corresponds to the affix point `A+pos1*xstep` for x-axis, and `A+pos1*ystep*i` for y-axis.

Dgradline

The axis plotting methods are based on the method **g:Dgradline({A, u}, options)**, where ⟨*{A,u}*⟩ represents the line passing through ⟨A⟩ (a complex number) and directed by the vector ⟨u⟩ (a non-zero complex number). The pair (A, u) serves as a reference point on this line (and orients this line), so each point M on this line has an abscissa x such that $M = A + xu$. This method allows you to draw this graduated line. The argument ⟨*options*⟩ is a table specifying the possible options, which are (with their default values):

- showaxe=1. This option specifies whether or not the axis should be plotted (1 or 0).
- arrows="-". This option allows you to add an arrow to the axis (no arrow by default; enter "->" to add an arrow).

- `limits="auto"`. This option allows you to specify the range of the two axes. The value `"auto"` means that it is the entire line, but you can specify the extreme abscissas, for example: `limits={-4.4}`.
- `gradlimits="auto"`. This option allows you to specify the range of the graduations on both axes. The value `"auto"` means that it is the entire line, but you can specify the extreme graduations, for example: `gradlimits={-2.3}`.
- `unit=1`. This option allows you to specify the number of graduations on the axis.
- `nbsubdiv=0`. This option specifies the number of subdivisions between two main ticks.
- `tickpos=0.5`. This option specifies the position of the ticks relative to the axis. These are two numbers between 0 and 1. The default value of 0.5 means they are centered on the axis. (0 and 1 represent the ends).
- `tickdir="auto"`. This option specifies the direction of the ticks on the axis. This direction is a non-zero vector (complex number). The default value `"auto"` means the ticks are orthogonal to the axis.
- `xyticks=0.2`. This option specifies the length of the ticks.
- `xylabelsep=defaultxylabelsep`. This option specifies the distance between the labels and the tick marks. `defaultxylabelse` is a global variable with a default value of 0.
- `originpos="center"`. This option specifies the position of the label at the origin on the axis. Possible values are: `"none"`, `"center"`, `"left"`, `"right"`.
- `originnum=0`. This option specifies the value of the tick mark at the origin A (tick mark number 0).

  The formula that defines the label at graduation number n (at point $A + nu$) is:

  ```
  (originnum + unit*n)"labeltext"/labelden.
  ```

- `legend=""`. This option allows you to specify a legend for the axis.
- `legendpos=0.975`. This option specifies the position (between 0 and 1) of the legend relative to the axis.
- `legendsep=ld.defaultlegendsep`. This option specifies the distance between the legend and the axis. The legend is on the other side of the axis from the graduations; `ld.defaultlegendsep` is a global variable that defaults to 0.2.
- `legendangle="auto"`. This option specifies the angle (in degrees) that the legend should form for the axis. The default value `"auto"` means that the legend must be parallel to the axis if the `labelstyle` option is also set to `"auto"`, otherwise the legend is horizontal.
- `legendstyle="auto"`. Specifies the label style for the legend (position relative to the anchor point); with the value *"auto"*, the style is determined automatically; otherwise, the following values can be used: `"center"`, `"N"`, `"NW"`, `"W"`, `"SW"`, `"S"`, `"SE"`, `"E"`, `"NE"`.
- `labelpos="bottom"`. This option specifies the position of the labels relative to the axis. The possible values are: `"none"`, `"bottom"`, or `"top"`. This position also determines the position of the legend: on the opposite side of the axis.
- `labelden=1`. This option specifies the denominator of the labels (integer) for the axis. The formula that defines the label at graduation number n is:

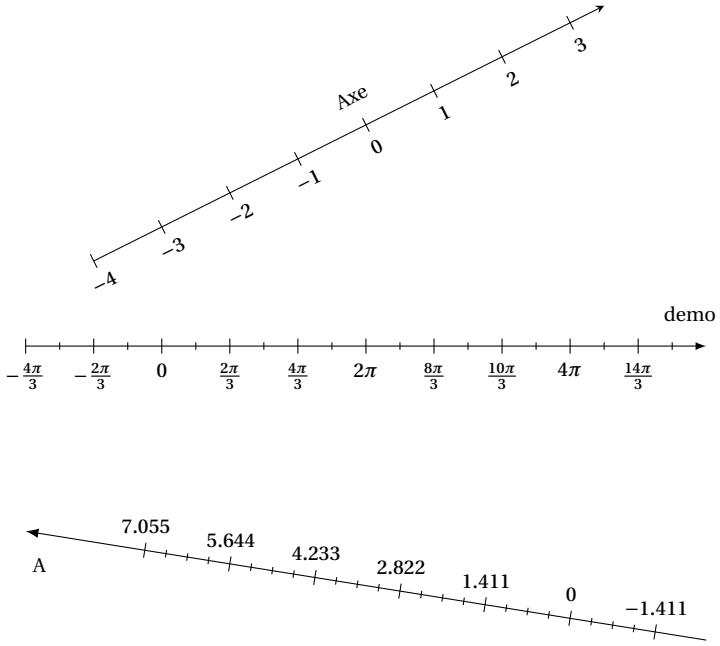
  ```
  (originnum + unit*n)"labeltext"/labelden.
  ```

- `labeltext=""`. This option defines the text that will be added to the numerator of the labels.
- `labelstyle="auto"`. This option defines the label style. Possible values are: `"auto"`, `"N"`, `"NW"`, `"W"`, `"SW"`, `"S"`, `"SE"`, `"E"`, `"NE"`.
- `labelangle=0`. This option defines the angle of the labels in degrees from the horizontal.
- `labelcolor=""`. This option allows you to choose a color for the labels. The empty string represents the current text color.
- `labelshift=0`. This option allows you to define a systematic offset for the labels on the axis (along axis offset).

- `nbdeci=2`. This option specifies the number of decimal places for numeric labels.
- `use_siunitx=ld.siunitx`. This option specifies whether numeric values should be formatted using the *siunitx* package; the default value is that of the global variable `ld.siunitx`, which is `false` by default.
- `mylabels=""`. This option allows you to impose custom labels. When any are present, the value passed to the option must be a list of the type: `{x1,"text1",x2,"text2",…}`. The numbers ⟨*x1*⟩, ⟨*x2*⟩, …, represent abscissas in the (A, u) coordinate system.

```
\begin{luadraw}{name=gradline}
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5},size={10,10}}
g:Labelsize("footnotesize")
local i = ld.cpx.I
g:Dgradline({3.25*i,1+i/2}, {limits={-4,4}, legend="Axe", legendpos=0.5, arrows="-stealth"})
g:Dgradline({-3,1}, {legend="demo", labeltext="\\pi", labelden=3, unit=2, nbsubdiv=1, arrows="-latex"})
g:Dgradline({3-4*i,-1.25+i/5}, {legend="A", labelstyle="N", gradlimits={-1,5},
nbsubdiv=3, unit=1.411, nbdeci=3, arrows="-Latex"})
g:Show()
\end{luadraw}
```

Figure 20: Examples of numbered lines



Dgrid

The **g:Dgrid({A, B}, options)** method allows you to draw a grid.

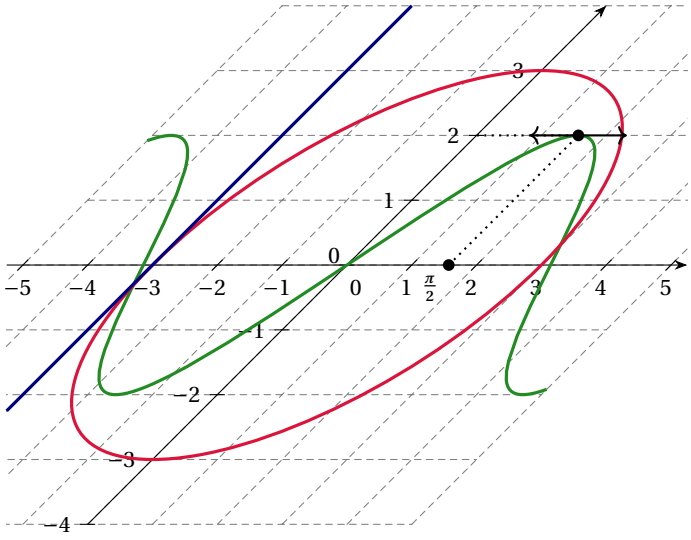
- The first argument is mandatory; it specifies the lower-left corner (this is the ⟨*A*⟩ complex number) and the upper-right corner (this is the ⟨*B*⟩ complex number) of the grid.
- The ⟨*options*⟩ argument is a table specifying the possible options. Here are these options with their default values:
  - `unit={1,1}`. This option defines the units on the axes for the main grid.
  - `showlines={true,true }`. This option allows you to display or hide the vertical lines of the grid (corresponding to the *x* axis) as well as the horizontal lines of the grid (corresponding to the *y* axis).
  - `gridwidth=4`. This option defines the line thickness of the main grid (0.4pt by default).
  - `gridcolor="gray"`. Grid color of the main grid.

- `gridstyle="solid"`. Line style for the primary grid.
- `nbsubdiv=0,0`. Number of subdivisions (for each axis) between two lines of the primary grid. These subdivisions determine the secondary grid.
- `subgridcolor ="lightgray"`. Color of the secondary grid.
- `subgridwidth=2`. Line thickness of the secondary grid (0.2pt by default).
- `subgridstyle="solid"`. Line style for the secondary grid.
- `originloc=A`. Location of the grid origin.

Example: It is possible to work in a non-orthogonal coordinate system. Here is an example where the x-axis is retained, but the first bisector becomes the new y-axis. We modify the graph's transformation matrix. Based on this modification, the affixes represent the coordinates in the new coordinate system.

```
\begin{luadraw}{name=axes_non_ortho}
local ld = luadraw
local g = ld.graph:new{window={-5.25,5.25,-4,4},size={10,10}}
local i, pi, Z = ld.cpx.I, math.pi, ld.cpx.Z
local f = function(x) return 2*math.sin(x) end
g:Setmatrix({0,1,1+i}); g:Labelsize("small")
g:Dgrid({-5-4*i,5+4*i},{gridstyle="dashed"})
g:Daxes({0,1,1}, {arrows="-Stealth"})
g:Lineoptions("solid","ForestGreen",12); g:Dcartesian(f,{x={-5,5}})
g:Dcircle(0,3,"Crimson")
g:DlineEq(1,0,3,"Navy") -- line with equation x=-3
g:Lineoptions("solid","black",8); g:DtangentC(f,pi/2,1.5,"<->")
g:Dpolyline({pi/2,pi/2+2*i,2*i},"dotted")
g:Ddots(Z(pi/2,2))
g:Dlabeldot("$\\frac{\\pi}2$",pi/2,{pos="SW"})
g:Show()
\end{luadraw}
```

Figure 21: Example of a non-orthogonal coordinate system



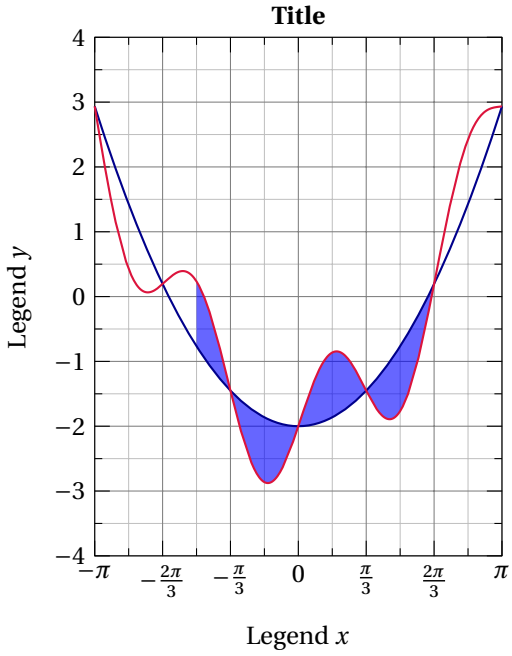
Dgradbox

The **g:Dgradbox({A, B, xstep, ystep}, options)** method allows you to draw a graduated box.

- The first argument is mandatory; it specifies the lower-left corner (this is the ⟨A⟩ complex number) and the upper-right corner (this is the ⟨B⟩ complex number) of the box, as well as the step on each axis.
- The ⟨*options*⟩ argument is a table specifying the possible options. These are the same as for the axes, except for some default values. In addition, the following option is added: `title=""`, which allows you to add a title at the top of the box; however, be careful to leave enough space for this.

```
\begin{luadraw}{name=gradbox}
local ld = luadraw
local g = ld.graph:new{window={-5,4,-5.5,5},size={10,10}}
local i, pi = ld.cpx.I, math.pi
local h = function(x) return x^2/2-2 end
local f = function(x) return math.sin(3*x)+h(x) end
g:Dgradbox({-pi-4*i,pi+4*i,pi/3,1},{grid=true,originloc=0, originnum={0,0}, labeltext={"\\pi",""},
labelden={3,1}, title="\\textbf{Title}", legend={"Legend $x$","Legend $y$"}})
g:Saveattr(); g:Viewport(-pi,pi,-4,4) -- The view is limited (clip).
g:Filloptions("full","blue",0.6); g:Linestyle("noline"); g:Ddomain2(f,h,{x={-pi/2,2*pi/3}})
g:Filloptions("none",nil,1); g:Lineoptions("solid",nil,8); g:Dcartesian(h,{x={-pi,pi}, draw_options="DarkBlue"})
g:Dcartesian(f,{x={-pi,pi},draw_options="Crimson"})
g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 22: Using Dgradbox



11) Set Drawings (Venn Diagrams)

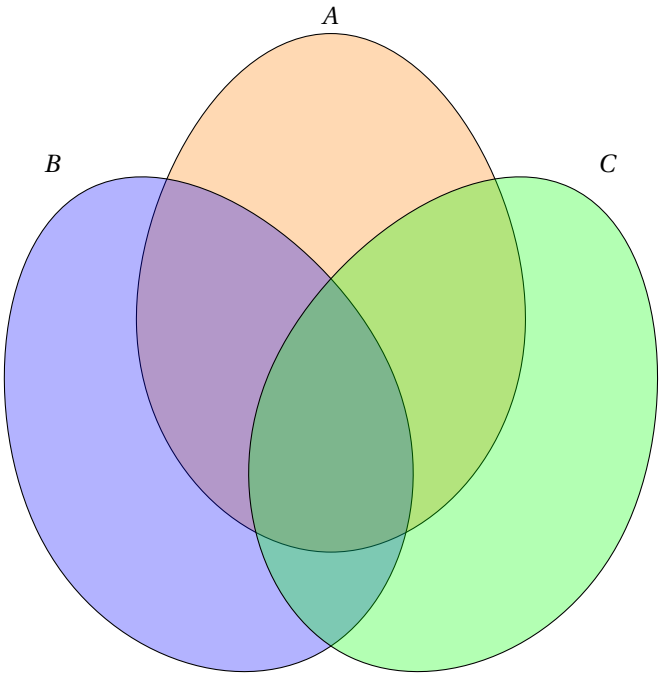
Drawing a Set

The function **ld.set(center [, angle, scale])** returns a path representing a set (egg-shaped), with the center being ⟨*center*⟩ (complex number), the argument ⟨*angle*⟩ representing the inclination (in degrees) of the set's vertical axis (0 by default), and the argument ⟨*scale*⟩ being a scale factor to modify the size of the set (1 by default). Such a path can be drawn with the method **g:Dpath()**.

```
\begin{luadraw}{name=set}
local ld = luadraw
local g = ld.graph:new{window={-5.25,5.25,-5,5},size={10,10}}
local i = ld.cpx.I
local A, B, C = ld.set(i,0), ld.set(-2-i,25), ld.set(2-i,-25)
g:Fillopacity(0.3)
g:Dpath(A,"fill=orange"); g:Dpath(B,"fill=blue")
g:Dpath(C,"fill=green")
g:Fillopacity(1)
g:Dlabel("$A$",5*i,{pos="N"},"$B$",-4+3*i,{pos="W"},"$C$",4+3*i,{pos="E"})
g:Show()
```

```
\end{luadraw}
```

Figure 23: Drawing a Set



Operations on Sets

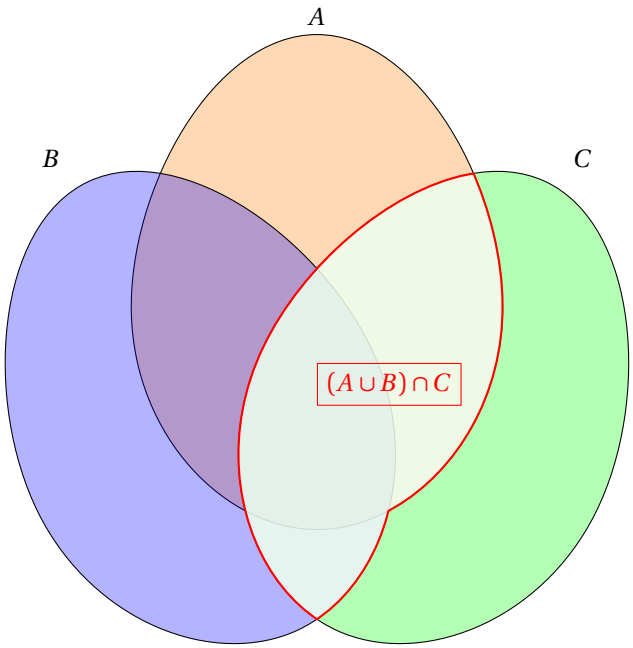
Let C_1 and C_2 be two lists of complex numbers representing the contour of two sets (simple closed curves, all in one piece). There are three possible operations:

- The function **ld.cap(C1, C2)** returns a list of complex numbers representing the contour of the intersection of the sets corresponding to C_1 and C_2.
- The function **ld.cup(C1, C2)** returns a list of complex numbers representing the contour of the union of the sets corresponding to C_1 and C_2.
- The function **ld.setminus(C1, C2)** returns a list of complex numbers representing the contour of the difference of the sets corresponding to C_1 and C_2 ($C_1 \setminus C_2$).

The result of these operations, being a list of complex numbers, can be drawn with the **g:Dpolyline()** method.

```
\begin{luadraw}{name=cap_and_cup}
local ld = luadraw
local g = ld.graph:new{window={-5.5,5.5,-5,5},size={10,10}}
local i = ld.cpx.I
local A, B, C = ld.set(i,0), ld.set(-2-i,25), ld.set(2-i,-25)
g:Fillopacity(0.3)
g:Dpath(A,"fill=orange"); g:Dpath(B,"fill=blue"); g:Dpath(C,"fill=green")
g:Fillopacity(1)
local C1, C2, C3 = ld.path(A), ld.path(B), ld.path(C) -- conversion: path -> list of complex numbers
local I = ld.cap(ld.cup(C1,C2),C3)
g:Linecolor("red"); g:Filloptions("full","white")
g:Dpolyline(I,true,"line width=0.8pt,fill opacity=0.8")
g:Dlabel("$A$",5*i,{pos="N"},"$B$",-4+3*i,{pos="W"},"$C$",4+3*i,{pos="E"},
"$(A\\cup B) \\cap C$",-i,{pos="NE",node_options="red,draw"})
g:Show()
\end{luadraw}
```

Figure 24: Set Operations



NB : The result is not always satisfactory when the contours become too complex, or when the contours share common sections.

12) Importing an Image

Placing an Image in the Graphic

To import an image into the current graphic (and possibly draw on top of it), use the method:

g:Dimage(filename, anchor, options)

where:

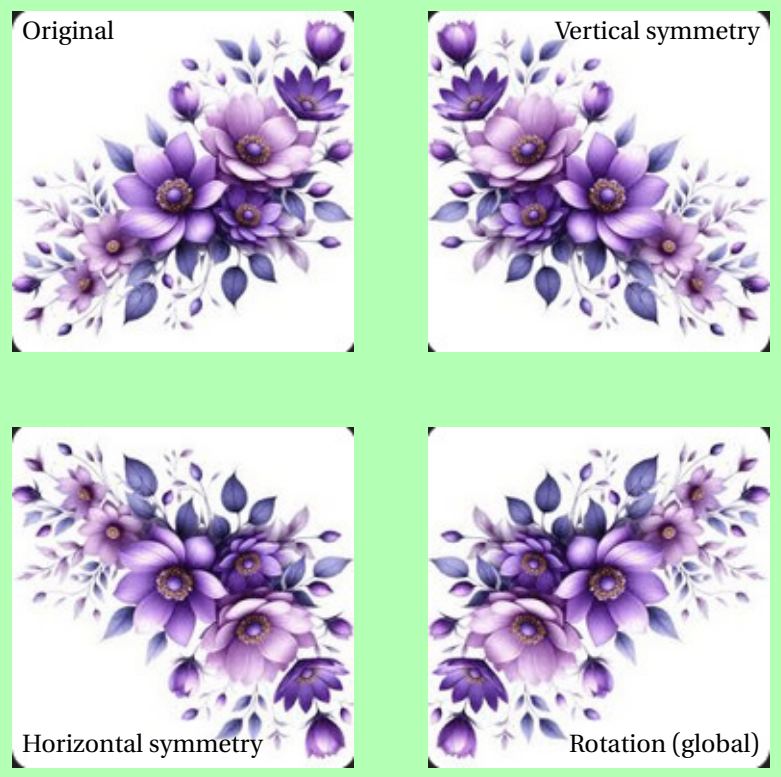
- ⟨*filename*⟩ is the full name of the image file. The image will be displayed using the `\includegraphics[]{}` command within a node.
- ⟨*anchor*⟩ is a complex number representing the anchor point of the image in the graphic.
- ⟨*options*⟩ is a table whose fields define the display parameters (with their default values):
  - `pos="center"` specifies the position of the image relative to the anchor point, following the same principle as for labels. Possible values are: `"center"`, `"N"`, `"NE"`, `"E"`, `"SE"`, `"S"`, `"SW"`, `"W"`, `"NW"`.
  - `name=""` optionally assigns a name to the node that will be created. This name can later be referenced in TikZ.
  - `graphics_options=""` which is a string containing options for the `\includegraphics` command (no default options).
  - `matrix=nil` is a table representing an affine transformation matrix. This matrix is applied locally to the created node (for which the origin is the anchor point). Note that the current global 2D transformation matrix of the graphic is also taken into account by this method.
  - `node_options=""` which is a string containing options for the `\node` command, these will be added after the default options like this:

    `\node[line width=0.3pt,inner sep=-0.15pt,cm=<matrix>,<position>,<node_options>].`

```
\begin{luadraw}{name=Dimage}
local ld = luadraw
local Z, i = ld.cpx.Z, ld.cpx.I
local g = ld.graph:new{margin={0,0,0,0}, size={10,10},bg="green!30"}
local filename = ld.cachedir.."flower.jpg"
```

```
local g_options = "width=4.5cm,height=4.5cm"
g:Dimage(filename,Z(-5,5),{pos="SE", graphics_options=g_options})
g:Dimage(filename,Z(5,5),{pos="SE", matrix={0,-1,i}, graphics_options=g_options})
g:Rotate(180)
g:Dimage(filename,Z(-5,5),{pos="SE", graphics_options=g_options})
g:IDmatrix()
g:Dimage(filename,Z(-5,-5),{pos="SE", matrix={0,1,-i}, graphics_options=g_options})
g:Dlabel( "Original", Z(-5,5),{pos="SE"},
"Vertical symmetry", Z(5,5), {pos="SW"},
"Rotation (global)", Z(5,-5), {pos="NW"},
"Horizontal symmetry", Z(-5,-5), {pos="NE"} )
g:Show()
\end{luadraw}
```

Figure 25: Importing an Image



Mapping an Image onto a Parallelogram

This can be done using the method: **g:Dmapimage(filename, parallelo, options)** (which uses the **g:Dimage()** method), where:

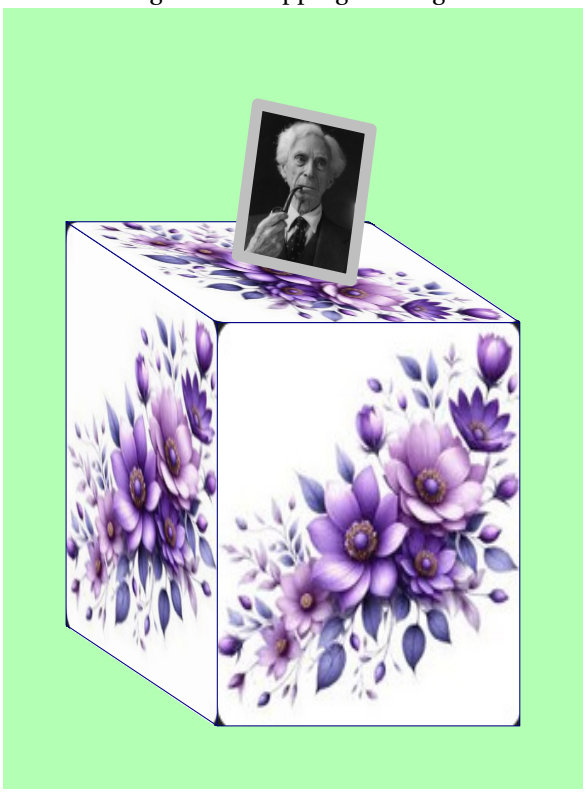
- ⟨*filename*⟩ is the full name of the image file.
- ⟨*parallelo*⟩ is a list of the form $\{a,u,v\}$, consisting of three complex numbers. This list defines the parallelogram with vertices $\{a, a+u, a+u+v, a+v\}$.
- ⟨*options*⟩ is a table whose fields specify the parameters (with their default values):
  - `name=""` optionally assigns a name to the node that is created. This name can later be referenced in TikZ.
  - `clip=false` is a Boolean flag indicating whether the image should be clipped to the parallelogram. In most cases, clipping is not required.
  - `border_options=nil`, when set to a value other than `nil`, must be a string containing the drawing options for the boundary of the parallelogram. The value of `border_options` is passed directly to the *\draw* command to render the outline.
  - `graphics_options=""` which is a string of options for the `\includegraphics` command, these will be added after the default options like this:

    `\includegraphics[width=1cm,height=1cm, <graphics_options>].`

– `node_options=""` which is a string containing options for the `\node` command, these will be added to the end of the default options as for **g:Dimage()**.

```
\begin{luadraw}{name=Dmapimage}
local ld = luadraw
local g = ld.graph:new{window={-2,3.5,-0.5,7}, margin={0,0,0,0}, size={10,10},bg="green!30"}
local filename = ld.cachedir.."flower.jpg"
local i = ld.cpx.I
local a, u, v, w = 0, 3, -1.5+i, 4*i
local facets = {{a,u,w}, {a+v,-v,w}, {a+v+w,-v,u} }
local portrait = {a+w+(2*v+u)/3+0.2, (u-v)/4, 1.5*i+0.2}
for _,f in ipairs(facets) do
    g:Dmapimage(filename,f,{border_options="Navy"})
end
g:Dmapimage(ld.cachedir.."russell.jpg", portrait, {border_options="lightgray,line width=1.5mm"})
g:Show()
\end{luadraw}
```

Figure 26: Mapping an Image



13) Colors

In the *luadraw* environment, colors are character strings that must correspond to colors known to TikZ. The *xcolor* package is strongly recommended so as not to be limited to basic colors.

Color Calculations

To be able to manipulate colors, they have been defined (in the *luadraw_colors.lua* module) as tables of three components: red, green, blue, each component being a number between 0 and 1, and with their names in the *svgnames* format of the *xcolor* package. For example, we find (among others) the declarations:

```
local ld = luadraw
ld.AliceBlue = {0.9412, 0.9725, 1}
ld.AntiqueWhite = {0.9804, 0.9216, 0.8431}
ld.Aqua = {0.0, 1.0, 1.0}
ld.Aquamarine = {0.498, 1.0, 0.8314}
```

You can refer to the *xcolor* documentation for a list of these colors.

To use these in the *luadraw* environment, you can:

- either use them by name if you have declared them in the preamble: `\usepackage[svgnames]{xcolor}`, for example: `g:Linecolor("AliceBlue")`,
- or use them with the *luadraw* **ld.rgb()** function, for example: `g:Linecolor(ld.rgb(AliceBlue))`. However, with this **ld.rgb()** function, to change the color locally, you must do the following (example):

  `g:Dpolyline(L,"color="..ld.rgb(AliceBlue))`, or `g:Dpolyline(L,"fill="..ld.rgb(AliceBlue))`. Because the `rgb()` function does not return a color name, but a color definition.

Functions for color management:

- The **ld.rgb(r, g, b)** or **ld.rgb({r, g, b})** function returns the color as a string understandable by TikZ in the `color=...` and `fill=...` options. The values of r, g, and b must be between 0 and 1.
- The **ld.hsb(h, s, b [, table])** function returns the color as a string understandable by TikZ. The ⟨h⟩ (hue) argument must be a number between 0 and 360, the ⟨s⟩ (saturation) argument must be between 0 and 1, and the ⟨b⟩ (brightness) argument must also be between 0 and 1. The (optional) argument ⟨*table*⟩ is a boolean (false by default) that indicates whether the result should be returned as a table $\{r, g, b\}$ or not (by default it is as a string).
- The function **ld.mixcolor(color1, proportion1, color2, proportion2, ..., colorN, proportionN)** mixes the colors ⟨*color1*⟩, ...,⟨*colorN*⟩ in the requested proportions and returns the resulting color as a string understandable by TikZ, followed by the same color as a table $\{r, g, b\}$. Each color must be a table of three components $\{r, g, b\}$.
- The function **ld.mixpalette(pal, percent, color)** returns a new palette after mixing each color from the palette ⟨*pal*⟩ with ⟨*color*⟩. The argument ⟨*pal*⟩ is a list of colors, each of which is a table of three components $\{r, g, b\}$, the argument ⟨*percent*⟩ is a number between 0 and 100, and the argument ⟨*color*⟩ is a color in the form of a table $\{r, g, b\}$.
- The function **ld.palette(colors, pos [, table])**: the argument ⟨*colors*⟩ is a list (table) of colors in the format $\{r, g, b\}$, the argument ⟨*pos*⟩ is a number between 0 and 1, the value 0 corresponds to the first color in the list and the value 1 to the last. The function calculates and returns the color corresponding to the position ⟨*pos*⟩ in the list by linear interpolation. The (optional) argument ⟨*table*⟩ is a boolean (false by default) that indicates whether the result should be returned as a table $\{r, g, b\}$ or not (by default it is as a string).
- The **ld.getpalette(colors, nb [, table])** function: the ⟨*colors*⟩ argument is a list (table) of colors in $\{r, g, b\}$ format, the ⟨*nb*⟩ argument indicates the desired number of colors. The function returns a list of ⟨*nb*⟩ colors evenly distributed in ⟨*colors*⟩. The (optional) ⟨*table*⟩ argument is a boolean (`false` by default) that indicates whether the colors are returned as $\{r, g, b\}$ tables or not (by default, they are as strings).
- The method **g:Newcolor(name, color)** allows you to define a new color in the TikZ export to rgb format. The name of this color will be ⟨*name*⟩ (string). The argument ⟨*color*⟩ can be either a table of three components: red, green, blue (between 0 and 1) defining this color, or a string representing a color. In the first case, the method uses a `\definecolor`, and in the second case, it uses a `\colorlet`.

You can also use all of TikZ's usual color management features.

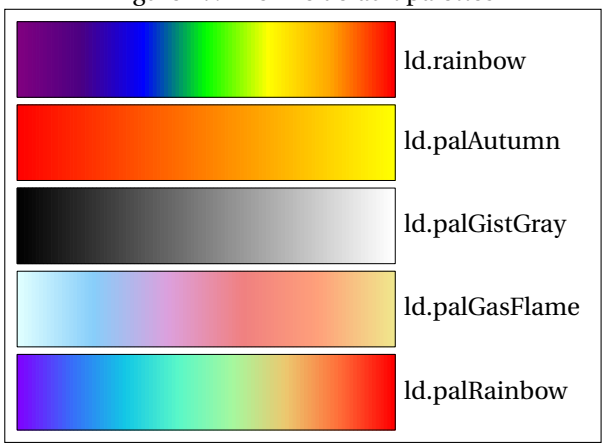
By default, there are five color palettes.[2].

```
\begin{luadraw}{name=palettes}
local ld = luadraw
local Z = ld.cpx.Z
local g = ld.graph:new{window={-5,5,-5,5},bbox=false, border=true}
g:Linewidth(1)
local Dpalette = function(pal,A,h,L,N,name)
    local dl = L/N
    for k = 1, N do
        local color = ld.palette(pal,(k-1)/(N-1))
        g:Drectangle(A,A+h,A+h+dl,"color="..color..",fill="..color)
        A = A+dl
    end
```

[2]A palette is a table of colors; these are themselves tables of numbers between 0 and 1 representing the red, green, and blue components.

```
    g:Drectangle(A,A+h,A+h-L); g:Dlabel(name,A+h/2,{pos="E"})
end
local A, h, dh, L, N = Z(-5,4), Z(0,-1), Z(0,-1.1), 5, 100
Dpalette(ld.rainbow,A,h,L,N,"ld.rainbow"); A = A+dh
Dpalette(ld.palAutumn,A,h,L,N,"ld.palAutumn"); A = A+dh
Dpalette(ld.palGistGray,A,h,L,N,"ld.palGistGray"); A = A+dh
Dpalette(ld.palGasFlame,A,h,L,N,"ld.palGasFlame"); A = A+dh
Dpalette(ld.palRainbow,A,h,L,N,"ld.palRainbow")
g:Show()
\end{luadraw}
```

Figure 27: The five default palettes



III Geometric Constructions

This section groups together functions that construct geometric figures without a corresponding dedicated graphical method.

1) circumcircle(), incircle()

- The function **ld.circumcircle(a, b, c)** (or **ld.circumcircle({a, b, c})**), where $\langle a\rangle$, $\langle b\rangle$, and $\langle c\rangle$ are three points (three complex numbers), returns the circumcircle of the triangle formed by these three points, in the form of a sequence: C, r, where C is the center of the circle (a complex number), and r is its radius.
- The function **ld.incircle(a, b, c)** (or **ld.incircle({a, b, c})**), where $\langle a\rangle$, $\langle b\rangle$, and $\langle c\rangle$ are three points (three complex numbers), returns the incircle of the triangle formed by these three points, as a sequence: C, r, where C is the center of the circle (a complex number), and r is its radius.

2) cvx_hull2d()

The function **ld.cvx_hull2d(L)**, where $\langle L\rangle$ is a list of complex numbers, computes and returns a list of complex numbers representing the convex hull of $\langle L\rangle$.

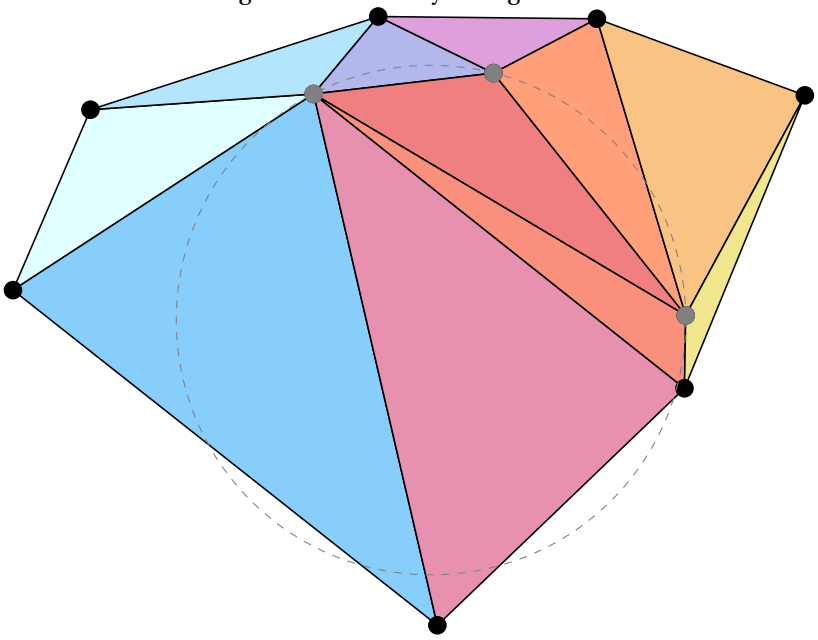
3) delaunay()

The function **ld.delaunay(L)** where $\langle L\rangle$ is a list of **distinct** complex numbers, returns a list of triangles (a triangle being a list of three complex numbers) obtained by Delaunay triangulation of the points of $\langle L\rangle$ (the circumcircle of each triangle does not contain any of the other points).

```
\begin{luadraw}{name=delaunay}
local ld = luadraw
local g = ld.graph:new{bbox=false, pictureoptions="scale=2"}
local i = ld.cpx.I; g:Linewidth(6)
local L = {0.285+1.46*i,1.556-0.142*i,2.344+1.313*i,-2.38+1.218*i,1.548-0.624*i,
0.969+1.819*i, -0.086-2.191*i,-0.477+1.834*i,-0.904+1.322*i,-2.892+0.025*i}
local T = ld.delaunay(L) -- list of triangles
```

```
local n = #T
local num = 7 --we choose a triangle
local colors = ld.getpalette(ld.palGasFlame,n)
for k = 1, n do
    g:Dpolyline(T[k],true,'fill='..colors[k])
    end
g:Ddots(L)
g:Dcircle( {ld.circumcircle(T[num])}, "line width=0.4pt,gray,dashed" )
g:Ddots(T[num],"gray")
g:Show()
\end{luadraw}
```

Figure 28: Delaunay Triangulation

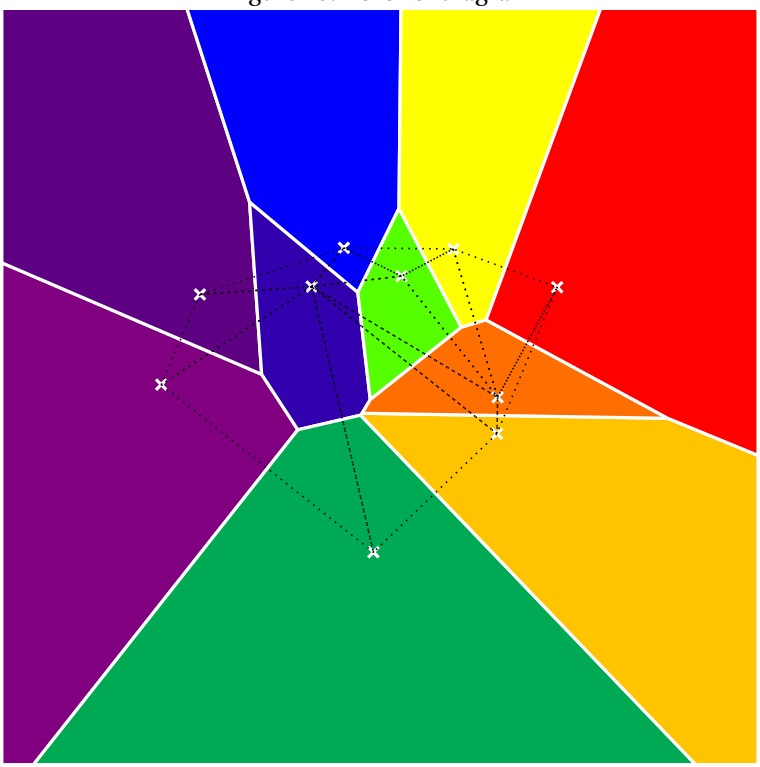


4) voronoi()

The function **ld.voronoi(L [, window])**, where ⟨*L*⟩ is a list of distinct complex numbers, determines the Voronoi diagram of the points in the list ⟨*L*⟩. This function returns a list of elements of the form `{A,polygon}`, where `A` is a point in the list ⟨*L*⟩, and `polygon` is a list of complex numbers representing the vertices of the cell associated with `A`. Thus, there is one cell for each point in ⟨*L*⟩. The cell for point `A` contains the points in the plane that are closer to `A` than to other points in ⟨*L*⟩. This function uses Delaunay triangulation. The optional argument ⟨*window*⟩, which defaults to `{-5,5,-5,5}`, is used to clip Voronoi cells that are unbounded; this window is automatically enlarged if necessary to contain all the points of ⟨*L*⟩ as well as all the centers of the circles circumscribed about the Delaunay triangles (note: this does not change the 2D window of the current graph).

```
\begin{luadraw}{name=voronoi}
local ld = luadraw
local g = ld.graph:new{ bbox=true, margin={0,0,0,0}, size={10,10}}
local i = ld.cpx.I
local S = {0.285+1.46*i,1.556-0.142*i,2.344+1.313*i,-2.38+1.218*i,1.548-0.624*i,
0.969+1.819*i,-0.086-2.191*i,-0.477+1.834*i,-0.904+1.322*i,-2.892+0.025*i}
local V = ld.voronoi(S)
local colors = ld.getpalette(ld.rainbow,#V)
for k,T in ipairs(V) do
    local A, polygon = table.unpack(T)
    g:Dpolyline(polygon,true,"color=white, line width=1.2pt,fill="..colors[k])
    g:Ddots(A,"mark=x,white,scale=2,line width=1.2pt")-- A is one of the points of S
end
g:Dpolyline(ld.delaunay(S),true,"dotted,line width=0.6pt") -- Delaunay triangles
g:Show()
\end{luadraw}
```

Figure 29: Voronoi diagram



5) line2strip()

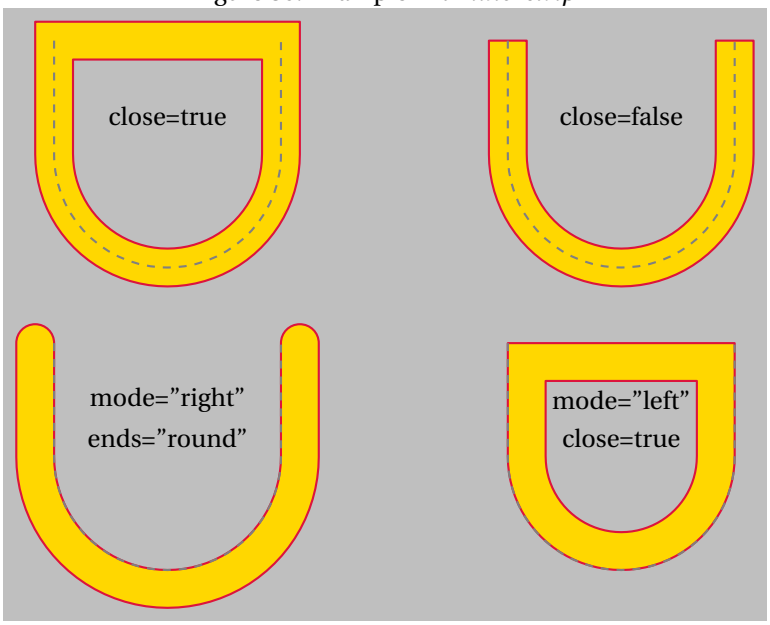
The function **ld.line2strip(L, width [, close, ends, mode])** where ⟨*L*⟩ is a list of complex numbers, or a list of lists of complex numbers, returns a path representing a "strip" calculated on ⟨*L*⟩ and of width ⟨*width*⟩. The optional argument ⟨*close*⟩ is a boolean that indicates whether ⟨*L*⟩ should be closed (`false` by default).

The optional argument ⟨*ends*⟩ specifies how the two ends of the strip are drawn. Possible values are: `"none"`, `"butt"` (default), or `"round"`. To maintain compatibility with the older version, this argument can also take the values `true` (equivalent to `"butt"`) or `false` (equivalent to `"none"`).

The argument ⟨*mode*⟩ can be either `"center"` (default value), or `"left"`, or `"right"`, in the first case the band is centered on ⟨*L*⟩, in the second case it is on the left of ⟨*L*⟩, and in the third case it is on the right of ⟨*L*⟩.

```
\begin{luadraw}{name=line2strip}
local ld = luadraw
local g = ld.graph:new{bbox=false, bg="lightgray"}
local i = ld.cpx.I; g:Linewidth(8)
local p = {-3+3*i,-3,"l",0,3,3,1,"ca", 3+3*i,"l"}
local P = ld.path(p)  -- p is first converted to polyline
g:Setmatrix({-3+3*i,0.5,0.5*i})
local L = ld.line2strip(P,1,true)
g:Dpath(L,"Crimson,fill=Gold"); g:Dpath(p,"gray,dashed")
g:Dlabel("close=true",i,{})
g:Setmatrix({3+3*i,0.5,0.5*i})
L = ld.line2strip(P,1,false)
g:Dpath(L,"Crimson,fill=Gold"); g:Dpath(p,"gray,dashed")
g:Dlabel("close=false",i,{})
g:Setmatrix({-3-i,0.5,0.5*i})
L = ld.line2strip(P,1,false,"round","right")
g:Dpath(L,"Crimson,fill=Gold"); g:Dpath(p,"gray,dashed")
g:Dlabel('mode="right"',1.5*i,{}); g:Dlabel('ends="round"',0.5*i,{})
g:Setmatrix({3-i,0.5,0.5*i})
L = ld.line2strip(P,1,true,false,"left")
g:Dpath(L,"Crimson,fill=Gold"); g:Dpath(p,"gray,dashed")
g:Dlabel('mode="left"',1.5*i,{}); g:Dlabel("close=true",0.5*i,{})
g:Show()
\end{luadraw}
```

Figure 30: Example with *line2strip*



6) parallel_polyline()

The function **ld.parallel_polyline(L, width [, close])** where ⟨*L*⟩ is a list of complex numbers, or a list of lists of complex numbers, returns a polygonal line parallel to ⟨*L*⟩ and located at a "distance" equal to ⟨*width*⟩. The argument ⟨*width*⟩ can be positive or negative to be on one side or the other of ⟨*L*⟩ (this depends on the direction of traversal of ⟨*L*⟩). The optional argument ⟨*close*⟩ is a boolean that indicates whether ⟨*L*⟩ should be closed (`false` by default).

7) sss_triangle()

The function **ld.sss_triangle(ab, bc, ca)**, where ⟨*ab*⟩, ⟨*bc*⟩, and ⟨*ca*⟩ are three lengths, computes and returns a list of three points (3 complex numbers) $\{A,B,C\}$ forming the vertices of a direct triangle whose side lengths are the arguments, i.e., $AB = ab$, $BC = bc$, and $CA = ca$, when possible. Vertex A is always the complex number 0 and vertex B is always the complex number ab. This triangle can be drawn with the **g:Dpolyline** method.

8) sas_triangle()

The function **ld.sas_triangle(ab, alpha, ca)**, where ⟨*ab*⟩ and ⟨*ca*⟩ are two lengths and ⟨*alpha*⟩ is an angle in degrees, calculates and returns a list of three points (3 complex numbers) $\{A,B,C\}$ forming the vertices of a triangle such that $AB = ab$, $CA = ca$, and angle $(\vec{AB},\vec{AC})$ has a measure of ⟨*alpha*⟩, whenever possible. Vertex A is always the complex number 0 and vertex B is always the complex number ab. This triangle can be drawn with the **g:Dpolyline** method.

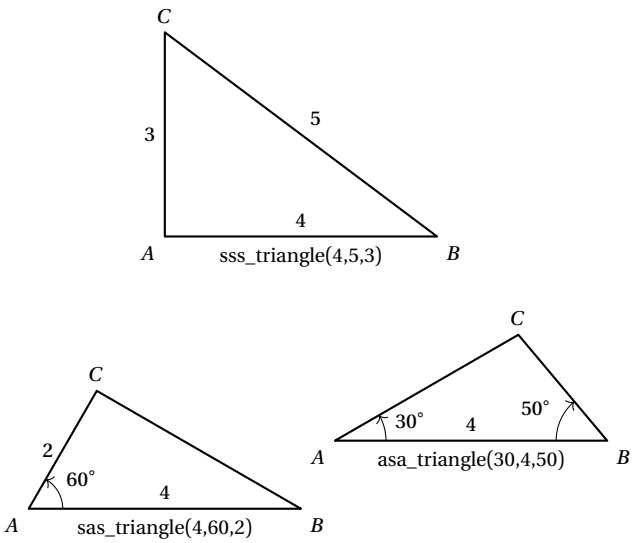
9) asa_triangle()

The function **ld.asa_triangle(alpha, ab, beta)**, where ⟨*ab*⟩ is a length, ⟨*alpha*⟩ and ⟨*beta*⟩ are two angles in degrees, calculates and returns a list of three points (3 complex numbers) $\{A,B,C\}$ forming the vertices of a triangle such that $AB = ab$, such that angle $(\vec{AB},\vec{AC})$ has measure ⟨*alpha*⟩, and such that angle $(\vec{BA},\vec{BC})$ has measure ⟨*beta*⟩, whenever possible. Vertex A is always the complex number 0 and vertex B is always the complex number ab. This triangle can be drawn using the **g:Dpolyline** method.

```
\begin{luadraw}{name=sss_triangles_and_co}
local ld = luadraw
local g = ld.graph:new{window={-5,5,-3,5},size={10,10}}
g:Labelsize("footnotesize"); g:Linewidth(8)
local Zp, i, deg = ld.cpx.Zp, ld.cpx.I, ld.deg
local T1 = ld.shift( ld.sss_triangle(4,5,3), 2*i-2)
local T2 = ld.shift( ld.sas_triangle(4,60,2), -4-2*i)
local T3 = ld.shift( ld.asa_triangle(30,4,50), 0.5-i)
g:Dpolyline({T1,T2,T3}, true)
```

```
g:Linewidth(4)
g:Darc(T2[2],T2[1],T2[3],0.5,1,"->")
g:Darc(T3[2],T3[1],T3[3],0.75,1,"->")
g:Darc(T3[1],T3[2],T3[3],0.75,-1,"->")
g:Dlabel(
  "$4$",(T1[1]+T1[2])/2,{pos="N"}, "$5$",(T1[2]+T1[3])/2,{pos="NE"},"$3$",(T1[1]+T1[3])/2,{pos="W"},
  "$4$",(T2[1]+T2[2])/2,{pos="N"},"$60^\\circ$", T2[1]+Zp(0.9,30*deg),{pos="center"},
  "$2$",(T2[1]+T2[3])/2,{pos="W"},
  "$4$",(T3[1]+T3[2])/2,{pos="N"}, "$30^\\circ$",T3[1]+Zp(1.15,15*deg),{pos="center"},
  "$50^\\circ$",T3[2]+Zp(1.15,155*deg),{pos="center"},
  "sss\\_triangle(4,5,3)",(T1[1]+T1[2])/2,{pos="S"}, "sas\\_triangle(4,60,2)",(T2[1]+T2[2])/2,{},
  "asa\\_triangle(30,4,50)",(T3[1]+T3[2])/2,{})
for _,T in ipairs({T1,T2,T3}) do
    g:Dlabel("$A$",T[1],{pos="SW"}, "$B$",T[2],{pos="SE"},"$C$",T[3],{pos="N"})
end
g:Show()
\end{luadraw}
```

Figure 31: sss_triangle, sas_triangle and asa_triangle



IV Computations on Lists

1) concat

The function **ld.concat(table1, table2, ...)** concatenates all the tables passed as arguments and returns the resulting table.

- Each argument can be a real number, a complex number, or a table.
- Example: The instruction `ld.concat(1,2,3,4,5,6,7)` returns the table `{1,2,3,4,5,6,7}`.

2) cut

The function **ld.cut(L, A [, before])** cuts ⟨*L*⟩ at the point ⟨*A*⟩, which is assumed to be located on the line ⟨*L*⟩ (which is either a list of complex numbers or a polygonal line, i.e., a list of lists of complex numbers). If the argument ⟨*before*⟩ is `false` (the default value), then the function returns the part before ⟨*A*⟩, followed by the part after ⟨*A*⟩; otherwise, the reverse is true.

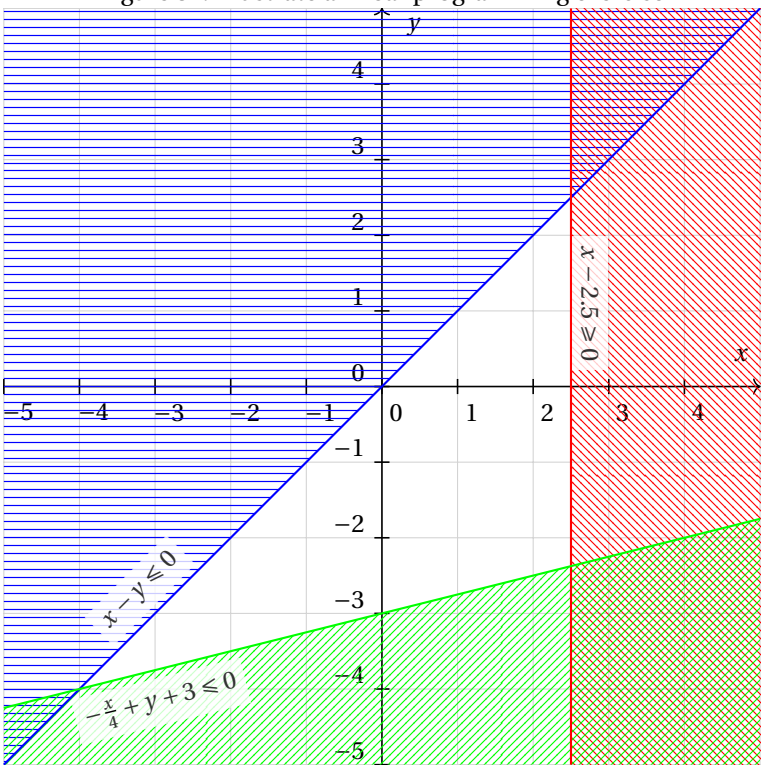
3) cutpolyline

The function **ld.cutpolyline(L, D [, close])** cuts the polygonal line ⟨*L*⟩ with the straight line ⟨*D*⟩. The argument ⟨*L*⟩ must be a list of complex numbers or a list of lists of complex numbers, the argument ⟨*D*⟩ is a list of the form `{A,u}` where *A* is a complex (point on the line) and *u* is a non-zero complex (direction vector of the line). The argument ⟨*close*⟩ indicates whether the line ⟨*L*⟩ should be closed (`false` by default). The function returns three things:

- The part of $\langle L\rangle$ that is in the half-plane defined by the line to the "left" of u (i.e., containing the point $A+iu$) (it is a polygonal line),
- followed by the part of $\langle L\rangle$ that is in the other half-plane (polygonal line),
- followed by the list of intersection points between $\langle L\rangle$ and the line $\langle D\rangle$.

```
\begin{luadraw}{name=cutpolyline}
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5}, size={10,10},margin={0,0,0,0}}
g:Linewidth(6)
local i = ld.cpx.I
local P = g:Box2d() -- polygon representing the 2D window
local D1 = ld.lineEq(1,-1,0,'<')     -- x-y < 0
local D2 = ld.lineEq(1,0,-2.5,'>')   -- x-2 > 0
local D3 = ld.lineEq(-1/4,1,3,'<')   -- x/4+y+3 < 0
local P1 = ld.cutpolyline(P,D1,true)
local P2 = ld.cutpolyline(P,D2,true)
local P3 = ld.cutpolyline(P,D3,true)
g:Daxes({0,1,1},{grid=true,gridcolor="LightGray",arrows="->",legend={"$x$","$y$"}})
g:Filloptions("horizontal","blue"); g:Dpolyline(P1,true,"draw=none")
g:Filloptions("fdiag","red"); g:Dpolyline(P2,true,"draw=none")
g:Filloptions("bdiag","green"); g:Dpolyline(P3,true,"draw=none")
g:Filloptions("none","black",1); g:Linewidth(8)
g:Dline(D1,"blue"); g:Dline(D2,"red"); g:Dline(D3,"green")
g:Dlabel(
  "$x-y\\leqslant 0$",-3-3*i,{pos="N",dir={1+i,-1+i},dist=0.1,node_options="fill=white,fill opacity=0.8"},
  "$x-2.5\\geqslant0$", 2.5+i,{dir={-i,1}},
  "$-\\frac{x}{4}+y+3\\leqslant0$", -3-15/4*i,{pos="S",dir={1+i/4,i-1/4}})
g:Show()
\end{luadraw}
```

Figure 32: Illustrate a linear programming exercise

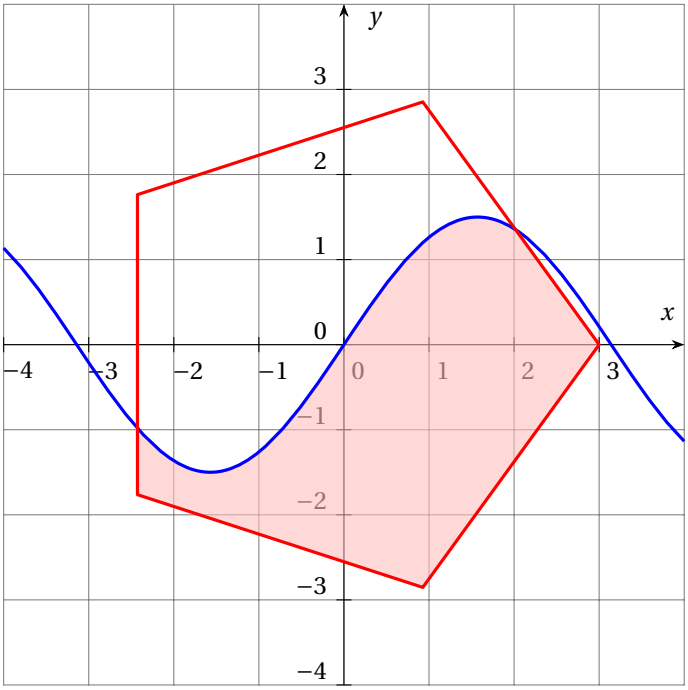


4) cutpolyline2

The function **ld.cutpolyline2(P, f, sg, [, close])** cuts the polygon $\langle P\rangle$ with the graph of the function $\langle f\rangle$. The argument $\langle P\rangle$ must be a list of complex numbers, and the argument $\langle f\rangle$ is a function $\langle f\rangle\colon x\mapsto f(x)\in\mathbf{R}$. The argument $\langle sg\rangle$ is either `">"` or `"<"`. In the first case, the function returns the subset of the polygon $\langle P\rangle$ whose points satisfy $y>f(x)$, and in the second case, it returns the points satisfying $y<f(x)$. The optional argument $\langle close\rangle$ indicates whether $\langle P\rangle$ should be closed.

```
\begin{luadraw}{name=cutpolyline2}
local ld = luadraw
local g = ld.graph:new{window = {-4, 4, -4, 4}, size ={10,10}}
g:Daxes({0,1,1}, {grid=true, arrows="-Stealth", legend={"$x$","$y$"}})
local f = function(x) return 1.5*math.sin(x) end
local P = ld.polyreg(0,3,5) -- pentagone
local L = ld.cutpolyline2(P,f,'<',true) -- partie du polygone située sous la courbe de f
g:Dpolyline(L,true,"draw=none,fill=pink,fill opacity=0.6")
g:Linewidth(12)
g:Dcartesian(f, {draw_options="blue"})
g:Dpolyline(P,true,"red")
g:Show()
\end{luadraw}
```

Figure 33: cutpolyline2



5) getbounds

- The function **ld.getbounds(L)** returns the bounds `xmin,xmax,ymin,ymax` of the polygonal line ⟨*L*⟩.
- Example: `local xmin,xmax,ymin,ymax=ld.getbounds(L)` (where ⟨*L*⟩ denotes a polygonal line).

6) getdot

The function **ld.getdot(x, L)** returns the point with abscissa ⟨*x*⟩ (real between 0 and 1) along the connected component ⟨*L*⟩ (list of complex numbers). The abscissa 0 corresponds to the first point and the abscissa 1 to the last. More generally, ⟨*x*⟩ corresponds to a percentage of the length of *L*.

7) insert

The function **ld.insert(table1, table2 [, pos])** inserts the elements of ⟨*table2*⟩ into ⟨*table1*⟩ at position ⟨*pos*⟩.

- The argument ⟨*table2*⟩ can be a real number, a complex number, or a table.
- The argument ⟨*table1*⟩ must be a variable that designates a table; this will be modified by the function.
- If the argument ⟨*pos*⟩ is `nil`, the insertion is performed at the end of ⟨*table1*⟩.

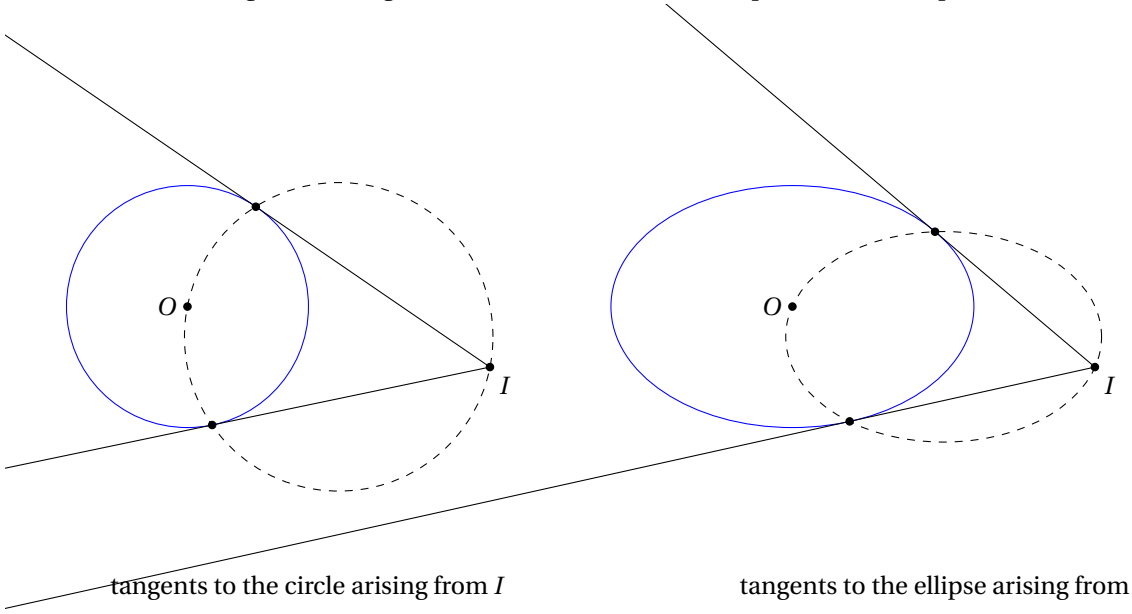
- Example: If a variable L is equal to `{1,2,6}`, then after the instruction `ld.insert(L,3,4,5,3)`, the variable L will be equal to `{1,2,3,4,5,6}`.

8) interCC

The function **ld.interCC(C1, C2)** returns the intersection of circle $\langle C1\rangle$ with circle $\langle C2\rangle$, where `C1={O1,r1}` (circle with center O_1 and radius r_1), and `C2={O2,r2}` (circle with center O_2 and radius r_2). The function returns a list containing 1 or 2 points or the entire circle if the intersection is not empty; otherwise, it returns `nil`.

```
\begin{luadraw}{name=interCC}
local ld = luadraw
local cpx = ld.cpx
local g = ld.graph:new{window={-10,10,-5,5}, margin={0,0,0,0},size={16,8}}
local i = cpx.I
-- pour le cercle {O,2}
g:Saveattr(); g:Viewport(-10,0,-5,5); g:Coordsystem(-4,6,-5,5)
local O = -1
local C1, I = {O, 2}, 4-i
local C2 = {(O+I)/2,cpx.abs(I-O)/2}
local rep = ld.interCC(C1,C2) -- points of tangency
g:Dcircle(C1,"blue"); g:Dcircle(C2,"dashed")
g:Dhline(I,rep[1],"red"); g:Dhline(I,rep[2],"red") --half-tangents
g:Ddots(rep); g:Ddots({O,I}); g:Dlabel("$I$",I,{pos="SE"},"$O$",O,{pos="W"},
  "tangents to the circle arising from $I$",1-5*i,{pos="N"})
g:Restoreattr()
-- pour l'ellipse (E) : {O,3,2}
g:Saveattr(); g:Viewport(0,10,-5,5); g:Coordsystem(-4,6,-5,5)
local mat = {0,1.5,i} -- This matrix transforms a circle {O1,2} into the ellipse (E)
local inv_mat = ld.invmatrix(mat) -- inverse matrix
local O1, I1 = table.unpack( ld.mtransform({O,I},inv_mat) ) -- antecedents of O and I
C1 = {O1, 2}
C2 = {(O1+I1)/2,cpx.abs(I1-O1)/2}
rep = ld.interCC(C1,C2) -- points of tangency (tangents from I1)
g:Composematrix(mat) -- The matrix is applied to find the ellipse; the tangency is preserved.
g:Dcircle(C1,"blue"); g:Dcircle(C2,"dashed")
g:Dhline(I1,rep[1],'red'); g:Dhline(I1,rep[2],"red")
g:Ddots(rep); g:Ddots({O1,I1}); g:Dlabel("$I$",I1,{pos="SE"},"$O$",O1,{pos="W"},
  "tangents to the ellipse arising from $I$",1-5*i,{pos="N"})
g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 34: Tangents to a circle O,2 and to an ellipse O,3,2 from a point



9) interD

The function **ld.interD(d1, d2)** returns the intersection point of the lines ⟨*d1*⟩ and ⟨*d2*⟩. A line is a list of two complex numbers: a point on the line and a direction vector.

10) interDC

The function **ld.interDC(d, C)** returns the intersection of the line ⟨*d*⟩ with the circle ⟨*C*⟩, where `d={A,u}` (a line passing through A and directed by u), and `C={O,r}` (a circle with center O and radius r). The function returns a list containing 1 or 2 points if the intersection is not empty; otherwise, it returns `nil`.

11) interDL

The function **ld.interDL(d, L)** returns the list of intersection points between the straight line ⟨*d*⟩ and the polygonal line ⟨*L*⟩.

12) interL

The function **ld.interL(L1, L2)** returns the list of intersection points of the polygonal lines defined by ⟨*L1*⟩ and ⟨*L2*⟩. These two arguments are two lists of complex numbers or two lists of lists of complex numbers.

13) interP

The function **ld.interP(P1, P2)** returns the list of intersection points of the paths defined by ⟨*P1*⟩ and ⟨*P2*⟩. These two arguments are two lists of complex numbers and instructions (see *Dpath* page 41).

14) linspace

The function **ld.linspace(a, b [, nbdots])** returns a list of ⟨*nbdots*⟩ equally distributed numbers from ⟨*a*⟩ to ⟨*b*⟩. By default, ⟨*nbdots*⟩ is 50.

Another possible syntax: **ld.linspace(a1, b1, n1, b2, n2, ..., bp, np)** returns a list of ⟨*n1*⟩ evenly distributed numbers from ⟨*a1*⟩ to ⟨*b1*⟩, followed by ⟨*n2*⟩ evenly distributed numbers from ⟨*b1*⟩ to ⟨*b2*⟩ (but ⟨*b1*⟩ is not repeated), etc.

15) map

The function **ld.map(f, list)** applies the function ⟨*f*⟩ to each element of the ⟨*list*⟩ and returns the table of results. When a result is `nil`, the complex `cpx.Jump` is inserted into the list.

16) merge

The function **ld.merge(L [, epsilon])** reassembles, if possible, the connected components of ⟨*L*⟩, which must be a list of lists of complex numbers. The function returns a new polygonal line. Comparisons are made to the nearest ⟨*epsilon*⟩, by default we have ⟨*epsilon*⟩= 10^{-10}.

17) range

The function **ld.range(a, b [, step])** returns the list of numbers from ⟨*a*⟩ to ⟨*b*⟩ with a step equal to ⟨*step*⟩, which is 1 by default.

18) read_csv_file

The function **ld.read_csv_file(file, options)** reads a *csv* file. The argument ⟨*file*⟩ is a string representing the file with the extension: `"<name>.csv"`. The argument ⟨*options*⟩ is an table whose fields represent the options, which are (with their default values):

- `header=true`, with the value `true` this means that the first line of the file contains the column names.

- `dic=false`, with the value `true` this means that the returned result will be a list of dictionaries (one per line), the keys of these dictionaries being the column names. With the value `false` (default value), the result is a list of lists of values (one list of values per line). When the `header` option is set to `false`, the `dic` option automatically remains `false`.
- `sep=","`, a string indicating the separator for the values on each line.
- `num=true`, a boolean indicating whether the values should be automatically converted to numeric values (when this conversion fails, the value remains a string).
- `comment="%%"`, a string announcing a comment (a line starting with these characters is ignored). **Warning**: the search uses Lua's *match* method, for which the % character is a special character; it must be doubled ("%%") to be considered a character.

The result returned by this function is a sequence consisting, in this order, of:

1. A list of result lists (one result list per line).
2. The list of values in the first line when the `header` option is set to `true`.

19) Clipping Functions

- The function **ld.clipseg(A, B, xmin, xmax, ymin, ymax)** clips the segment $[A;B]$ (complex numbers) with the window $[x_{\min};x_{\max}]\times[y_{\min};y_{\max}]$ and returns the result.
- The function **ld.clipline(d, xmin, xmax, ymin, ymax)** clips the line $\langle d\rangle$ with the window $[x_{\min};x_{\max}]\times[y_{\min};y_{\max}]$ and returns the result. The line $\langle d\rangle$ is a list of two complex numbers: a point and a direction vector.
- The function **ld.clippolyline(L, xmin, xmax, ymin, ymax [, close])** clips the polygonal line $\langle L\rangle$ with the window $[x_{\min};x_{\max}]\times[y_{\min};y_{\max}]$ and returns the result. The argument $\langle L\rangle$ is a list of complex numbers or a list of lists of complex numbers. The optional argument $\langle close\rangle$ (`false` by default) indicates whether the polygonal line should be closed.
- The function **ld.clipdots(L, xmin, xmax, ymin, ymax)** clips the point list (complex numbers) $\langle L\rangle$ with the window $[x_{\min};x_{\max}]\times[y_{\min};y_{\max}]$ and returns the result (exterior points are simply excluded). The argument $\langle L\rangle$ is a list of complex numbers or a list of lists of complex numbers.

20) Adding Mathematical Functions

In addition to the functions associated with graphics methods that perform calculations and return a polygonal line (such as **ld.cartesian**, **ld.periodic**, **ld.implicit**, **ld.odesolve**, etc), the *luadraw* package adds some mathematical functions that are not natively provided in the *math* module.

Protected Evaluation: evalf

The **ld.evalf(f, ...)** function allows you to evaluate `f(...)` and return the result if there is no runtime error in Lua; otherwise, the function returns `nil`. For example, executing:

```
local Z = cpx.Z
local f = function(a,b)
    return 2*Z(a,1/b)
end
print(f(1,0))
```

causes the runtime error `attempt to perform arithmetic on a nil value` (in the console), because here `Z(1,1/0)` returns `nil`, and Lua does not accept an argument equal to `nil` in a calculation. On the other hand, executing:

```
local Z = cpx.Z
local f = function(a,b)
    return 2*Z(a,1/b)
end
print(ld.evalf(f,1,0))
```

does not cause an error from Lua, and there is no output to the console either since the value to be displayed is `nil`.

int

The function **ld.int(f, a, b)** returns an approximate value of the integral of the function ⟨*f*⟩ over the interval [*a*; *b*] (two complex numbers). The function ⟨*f*⟩ is a real variable and has real or complex values. The method used is Simpson's method, accelerated twice with the Romberg method.

Example:

```
$\int_0^1 e^{t^2}\mathrm d t \approx \directlua{tex.sprint(int(function(t) return math.exp(t^2) end, 0, 1))}$
```

Result: $\int_0^1 e^{t^2}\,\mathrm{d}t \approx 1.4626517459589$.

gcd

The function **ld.gcd(a, b)** returns the greatest common divisor between ⟨*a*⟩ and ⟨*b*⟩, two integers.

lcm

The function **ld.lcm(a, b)** returns the smallest positive common divisor between ⟨*a*⟩ and ⟨*b*⟩, two integers.

nth_root

The function **ld.nth_root(n, x)** returns the nth root of the real number ⟨*x*⟩, the argument ⟨*n*⟩ must be an integer.

solve

The function **ld.solve(f, a, b [, n, df])** numerically solves the equation $f(x) = 0$ within the interval [*a*; *b*], where ⟨*f*⟩ is the function and ⟨*a*⟩ and ⟨*b*⟩ are real numbers. The interval is subdivided into *n* segments (25 by default). The optional argument ⟨*df*⟩ can be:

- The string `"bisection"`, in which case the method used is the bisection method.
- A function representing the derivative of ⟨*f*⟩, in which case the method used is Newton's method.
- `nil` (the default value), in which case the method used is a variant of Newton's method.

The function returns a list of numbers or `nil`.

Example 1:

```
\begin{luacode}
local ld = luadraw
resol = function(f,a,b)
    local y = ld.solve(f,a,b)
    if y == nil then tex.sprint("\\emptyset")
    else
        local str = y[1]
        for k = 2, #y do
            str = str..", "..y[k]
        end
        tex.sprint(str)
    end
end
\end{luacode}
\def\solve#1#2#3{\directlua{resol(#1,#2,#3)}}%
\begin{luacode}
f1 = function(x) return math.cos(x)-x end
f2 = function(x) return x^3-2*x^2+1/2 end
\end{luacode}
Solving the equation $\cos(x)=x$ in $[0;\frac{\pi}2]$ gives $\solve{f1}{0}{math.pi/2}$.\par
Solving the equation $\cos(x)=x$ in $[\frac{\pi}2;\pi]$ gives $\solve{f1}{math.pi/2}{math.pi}$.\par
Solving the equation $x^3-2x^2+\frac12=0$ in $[-1;2]$ gives: $\{\solve{f2}{-1}{2}\}$.
```

Result:

Solving the equation $\cos(x) = x$ in $[0;\frac{\pi}{2}]$ gives 0.73908513321516.
Solving the equation $\cos(x) = x$ in $[\frac{\pi}{2};\pi]$ gives $\emptyset$.
Solving the equation $x^3 - 2x^2 + \frac{1}{2} = 0$ in $[-1;2]$ gives: $\{-0.45160596295578, 0.59696828323732, 1.8546376797185\}$.

Example 2: We want to plot the curve of the function f defined by the condition:

$$\forall x \in \mathbf{R},\ \int_x^{f(x)} \exp(t^2)\mathrm{d}t = 1.$$

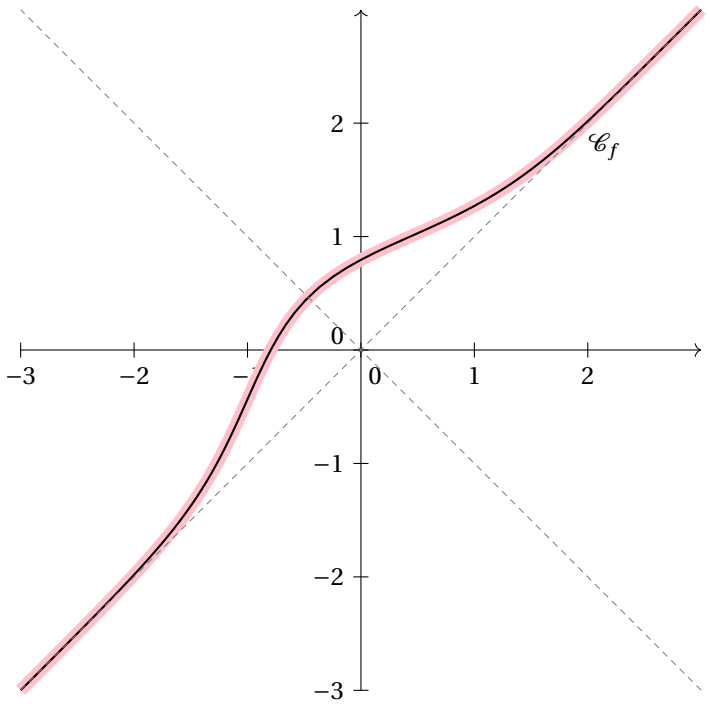
We have two possible methods:

1. We consider the function $G\colon (x,y) \mapsto \int_x^y \exp(t^2)\mathrm{d}t - 1$, and we draw the implicit curve with equation $G(x,y) = 0$.
2. We determine a real number y_0 such that $\int_0^{y_0} \exp(t^2)\mathrm{d}t = 1$ and we draw the solution to the differential equation $y' = e^{x^2-y^2}$ satisfying the initial condition $y(0) = y_0$.

Let's draw both:

```
\begin{luadraw}{name=int_solve}
local ld = luadraw
local Z = ld.cpx.Z
local g = ld.graph:new{window={-3,3,-3,3},size={10,10}}
local h = function(t) return math.exp(t^2) end
local G = function(x,y) return ld.int(h,x,y)-1 end
local H = function(y) return G(0,y) end
local F = function(x,y) return math.exp(x^2-y^2) end
local y0 = ld.solve(H,0,1)[1] -- solution of H(x)=0
g:Daxes({0,1,1}, {arrows="->"})
g:Dimplicit(G, {draw_options="line width=4.8pt,Pink"})
g:Dodesolve(F,0,y0,{draw_options="line width=0.8pt"})
g:Lineoptions("dashed","gray",4); g:DlineEq(1,-1,0); g:DlineEq(1,1,0) -- bisectors
g:Dlabel("${\\mathcal C}_f$",Z(2.15,2),{pos="S"})
g:Show()
\end{luadraw}
```

Figure 35: Function f defined by $\int_x^{f(x)} \exp(t^2)\mathrm{d}t = 1$.



We see that the two curves overlap well, however the first method (implicit curve) is much more computationally intensive, so method 2 is preferable.

V Transformations

In the following:

- the argument ⟨*L*⟩ is either a complex number, a list of complex numbers, or a list of lists of complex numbers,
- the line ⟨*d*⟩ is a list of two complex numbers: a point on the line and a direction vector.

1) affin

The function **ld.affin(L, d, v, k)** returns the image of ⟨*L*⟩ by the affinity of base line ⟨*d*⟩, parallel to the vector ⟨*v*⟩ and of ratio ⟨*k*⟩.

2) ftransform

The function **ld.ftransform(L, f)** returns the image of ⟨*L*⟩ by the function ⟨*f*⟩, which must be a function of the complex variable. If one of the elements of ⟨*L*⟩ is the complex number `cpx.Jump`, then it is returned as is in the result.

3) hom

The function **ld.hom(L, factor [, center])** returns the image of ⟨*L*⟩ by the homothety with center ⟨*center*⟩ and ratio ⟨*factor*⟩. By default, the argument ⟨*center*⟩ is 0.

4) inv

The function **ld.inv(L, radius [, center])** returns the image of ⟨*L*⟩ by the inversion with respect to the circle with center ⟨*center*⟩ and radius ⟨*radius*⟩. By default, the argument ⟨*center*⟩ is 0.

5) proj

The function **ld.proj(L, d)** returns the image of ⟨*L*⟩ by the orthogonal projection onto the line ⟨*d*⟩.

6) projO

The function **ld.projO(L, d, v)** returns the image of ⟨*L*⟩ by projection onto the line ⟨*d*⟩ parallel to the vector ⟨*v*⟩.

7) rotate

The function **ld.rotate(L, angle [, center])** returns the image of ⟨*L*⟩ by rotation with center *center* and angle ⟨*angle*⟩ (in degrees). By default, the argument ⟨*center*⟩ is 0.

8) shift

The function **ld.shift(L, u)** returns the image of ⟨*L*⟩ by translation of vector ⟨*u*⟩.

9) simil

The function **ld.simil(L, factor, angle [, center])** returns the image of ⟨*L*⟩ by the similarity of center ⟨*center*⟩, ratio ⟨*factor*⟩, and angle ⟨*angle*⟩ (in degrees). By default, the argument *center* is 0.

10) sym

The function **ld.sym(L, d)** returns the image of ⟨*L*⟩ by the orthogonal symmetry of axis ⟨*d*⟩.

11) symG

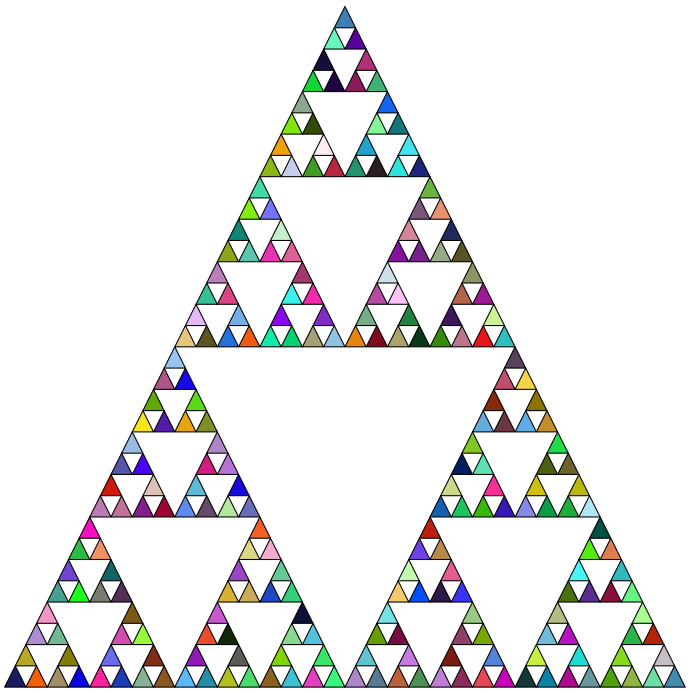
The function **ld.symG(L, d, v)** returns the image of ⟨*L*⟩ by the symmetry about the line ⟨*d*⟩ followed by the translation of vector ⟨*v*⟩ (sliding symmetry).

12) symO

The function **ld.symO(L, d, v)** returns the image of ⟨*L*⟩ by symmetry with respect to the line ⟨*d*⟩ and parallel to the vector ⟨*v*⟩ (oblique symmetry).

```
\begin{luadraw}{name=Sierpinski}
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5},size={10,10}}
local i, hom = ld.cpx.I, ld.hom
local rand = math.random
local A, B, C = 5*i, -5-5*i, 5-5*i -- triangle initial
local T, niv = {{A,B,C}}, 5
for k = 1, niv do
    T = ld.concat( hom(T,0.5,A), hom(T,0.5,B), hom(T,0.5,C) )
end
for _,cp in ipairs(T) do
    g:Filloptions("full", ld.rgb(rand(),rand(),rand()))
    g:Dpolyline(cp,true)
end
g:Show()
\end{luadraw}
```

Figure 36: Using Transformations



VI Matrix Calculus

If f is an affine application of the complex plane, we will call the list (table) of f the matrix:

`{f(0),Lf(1),Lf(i)}`

where Lf denotes the linear part of f (we have $Lf(1) = f(1) - f(0)$ and $Lf(i) = f(i) - f(0)$). The identity matrix is the global variable denoted `ID` in the *luadraw* package; it simply corresponds to the list $\{0, 1, i\}$.

1) Matrix Calculations

applymatrix and applyLmatrix

- The function **ld.applymatrix(z, M)** applies the matrix ⟨*M*⟩ to the complex ⟨*z*⟩ and returns the result (which is equivalent to calculating $f(z)$ if ⟨*M*⟩ is the matrix of f). When ⟨*z*⟩ is the complex `cpx.Jump` then the result is `cpx.Jump`. When ⟨*z*⟩ is a string then the function returns ⟨*z*⟩.

- The function **ld.applyLmatrix(z, M)** applies the linear part of the matrix ⟨*M*⟩ to the complex ⟨*z*⟩ and returns the result (which is equivalent to calculating $Lf(z)$ if ⟨*M*⟩ is the matrix of f). When ⟨*z*⟩ is the complex `cpx.Jump` then the result is `cpx.Jump`.

composematrix

The function **ld.composematrix(M1, M2)** performs the matrix product ⟨*M1*⟩×⟨*M2*⟩ and returns the result.

getmatrix

The function **ld.getmatrix(T1, T2)** calculates and returns the matrix that transforms triangle ⟨*T1*⟩ into triangle ⟨*T2*⟩, with ⟨*T1*⟩= $\{a_1, b_1, c_1\}$ (a list of three non-collinear 2D points) and ⟨*T2*⟩= $\{a_2, b_2, c_2\}$ (a list of three 2D points).

invmatrix

The function **ld.invmatrix(M)** calculates and returns the inverse of the matrix ⟨*M*⟩ when possible.

matrixof

- The function **ld.matrixof(f)** calculates and returns the matrix of ⟨f⟩ (which must be an affine application of the complex plane).
- Example: `ld.matrixof(function(z) return ld.proj(z,0,Z(1,-1)) end)` returns `{0,Z(0.5,-0.5),Z(-0.5,0.5)}` (matrix of the orthogonal projection onto the second bisector).

mtransform and mLtransform

- The function **ld.mtransform(L, M)** applies the matrix ⟨*M*⟩ to the list ⟨*L*⟩ and returns the result. ⟨*L*⟩ must be a list of complex numbers or a list of lists of complex numbers. If one of them is the complex `cpx.Jump` or a string, then it is returned as is.
- The function **ld.mLtransform(L, M)** applies the linear part of the matrix ⟨*M*⟩ to the list ⟨*L*⟩ and returns the result. ⟨*L*⟩ must be a list of complex numbers or a list of lists of complex numbers. If one of them is the complex `cpx.Jump` or a string, then it is returned as is.

2) Matrix associated with the graph

When creating a graph in the *luadraw* environment, for example:

```
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5},size={10,10}}
```

The created *g* object has a transformation matrix that is initially the identity. All graphics methods used automatically apply the graph's transformation matrix. This matrix is designated `g.matrix`, but to manipulate it, the following methods are available.

g:Composematrix()

The **g:Composematrix(M)** method multiplies the graph matrix *g* by the matrix ⟨*M*⟩ (with ⟨*M*⟩ on the right) and assigns the result to the graph matrix. The argument ⟨*M*⟩ must therefore be a matrix.

g:Det2d()

The **g:Det2d()** method returns 1 when the transformation matrix has a positive determinant, and −1 otherwise. This information is useful when we need to know whether the plane orientation has been changed or not.

g:IDmatrix()

The **g:IDmatrix()** method reassigns the identity to the graph matrix *g*.

g:Mtransform()

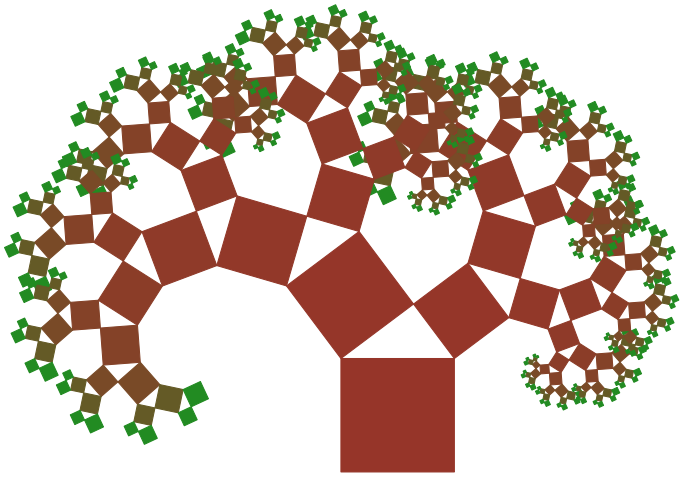
The **g:Mtransform(L)** method applies the graph matrix g to $\langle L \rangle$ and returns the result. The argument $\langle L \rangle$ must be a list of complex numbers, or a list of lists of complex numbers.

g:MLtransform()

The **g:MLtransform(L)** method applies the linear part of the graph matrix g to $\langle L \rangle$ and returns the result. The argument $\langle L \rangle$ must be a list of complex numbers, or a list of lists of complex numbers.

```
\begin{luadraw}{name=Pythagore}
local ld = luadraw
local g = ld.graph:new{window={-15,15,0,22},size={10,10}}
local a, b, c = 3, 4, 5 -- un triplet de Pythagore
local i, arccos, exp = ld.cpx.I, math.acos, ld.cpx.exp
local f1 = function(z)
    return (z-c)*a/c*exp(-i*arccos(a/c))+c+i*c end
local M1 = ld.matrixof(f1)
local f2 = function(z)
    return z*b/c*exp(i*arccos(b/c))+i*c end
local M2 = ld.matrixof(f2)
local arbre
arbre = function(n)
    local color = ld.mixcolor(ld.ForestGreen,1,ld.Brown,n)
    g:Linecolor(color); g:Dsquare(0,c,1,"fill="..color)
    if n > 0 then
        g:Savematrix(); g:Composematrix(M1); arbre(n-1)
        g:Restorematrix(); g:Savematrix(); g:Composematrix(M2)
        arbre(n-1); g:Restorematrix()
    end
end
arbre(8)
g:Show()
\end{luadraw}
```

Figure 37: Using the Graph Matrix



g:Rotate()

The **g:Rotate(angle [, center])** method modifies the transformation matrix of the graph g by composing it with the rotation matrix with angle $\langle angle \rangle$ (in degrees) and center $\langle center \rangle$. The argument $\langle center \rangle$ is a complex matrix that defaults to 0.

g:Scale()

The **g:Scale(factor [, center])** method modifies the transformation matrix of the graph *g* by composing it with the homothety matrix with ratio ⟨*factor*⟩ and center ⟨*center*⟩. The argument ⟨*center*⟩ is a complex matrix that defaults to 0.

g:Savematrix() and g:Restorematrix()

- The **g:Savematrix()** method saves the transformation matrix of graph *g* to a stack.
- The **g:Restorematrix()** method restores the transformation matrix of graph *g* to its last saved value.

g:Setmatrix()

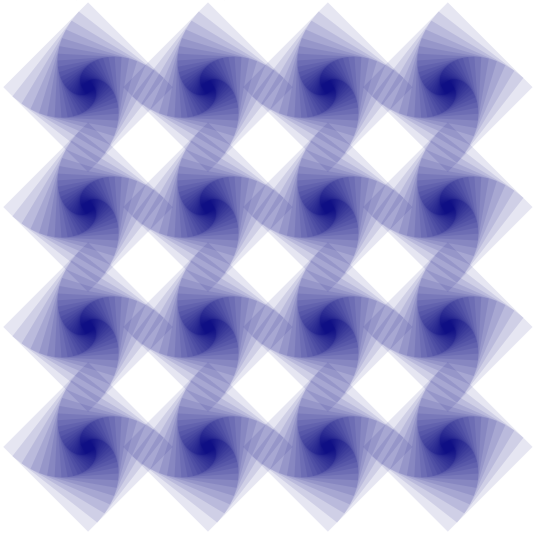
The **g:Setmatrix(M)** method assigns matrix ⟨*M*⟩ to the transformation matrix of graph *g*.

g:Shift()

The **g:Shift(v)** method modifies the transformation matrix of graph *g* by compositing it with the translation matrix of vector ⟨*v*⟩, which must be a complex matrix.

```
\begin{luadraw}{name=free_art}
local ld = luadraw
local du = math.sqrt(2)/2
local g = ld.graph:new{window={1-du,4+du,1-du,4+du},
    margin={0,0,0,0},size={7,7}}
local i = ld.cpx.I
g:Linestyle("noline")
g:Filloptions("full","Navy",0.1)
for X = 1, 4 do
    for Y = 1, 4 do
        g:Savematrix()
        g:Shift(X+i*Y); g:Rotate(45)
        for k = 1, 25 do
            g:Dsquare((1-i)/2,(1+i)/2,1)
            g:Rotate(7); g:Scale(0.9)
        end
        g:Restorematrix()
    end
end
g:Show()
\end{luadraw}
```

Figure 38: Using Shift, Rotate and Scale



3) View change. Change of coordinate system

View change: when creating a new graph, for example:

```
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5},size={10,10}}
```

The option `window={xmin,xmax,ymin,ymax}` sets the view for graph g. This will be the $[x_{\min};x_{\max}]\times[y_{\min};y_{\max}]$ block of $\mathbf{R}^2$, and all plots will be clipped by this window (except labels, which can overflow into the margins, but not beyond). Within this box, it is possible to define another box to create a new view, using the **g:Viewport(x1, x2, y1, y2)** method. The values of $\langle x1\rangle$, $\langle x2\rangle$, $\langle y1\rangle$, $\langle y2\rangle$ refer to the initial window defined by the `window` option. From then on, everything outside this new area will be clipped, and the graph matrix will be reset to the same value. Therefore, you must first save the current graphics settings:

```
g:Saveattr()
g:Viewport(x1,x2,y1,y2)
```

To return to the previous view with the previous matrix, simply restore the graphics settings with the **g:Restoreattr()** method.

Warning: Each **g:Saveattr()** instruction must correspond to a **g:Restoreattr()** instruction, otherwise a compilation error will occur.

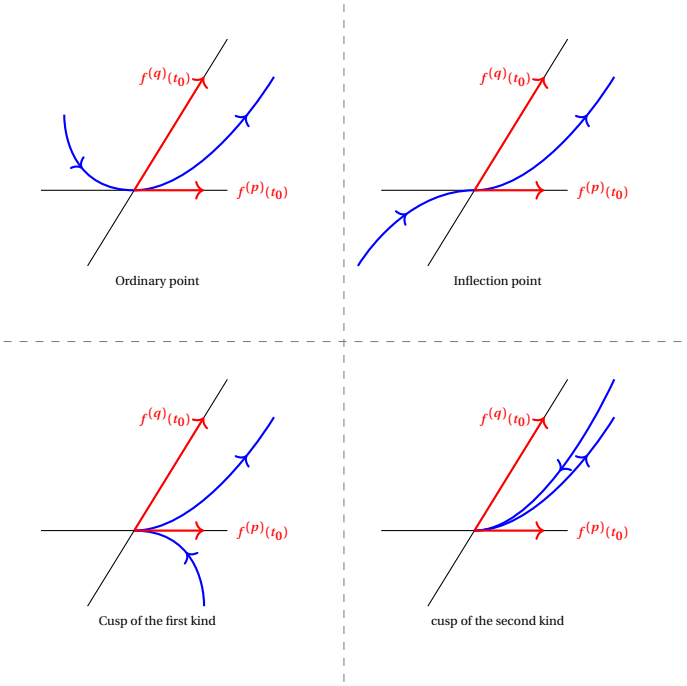
Changing the coordinate system: The coordinate system of the current view can be changed with

g:Coordsystem(x1, x2, y1, y2 [, ortho]).

This method will modify the graph matrix so that everything occurs as if the current view corresponded to the $[x_1;x_2]\times[y_1;y_2]$ box. The optional Boolean argument $\langle ortho\rangle$ indicates whether the new coordinate system should be orthonormal or not (`false` by default). Since the graph matrix is modified, it is best to save the graphical parameters first and restore them later. This can be used, for example, to create multiple figures in the current graph.

```
\begin{luadraw}{name=viewport_changewin}
local ld = luadraw
local g = ld.graph:new{window={-5,5,-5,5},size={10,10}}
local i, Z = ld.cpx.I, ld.cpx.Z
g:Labelsize("tiny")
local styleA = "\\tikzset{->-/.style={decoration={markings, mark="
local styleB = "at position #1 with {\\arrow{>}}}, postaction={decorate}}}"
g:Writeln(styleA..styleB)
g:Dline({0,1},"dashed,gray"); g:Dline({0,i},"dashed,gray")
local legende = {"Ordinary point", "Inflection point", "Cusp of the first kind", "cusp of the second kind"}
local A, B, C =(1+i)*0.75, 0.75, 0
local A2, B2 ={-1.25+i*0.5,-0.75-i*0.5,1.25-0.5*i, 0.5+i}, {-0.75,-0.75,0.75,0.75}
local u = {Z(-5,0),Z(0,0),-5-5*i,-5*i}
for k = 1, 4 do
    g:Saveattr(); g:Viewport(u[k].re,u[k].re+5,u[k].im,u[k].im+5)
    g:Coordsystem(-1.4,2.25,-1,1.25)
    g:Composematrix({0,1,1+i}) -- to tilt the Oy axis
    g:Dpolyline({{-1,1},{-i*0.5,i}}) -- axes
    g:Lineoptions(nil,"blue",8)
    g:Dpath({A2[k],(B2[k]+2*A2[k])/3,(C+5*B2[k])/6, C,"b"},"->-=0.5")
    g:Dpath({C,(C+5*B)/6,(B+2*A)/3,A,"b"},"->-=0.75")
    g:Dpolyline({{0,0.75},{0,0.75*i}},false,"->,red")
    g:Dlabel(
    legende[k],0.75-0.5*i, {pos="S"},
        "$f^{(p)}(t_0)$",1,{pos="E",node_options="red"},
        "$f^{(q)}(t_0)$",0.75*i,{pos="W",dist=0.05})
    g:Restoreattr()
end
g:Show()
\end{luadraw}
```

Figure 39: Classification of the points of a parametric curve



VII Adding Your Own Methods

Without having to modify the Lua source files associated with the *luadraw* package, you can add your own methods to the *ld.graph* class, or modify an existing method. This is only useful if these modifications will be used in different graphs and/or different documents (otherwise, you can simply write a function locally in the graph where it's needed).

1) An Example

In the graph on page 18, we drew a vector field. To do this, we wrote a function that calculates the vectors before drawing, but this function is local. We could make it a global function in the namespace *luadraw*, which would then be usable throughout the document, but not in another document!

To generalize this function, we will need to create a Lua file that can then be imported into documents if necessary. To make the example a bit more consistent, we'll create a file that defines a function that calculates the vectors of a field, and that will add two new methods to the *ld.graph* class: one to draw a vector field of a function $f:(x,y)\to f(x,y)\in\mathbf{R}^2$, we'll name it **ld.graph:Dvectorfield**, and another to draw a gradient field of a function $f:(x,y)\to f(x,y)\in\mathbf{R}$, we'll name it **ld.graph:Dgradientfield**. Therefore, we'll call this file: *luadraw_fields.lua*.

File contents:

```
-- luadraw_fields.lua
-- added methods to the graph class of the luadraw package
-- to draw vector or gradient fields
local ld = luadraw
local graph = ld.graph
local cpx = ld.cpx
local Z = cpx.Z

function ld.field(f,x1,x2,y1,y2,grid,long)  -- mathematical function, independent of the graph
-- calculates a vector field in the tile [x1,x2]x[y1,y2]
-- f function of two variables with values in R^2
-- grid = {nbx, nby} : number of vectors along x and along y
-- long = length of a vector
    if grid == nil then grid = {25,25} end
    local deltax, deltay = (x2-x1)/(grid[1]-1), (y2-y1)/(grid[2]-1) -- x and y step
    if long == nil then long = math.min(deltax,deltay) end -- default length
```

```
    local vectors = {} -- will contain the list of vectors
    local x, y, v = x1
    for _ = 1, grid[1] do -- a loop on x
        y = y1
        for _ = 1, grid[2] do -- a loop on y
            v = f(x,y) -- we assume that v is well defined
            v = Z(v[1],v[2]) -- to complex number
            v = cpx.normalize(v)
            if v ~= nil then
                table.insert(vectors, {Z(x,y)-long/2*v, Z(x,y)+long/2*v} ) -- we add the vector
            end
            y = y+deltay
        end
        x = x+deltax
    end
    return vectors -- we return the result (polygonal line)
end

function graph:Dvectorfield(f,args) -- added a method to the graph class
-- draws a vector field
-- f is a function of two variables with values in R^2
-- args is a 4-field table:
-- { view={x1,x2,y1,y2}, grid={nbx,nby}, long=, draw_options=""}
    args = args or {}
    local view = args.view or {self:Xinf(),self:Xsup(),self:Yinf(),self:Ysup()} -- default user reference
    local vectors = ld.field(f,view[1],view[2],view[3],view[4],args.grid,args.long) -- field calculation
    self:Dpolyline(vectors,false,args.draw_options) -- the drawing (non-closed polygonal line)
end

function graph:Dgradientfield(f,args) -- added another method to the graph class
-- draws a gradient field
-- f is a function of two variables with values in R
-- args is a 4-field table:
-- { view={x1,x2,y1,y2}, grid={nbx,nby}, long=, draw_options=""}
    local h = 1e-6
    local grad_f = function(x,y) -- gradient function of f
        return { (f(x+h,y)-f(x-h,y))/(2*h), (f(x,y+h)-f(x,y-h))/(2*h) }
    end
    self:Dvectorfield(grad_f,args) -- we use the previous method
end
```

2) How to import the file

There are two methods for this:

1. With the Lua instruction *dofile*. This can be written, for example, in the preamble after the package declaration:

   ```
   \usepackage[]{luadraw}
   \directlua{dofile("<path>/luadraw_fields.lua")}
   ```

   Of course, you will need to replace `<path>` with the path to this file.

   The instruction `\directlua{dofile("<path>/luadraw_fields.lua")}` can be placed elsewhere in the document, as long as it is after the package has been loaded (otherwise the *graph* class will not be recognized when reading the file). We can also place the instruction `dofile("<path>/luadraw_fields.lua")` in a *luacode* environment, and therefore in particular in a *luadraw* environment.

   As soon as the file is imported, the new methods are available for the rest of the document.

   This approach has at least two drawbacks: you need to remember the path each time you use it, and secondly, the *dofile* instruction does not check whether the file has already been read. For these reasons, we prefer the following method.

2. With the Lua instruction *require*. For example, we can write it in the preamble after the package declaration:

```
\usepackage[]{luadraw}
\directlua{require "luadraw_fields"}
```

Note the absence of the path (and the lua extension is unnecessary).

The `\directlua{require "luadraw_fields"}` instruction can be placed elsewhere in the document, provided it is after the package has been loaded (otherwise the *graph* class will not be recognized when reading the file). We can also place the `require "luadraw_fields"` instruction in a *luacode* environment, and therefore in particular in a *luadraw* environment.

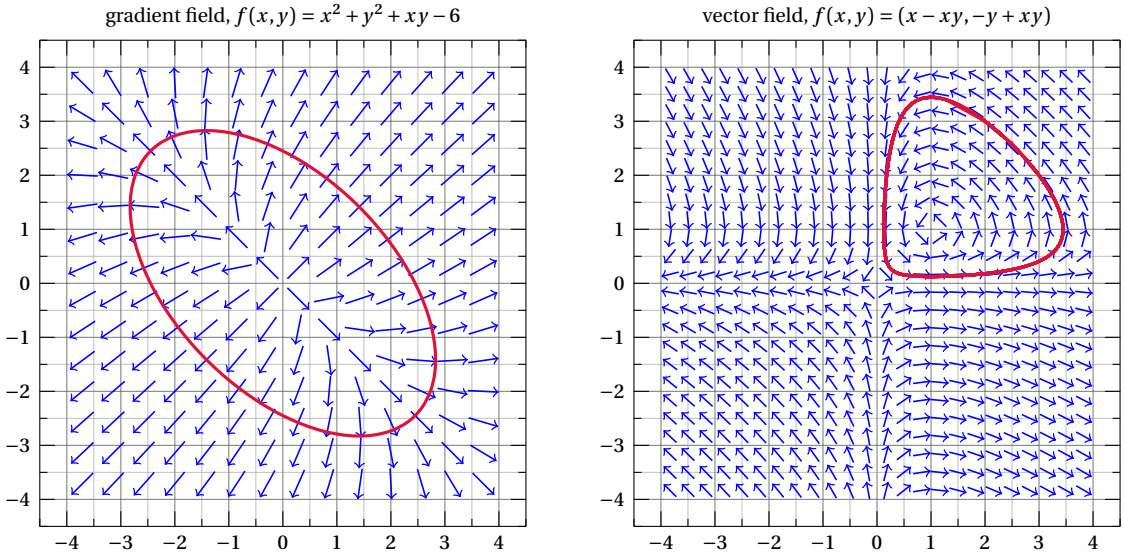
The *require* instruction checks whether the file has already been loaded or not, which is preferable. However, Lua must be able to find this file, and the easiest way to do this is for it to be somewhere in a tree structure known to TeX. For example, you can create the following path in your local *texmf*:

```
texmf/tex/lualatex/myluafiles/
```

then copy the file *luadraw_fields.lua* into the *myluafiles* folder.

```
\begin{luadraw}{name=fields}
require "luadraw_fields" -- import of new methods
local ld = luadraw
local g = ld.graph:new{window={0,21,0,10},size={16,10}}
local i = ld.cpx.I
g:Labelsize("footnotesize")
local f = function(x,y) return {x-x*y,-y+x*y} end -- Volterra
local F = function(x,y) return x^2+y^2+x*y-6 end
local H = function(t,Y) return f(Y[1],Y[2]) end
-- top
g:Saveattr();g:Viewport(0,10,0,10);g:Coordsystem(-5,5,-5,5)
g:Dgradbox({-4.5-4.5*i,4.5+4.5*i,1,1}, {originloc=0,originnum={0,0},grid=true,
    title="gradient field, $f(x,y)=x^2+y^2+xy-6$"})
g:Arrows("->"); g:Lineoptions(nil,"blue",6)
g:Dgradientfield(F,{view={-4,4,-4,4},grid={15,15},long=0.5})
g:Arrows("-"); g:Lineoptions(nil,"Crimson",12); g:Dimplicit(F, {view={-4,4,-4,4}})
g:Restoreattr()
-- bottom
g:Saveattr();g:Viewport(11,21,0,10);g:Coordsystem(-5,5,-5,5)
g:Dgradbox({-4.5-4.5*i,4.5+4.5*i,1,1}, {originloc=0,originnum={0,0},grid=true,
    title="vector field, $f(x,y)=(x-xy,-y+xy)$"})
g:Arrows("->"); g:Lineoptions(nil,"blue",6); g:Dvectorfield(f,{view={-4,4,-4,4}})
g:Arrows("-");g:Lineoptions(nil,"Crimson",12)
g:Dodesolve(H,0,{2,3},{t={0,50},out={2,3},nbdots=250})
g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 40: Using the new methods



3) Modifying an existing method

Let's take for example the method **g:DplotXY(X, Y [, draw_options])**, which takes two lists (tables) of real numbers as arguments and draws the polygonal line formed by the points with coordinates $(X[k], Y[k])$. We'll modify it to account for the case where $\langle X \rangle$ is a list of names (strings). In this case, the names will be displayed below the x-axis (with x-axis k for the kth name) and the polygonal line formed by the points with coordinates $(k, Y[k])$ will be drawn. Otherwise, we'll use the same method as the old method. To do this, simply rewrite the method (in a Lua file so that it can be imported later):

```
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

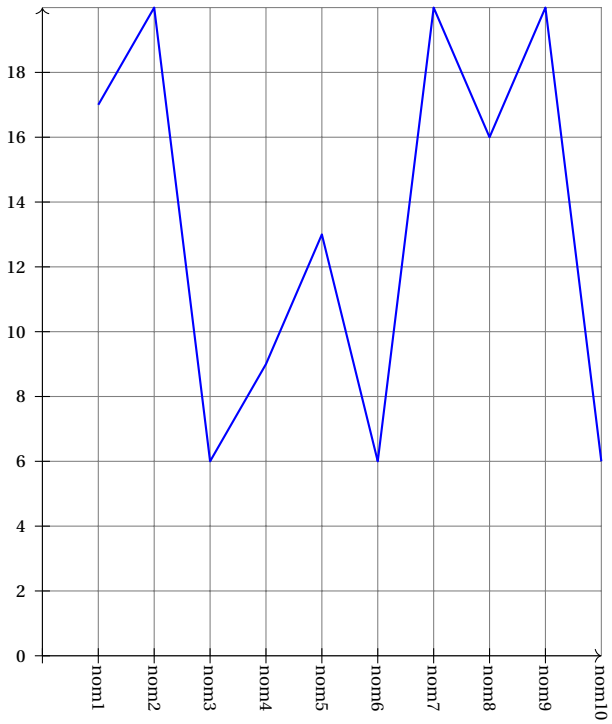
function ld.graph:DplotXY(X,Y,draw_options)
-- X is a list of real numbers or strings
-- Y is a list of real numbers of the same length as X
    local L = {} -- list of points to draw
    if type(X[1]) == "number" then -- list of real numbers
        for k,x in ipairs(X) do
            table.insert(L,Z(x,Y[k]))
        end
    else
        local noms = {} -- list of labels to place
        for k = 1, #X do
            table.insert(L,Z(k,Y[k]))
            insert(noms,{X[k],k,{pos="E",node_options="rotate=-90"}})
        end
        self:Dlabel(table.unpack(noms)) -- drawing labels
    end
    self:Dpolyline(L,draw_options) -- drawing the curve
end
```

As soon as the file is imported, this new definition will overwrite the old one (for the rest of the document). Of course, you could imagine adding other options to the drawing style, for example (lines, bars, dots, etc.).

```
\begin{luadraw}{name=newDplotXY}
require "luadraw_fields" -- import of the modified method
local ld = luadraw
local g = ld.graph:new{window={-0.5,11,-1,20}, margin={0.5,0.5,0.5,1}, size={10,10,0}}
g:Labelsize("scriptsize")
local X, Y = {}, {} -- we define two lists X and Y, we could also read them in a file
for k = 1, 10 do
    table.insert(X,"nom"..k)
```

```
    table.insert(Y,math.random(1,20))
end
g:Daxes({0,1,2},{limits={{0,10},{0,20}}, labelpos={"none","left"},
    labelshift={0,0}, originpos={"none","center"}, arrows="->", grid=true})
g:DplotXY(X,Y,"line width=0.8pt, blue")
g:Show()
\end{luadraw}
```

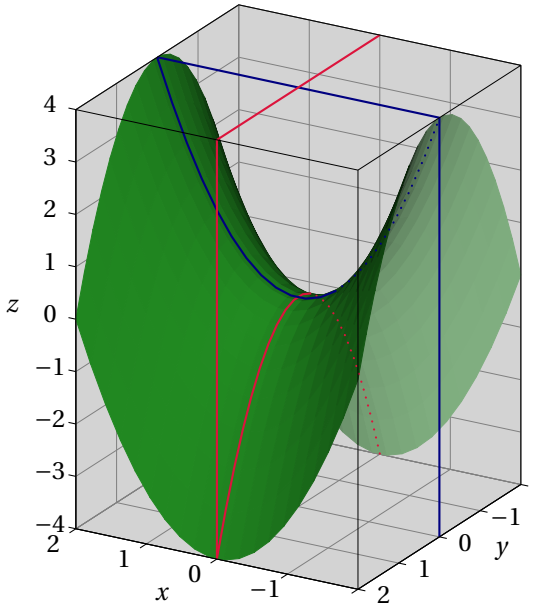
Figure 41: Modifying an existing method



Chapter 2

3D Drawing

Figure 1: Saddle point at $M(0,0,0)$ $(z = x^2 - y^2)$



I Introduction

As a reminder, we use the following shortcuts:

```
local ld = luadraw -- alias on the namespace
local cpx = ld.cpx -- shorthand for the cpx class
local pt3d = ld.pt3d -- shorthand for the pt3d class
```

1) Prerequisites

- This chapter presents the use of the *luadraw* package with the *3d* global option: `\usepackage[3d]{luadraw}`.
- The package loads the *luadraw_graph2d.lua* module, which defines the *ld.graph* class and provides the *luadraw* environment for creating graphs in Lua. Everything said in the previous chapter (Drawing 2D) therefore applies, and is assumed to be known here.
- The *3d* global option also allows the loading of the *luadraw_graph3d.lua* module. This also defines the *ld.graph3d* class (which relies on the *ld.graph* class) for 3D drawings.

2) Some reminders

- Another global package option: noexec. When this global option is mentioned, the default value of the exec option for the *luadraw* environment will be false (and no longer true).
- When a graph is finished, it is exported in TikZ format, so this package also loads the TikZ package and the libraries:
  - *patterns, patterns.meta*
  - *plotmarks*
  - *arrows.meta, bending*
  - *decorations.markings*
  - *backgrounds*
- Graphs are created in a *luadraw* environment, which calls *luacode*, so the **Lua language** must be used in this environment.
- Saving the **.tkz* file: the chart is exported in TikZ format to a file (with the *tkz* extension). By default, it is saved in the *_luadraw* folder, which is a subfolder of the current folder (containing the master document), but it is possible to specify a path to another subfolder, with the global option cachedir=....
- The environment options are:
  - name=...: allows you to name the resulting TikZ file. It is given a name without an extension (the extension will be automatically added; it is *.tkz*). If this option is omitted, then a default name is used, which is the name of the master file followed by a number.
  - exec=true/false: allows you to execute or not the Lua code included in the environment. By default, this option is true, **EXCEPT** if the global option noexec was mentioned in the preamble with the package declaration. When a complex graph that requires a lot of calculations is developed, it may be useful to add the option exec=false; this will avoid recalculating the same graph for future compilations.
  - auto=true/false: allows you to automatically include or not the TikZ file in place of the *luadraw* environment when the exec option is false. By default, the auto option is true.

3) Creating a 3D Graph

```
\begin{luadraw}{ name=<filename>, exec=true/false, auto=true/false }
local ld = luadraw
-- create a new graph and give it a local name
local g = ld.graph3d:new{ window3d={x1,x2,y1,y2,z1,z2 [,xscale,yscale,zscale]}, adjust2d=true/false, viewdir={30,60},
 ↪  window={x1,x2,y1,y2 [,xscale,yscale]}, margin={top,right,bottom,left}, size={width,height,ratio}, bg="color",
 ↪  border=true/false }
-- build graph g
graph instructions in Lua language ...
-- display graph g and save it in the file <filename>.tkz
g:Show()
-- or Save only in the <filename>.tkz file
g:Save()
\end{luadraw}
```

Important: throughout the rest of this chapter, points in space are called *3D points*, they are triplets of $\mathbf{R}^3$. In the *luadraw* environment, the 3D point (x,y,z) will be denoted **pt3d.M(x, y, z)** (**pt3d.M** is a function that creates and returns a 3D point).

Creation is done in a *luadraw* environment. This creation is done on the first line inside the environment by naming the graph:

```
local ld = luadraw
local g = ld.graph3d:new{ window3d={x1,x2,y1,y2,z1,z2 [,xscale,yscale,zscale]}, adjust2d=true/false, viewdir={30,60},
 ↪  window={x1,x2,y1,y2 [,xscale,yscale]}, margin={left,right,top,bottom}, size={width,height,ratio}, bg="color",
 ↪  border=true/false }
```

The *graph3d* class is defined in the *luadraw* package using the global option `3d`. This class is instantiated by invoking its constructor and giving it a name (here it's *g*). This is done locally so that the graph *g* thus created will no longer exist once it leaves the environment (otherwise *g* would remain in memory until the end of the document).

- The option `window3d={x1,x2,y1,y2,z1,z2 [,xscale,yscale,zscale]}` defines the $\mathbf{R}^3$ parallelepiped corresponding to the graph: it is $[x_1;x_2]\times[y_1;y_2]\times[z_1;z_2]$, as well as the scale on the three axes: *xscale*, *yscale*, and *zscale*, these are optional and default to 1. The default parallelepiped is $[-5;5]\times[-5;5]\times[-5;5]$.

  Caution: The three scales determine the initial 3D matrix of the graph. When one of them is not 1, this matrix is not the identity matrix. If you need to change the graph 3D matrix later, you must use the method **g:Composematrix3d()** and not **g:Setmatrix3d()**, and remember to save the initial matrix beforehand with the method **g:Savematrix()**, you can then restore it with the method **g:Restorematrix()**.

- The option `adjust2d` indicates whether the 2D window that will contain the projection of the 3D drawing should be determined automatically (`false` by default). This 2D window corresponds to the `window` option.

- The option `viewdir` is a table that defines the projection method and the two viewing angles (in degrees). By default `viewdir={"ortho",30,60}` (orthographic projection). The following figure shows what these two angles correspond to.

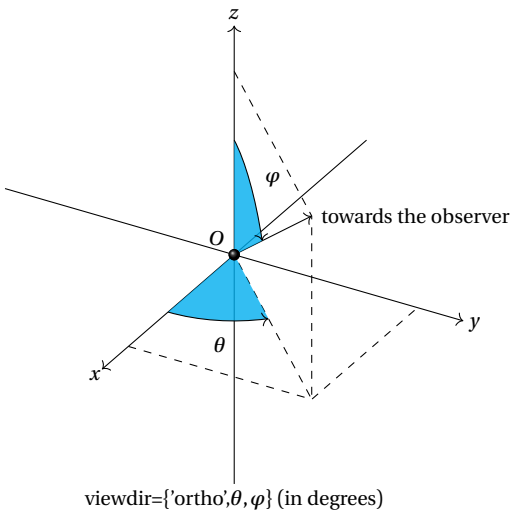


Figure 2: Viewing Angles

- The other options are those of the *ld.graph* class, described in Chapter 1.

Graph construction.

- The instantiated object (*g* in the example) has all the methods of the *ld.graph* class, plus methods specific to 3D.
- The *ld.graph3d* class also provides a number of mathematical functions specific to 3D.

4) Affine Projection Modes

- Orthogonal Projections:
  - `viewdir={"ortho",theta,phi}`, or `viewdir={theta,phi}`: orthographic projection (orthogonal projection onto the screen), this is the default projection with theta=30 and phi=60 (degrees),
  - `viewdir="xOy"`: projection onto the xy plane (observer on the positive z side),
  - `viewdir="yOx"`: projection onto the xy plane (observer on the negative z side),
  - `viewdir="xOz"`: projection onto the xz plane (observer on the positive y side),

  - `viewdir="zOx"`: projection onto the xz plane (observer on the y side) negative),
  - `viewdir="yOz"`: projection onto the yz plane (observer on the side of the positive x),
  - `viewdir="zOy"`: projection onto the yz plane (observer on the side of the negative x),

- Non-orthogonal projections:

  - `viewdir={"yz",k,alpha}`: cavalier perspective onto the yz plane (observer on the side of the positive x),
  - `viewdir={"zy",k,alpha}`: cavalier perspective onto the yz plane (observer on the side of the negative x),
  - `viewdir={"xz",k,alpha}`: cavalier perspective onto the xz plane (observer on the side of the positive y),
  - `viewdir={"zx",k,alpha}`: cavalier perspective on the xz plane (observer on the side of the negative y),
  - `viewdir={"xy",k,alpha}`: cavalier perspective on the xy plane (observer on the side of the positive z),
  - `viewdir={"yx",k,alpha}`: cavalier perspective on the xy plane (observer on the side of the negative z),
  - `viewdir="iso"`: isometric perspective (observer on the side of the positive x and y),
  - `viewdir="osi"`: isometric perspective (observer on the side of the negative x and y).

  Cavalier perspectives are defined using two parameters: a positive number k and an angle in degrees *alpha*, which are highlighted in the following figure.

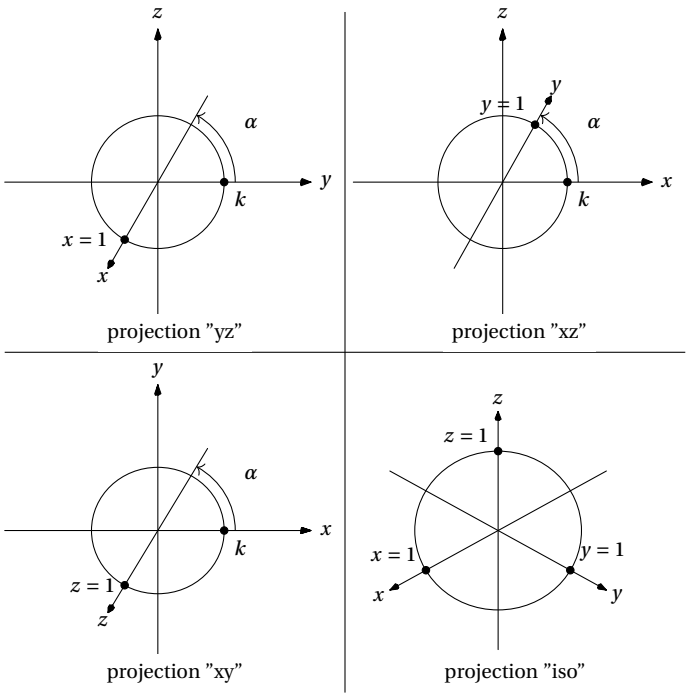


Figure 3: Affine Projection Modes

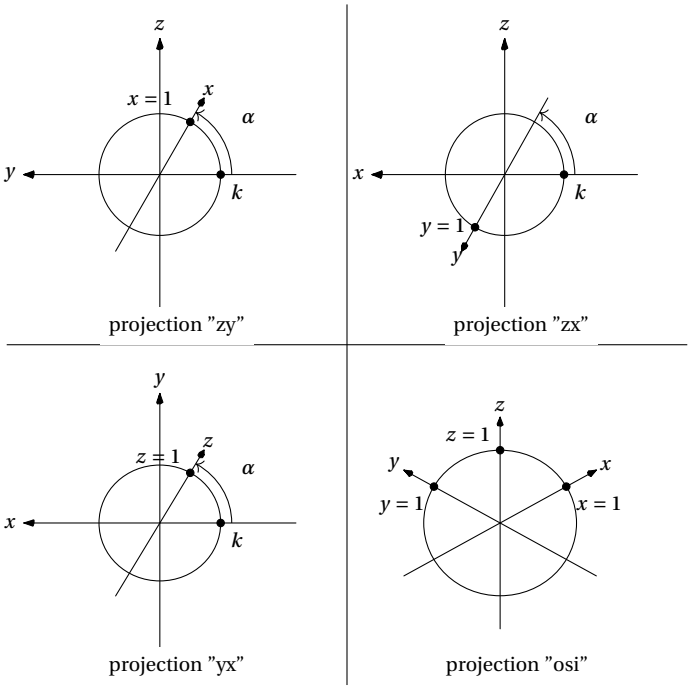


Figure 4: "Symmetrical" modes of the previous ones

All values that can be passed to the `viewdir` option during creation can also be used in the **g:Setviewdir()** method during graphing.

5) Central Projection

Since version 2.4, *luadraw* also offers central projection. Unlike previous modes, **this projection is not affine**, and furthermore, it is not defined for all points in space, which can lead to errors, thus requiring thought and adjustments. This projection is defined by:

- A camera, which is a point in space stored in a global variable called `ld.camera` and which should not be modified directly.
- A target, which is a point in space stored in a global variable called `ld.target` and which should not be modified directly.

The plane passing through the `ld.target` and orthogonal to the `ld.target` - `ld.camera` axis is the projection plane; it represents the screen. As with the previous modes, the central projection is accessed via the `viewdir` option, or the **g:Setviewdir** method:

viewdir = {"central" [, camera, target]}

or

viewdir = {"central" [, theta, phi, d, target]}

In the first case, the values of ⟨*camera*⟩ and ⟨*target*⟩ are given (3D points; by default, ⟨*target*⟩ is the origin). In the second case, the three arguments ⟨*theta*⟩, ⟨*phi*⟩, and ⟨*d*⟩ are used to calculate the position of the camera according to the following diagram:

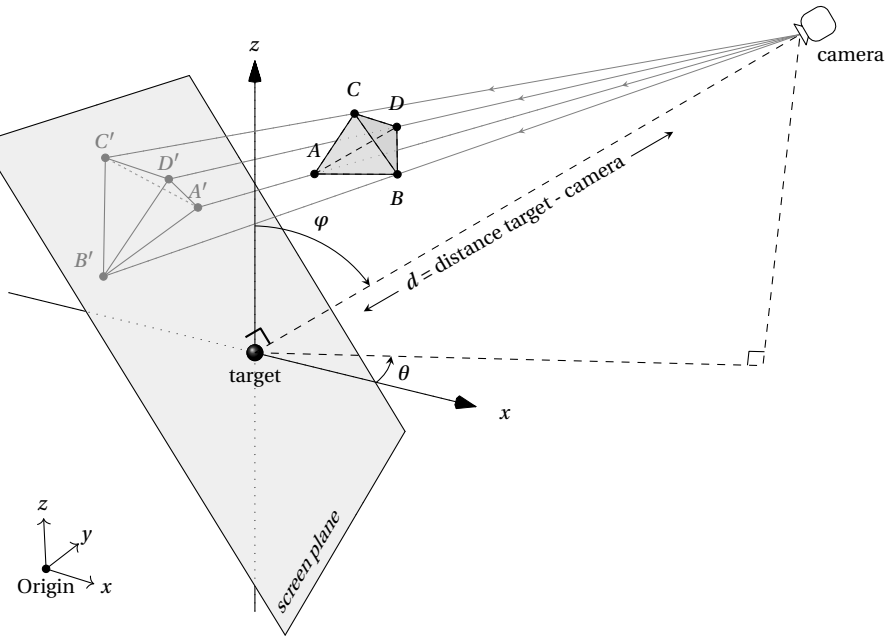


Figure 5: Central projection

The default values are: ⟨*theta*⟩=30 (degrees), ⟨*phi*⟩=60, ⟨*d*⟩=15, ⟨*target*⟩=`pt3d.Origin` (the variable `pt3d.Origin` is the 3D point $(0,0,0)$).

II The pt3d Class

1) Representation of Points and Vectors

The usual space is $\mathbf{R}^3$, so points and vectors are triplets of real numbers, we will call them: 3D points. The class *pt3d* (which is automatically loaded) allows you to manage 3D points, possible operations, and a number of methods and constants.

- Four triplets have specific names (global variables), namely:
  - `pt3d.Origin`, which represents the triplet $(0,0,0)$.
  - `pt3d.vecI`, which represents the triplet $(1,0,0)$.
  - `pt3d.vecJ`, which represents the triplet $(0,1,0)$.
  - `pt3d.vecK`, which represents the triplet $(0,0,1)$.
- To create a 3D point, there are three methods:
  - Cartesian definition: the function **pt3d.M(x, y, z)** returns the triplet (x,y,z). This triplet can also be obtained by doing: `x*vecI+y*vecJ+z*vecK`.
  - Cylindrical definition: the function **pt3d.Mc(r, theta, z)** (angle expressed in radians) returns the triplet $(r\cos(\theta), r\sin(\theta), z)$.
  - Spherical definition: the function **pt3d.Ms(r, theta, varphi)** returns the triplet $(r\cos(\theta)\sin(\varphi), r\sin(\theta)\sin(\varphi), r\cos(\varphi))$ (angles expressed in radians).

  Accessing the components of a 3D point: if a variable A denotes a 3D point, then its three components are $A.x$, $A.y$, and $A.z$.

  To test whether a variable A designates a 3D point, we use the **pt3d.isPoint3d()** function, which returns a Boolean.

  Conversion: To convert a real or complex number into a 3D point, we use the **pt3d.toPoint3d()** function.

2) Operations on 3D Points

These operations are the usual operations with the usual symbols:

- Addition (+), difference (-), and negative (-).

- The product by a scalar, if k is a real number, `k*M(x,y,z)` returns `M(k*x,k*y,k*z)`.
- A 3D point can be divided by a scalar; for example, if A and B are two 3D points, then the midpoint is simply written $(A+B)/2$.
- The equality of two 3D points can be tested with the symbol =.

3) Methods of the class *pt3d*

These are:

- **pt3d.abs(u)**: Returns the Euclidean norm of the 3D point ⟨u⟩.
- **pt3d.abs2(u)**: Returns the squared Euclidean norm of the 3D point ⟨u⟩.
- **pt3d.N1(u)**: Returns the 1-norm of the 3D point ⟨u⟩. If $u = M(x,y,z)$, then `pt3d.N1(u)` returns $|x|+|y|+|z|$.
- **pt3d.dot(u, v)**: Returns the dot product between the vectors (3d points) ⟨u⟩ and ⟨v⟩.
- **pt3d.det(u, v, w)**: Returns the determinant between the vectors (3D points) ⟨u⟩, ⟨v⟩, and ⟨w⟩.
- **pt3d.prod(u, v)**: Returns the cross product between the vectors (3D points) ⟨u⟩ and ⟨v⟩.
- **pt3d.angle3d(u, v [, epsilon])**: Returns the angular difference (in radians) between the vectors (3D points) ⟨u⟩ and ⟨v⟩, assumed to be non-zero. The (optional) argument ⟨*epsilon*⟩ is 0 by default; it indicates how close a given equality test is to a floating point.
- **pt3d.normalize(u)**: Returns the normalized vector (3D point) ⟨u⟩ (returns `nil` if u is zero).
- **pt3d.round(u, nbDeci)**: Returns a 3D point whose components are those of the 3D point ⟨u⟩ rounded to ⟨*nbDeci*⟩ decimal places.
- **pt3d.isobar3d(L)**: Returns the isobarycenter of the 3D points in the list (table) ⟨L⟩ (elements of ⟨u⟩ that are not 3D points are ignored).
- **pt3d.insert3d(L, A [, epsilon])**: This function inserts the 3D point ⟨A⟩ into the list ⟨L⟩, which must be a **variable** (and which will therefore be modified). Point ⟨A⟩ is inserted **without duplicates** and the function returns its position (index) in list ⟨L⟩ after insertion. The (optional) argument ⟨*epsilon*⟩ is 0 by default, indicating how closely the comparisons are made.

4) Displaying a Variable in the Terminal

The instruction **ld.whatis(variable [, msg])** displays the type of the ⟨*variable*⟩ and its contents in the terminal during compilation. Recognized types include the predefined types plus: *complex number*, *list of (complex) numbers*, and *list of lists of (complex) numbers*, *3D point*, *list of 3D points*, *list of lists of 3D points*. The argument ⟨*msg*⟩ is an optional string (empty by default) which is displayed with the type to locate the variable in the terminal.

III Graphics Methods

All 2D graphics methods apply. Added to this is the ability to draw polygonal lines, segments, straight lines, curves, paths, points, labels, planes, and solids in space. With solids, we also have the concept of facets, which was not found in 2D.

3D graphics methods will automatically calculate the projection onto the screen plane. After applying the 3D transformation matrix associated with the graphic (which is the default identity) to the objects, the 2D graphics methods will then take over.

The method that applies the 3D matrix and performs the projection onto the screen (plane passing through the origin and normal to the unit vector directed towards the observer and defined by the viewing angles) is: **g:Proj3d(L)** where ⟨L⟩ is either a 3D point, a list of 3D points, or a list of lists of 3D points. This function returns complex numbers (affixes of the projected points onto the screen).

Warning: when the 3D matrix of the graph is an affine transformation but not linear, the projection onto the screen of a vector u in space is not `g:Proj3d(u)`, but `g:Proj3d(A+u)-g:Proj3d(A)` where A denotes any point in space. To

avoid these calculations, the method **g:Proj3dV()** has been introduced, it projects the **vectors** onto the screen, and returns complex numbers (affixes of the projected vectors onto the screen).

Note on colors: In certain situations, the color specified by the user is slightly modified (for example, for facets depending on their inclination); consequently, the color must be either a color name, a table in `{r,g,b}` format, or a string containing a pre-modified color, provided the syntax is complete. For instance, the syntax `"orange!50"` is incomplete and may cause an error, whereas the correct syntax is `"orange!50!white"` (which will not cause an error).

1) Line Drawing

Polygonal Line: Dpolyline3d

The method **g:Dpolyline3d(L [, close, draw_options, clip])** (where g denotes the graph being created), ⟨*L*⟩ is a 3D polygonal line (list of 3D point lists), ⟨*close*⟩ is an optional argument that is `true` or `false` indicating whether the line should be closed or not (`false` by default), and ⟨*draw_options*⟩ is a string that will be passed directly to the *\draw* instruction in the export. The ⟨*clip*⟩ argument is set to `false` by default, it indicates whether the line ⟨*L*⟩ should be clipped to the current 3D window.

Right Angle: Dangle3d

The **g:Dangle3d(B, A, C [, r, draw_options, clip])** method draws the angle BAC with a parallelogram (only two sides are drawn). The optional argument ⟨*r*⟩ specifies the length of one side (0.25 by default). The parallelogram is in the plane defined by points ⟨*A*⟩, ⟨*B*⟩, and ⟨*C*⟩, so they should not be aligned. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The ⟨*clip*⟩ argument is set to `false` by default; it indicates whether the plot should be clipped to the current 3D window.

Segment: Dseg3d, Dmarkseg3d

The **g:Dseg3d(seg [, scale, draw_options, clip])** method draws the segment defined by the ⟨*seg*⟩ argument, which must be a list of two 3D points. The optional ⟨*scale*⟩ argument (1 by default) is a number that allows you to increase or decrease the length of the segment (the natural length is multiplied by ⟨*scale*⟩). The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The ⟨*clip*⟩ argument is set to `false` by default; it indicates whether the plot should be clipped to the current 3D window.

The method **g:Dmarkseg3d(A, B, V, n, [long, space, angle, draw_options])** marks the segment defined by the 3D points ⟨*A*⟩ and ⟨*B*⟩ with ⟨*n*⟩ small slanted segments of ⟨*angle*⟩ degrees (45° by default). The drawing is performed in the plane containing the points ⟨*A*⟩, ⟨*B*⟩ and the 3D vector ⟨*V*⟩ (which must therefore not be collinear with the vector $\vec{AB}$). By default, the length (argument ⟨*long*⟩) is 0.25, and the spacing (argument ⟨*space*⟩) is 0.125. The argument ⟨*draw_options*⟩ is a string (empty by default) which will be passed as is to the *\draw* instruction.

Line: Dline3d

The method **g:Dline3d(d [, draw_options, clip])** draws the line ⟨*d*⟩, which is a list of type ⟨*d*⟩={*A*, *u*} where A represents a point on the line (3D point) and u a direction vector (a non-zero 3D point).

Variant: the method **g:Dline3d(A, B [, draw_options, clip])** draws the line passing through the points ⟨*A*⟩ and ⟨*B*⟩ (two 3D points). The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The ⟨*clip*⟩ argument is set to `false` by default; it indicates whether the plot should be clipped to the current 3D window.

The **g:Line3d2seg(d [, scale])** method returns a table consisting of two 3D points representing a segment. This segment is the portion of the line ⟨*d*⟩ inside the current 3D window. The ⟨*scale*⟩ argument (1 by default) allows you to vary the size of this segment. When the window is too small, the intersection may be empty.

Circular arc: Darc3d

- The method **g:Darc3d(B, A, C, r, sens [, normal, draw_options, clip])** draws a circular arc with center ⟨*A*⟩ (3D point), radius ⟨*r*⟩, going from ⟨*B*⟩ (3D point) to ⟨*C*⟩ (3D point) in the forward direction if the argument ⟨*sens*⟩ is 1, and in the reverse direction otherwise. This arc is drawn in the plane containing the three points ⟨*A*⟩, ⟨*B*⟩, and ⟨*C*⟩. When these three points are aligned, the argument ⟨*normal*⟩ (non-zero 3D point) must be specified, which represents a vector normal to the plane. This plane is oriented by the vector product $\vec{AB} \wedge \vec{AC}$ or by the vector ⟨*normal*⟩ if it is specified.

The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The ⟨*clip*⟩ argument is set to `false` by default; it indicates whether the plot should be clipped to the current 3D window.

- The **ld.arc3d(B, A, C, r, sens [, normal])** function returns the list of points of this arc (3D polygonal line).
- The **ld.arc3db(B, A, C, r, sens [, normal])** function returns this arc as a 3D path (see *Dpath3d* page 93)) using Bézier curves.

Circle: Dcircle3d

- The method **g:Dcircle3d(I, R, normal [, draw_options, clip])** draws the circle with center ⟨*I*⟩ (3D point) and radius ⟨*R*⟩, in the plane containing ⟨*I*⟩ and normal to the vector defined by the argument ⟨*normal*⟩ (non-zero 3D point). The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The ⟨*clip*⟩ argument is set to `false` by default; it indicates whether the plot should be clipped to the current 3D window.

  Another possible syntax: **g:Dcircle3d(C [, draw_options, clip])** where ⟨*C*⟩={⟨*I*⟩,⟨*R*⟩,⟨*normal*⟩}.
- The **ld.circle3d(I, R, normal)** function returns the list of points on this circle (3D polygonal line).
- The **ld.circle3db(I, R, normal)** function returns this circle as a 3D path (see *Dpath3d* page 93) using Bézier curves.

Elliptical arc: Dellipticarc3d

- The method **g:Dellipticarc3d(B, A, C, r1, r2, sens, dir1, [, normal, draw_options, clip])** draws an elliptical arc centered at ⟨*A*⟩ (3D point) with radii ⟨*r1*⟩ and ⟨*r2*⟩, extending from ⟨*B*⟩ (3D point) to ⟨*C*⟩ (3D point) in the counter-clockwise direction if the argument ⟨*sens*⟩ is 1, or the clockwise direction otherwise. This arc is drawn in the plane containing the three points ⟨*A*⟩, ⟨*B*⟩, and ⟨*C*⟩; if these three points are collinear, the argument ⟨*normal*⟩ (a non-zero 3D point representing a vector normal to the plane) must be specified. This plane is oriented by the cross product $\vec{AB} \wedge \vec{AC}$ or by the vector ⟨*normal*⟩ if the latter is specified. The argument ⟨*dir1*⟩ is a direction vector for the axis carrying the radius ⟨*r1*⟩ (it must be orthogonal to the vector ⟨*normal*⟩); the second axis is determined by the cross product of ⟨*dir1*⟩ and ⟨*normal*⟩.

  The argument ⟨*draw_options*⟩ is a string (empty by default) that is passed as-is to the *\draw* command. The argument ⟨*clip*⟩ defaults to `false`; it indicates whether the drawing should be clipped to the current 3D window.
- The function **ld.ellipticarc3d(B, A, C, r1, r2, sens, dir1, [, normal])** returns the list of points defining this arc (as a 3D polyline).
- The function **ld.ellipticarc3db(B, A, C, r1, r2, sens, dir1, [, normal])** returns this arc as a 3D path (see *Dpath3d* on page 93) using Bézier curves.

Ellipse: Dellipse3d

- The method **g:Dellipse3d(C, r1, r2, dir1, normal [, draw_options, clip])** draws the ellipse centered at ⟨*C*⟩ (3D point) with radii ⟨*r1*⟩ and ⟨*r2*⟩. This ellipse is drawn in the plane containing ⟨*C*⟩ and orthogonal to the vector ⟨*normal*⟩ (a non-zero 3D point). The argument ⟨*dir1*⟩ is a direction vector for the axis corresponding to radius ⟨*r1*⟩ (it must be orthogonal to the vector ⟨*normal*⟩); the second axis is determined by the cross product of ⟨*dir1*⟩ and ⟨*normal*⟩.

  The argument ⟨*draw_options*⟩ is a string (empty by default) that is passed as-is to the *\draw* command. The argument ⟨*clip*⟩ defaults to `false`; it indicates whether the drawing should be clipped to the current 3D window.
- The function **ld.ellipse3d(C, r1, r2, dir1, normal)** returns the list of points defining this ellipse (as a 3D polyline).
- The function **ld.ellipse3db(C, r1, r2, dir1, normal)** returns this ellipse as a 3D path (see *Dpath3d* on page 93) using Bézier curves.

3D Path: Dpath3d

The method **g:Dpath3d(path [, draw_options, clip])** draws the ⟨*path*⟩. The ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed as is to the *\draw* instruction. The ⟨*clip*⟩ argument is set to `false` by default; it indicates whether the plot should be clipped to the current 3D window. The ⟨*path*⟩ argument is a list of 3D points followed by instructions (strings) that function **on the same principle as in 2D**. Instructions available and their syntax, the word *last* represents the last point of the previous piece:

- `p1,"m"` (moveto), which starts a new component of the path at the 3D point p_1.
- `p1,...,pn,"l"` (lineto) draws the 3D polygonal line `{last,p1,...,pn}`.
- `c1,c2,p2,"b"` (Bézier) draws the Bézier curve `{last,c1,c2,p2}`, where c_1 and c_2 are the two control 3D points.
- `p1,n,"c"` (circle) draws the circle centered at p_1 and passing through the point *last*, and normal to the 3D vector n.
- `p1,p2,r,sens,n,"ca"` (circle arc) draws a circular arc centered at p_1, with radius r, extending from *last* to p_2, in the clockwise direction when *sens*=1 (and therefore in the counterclockwise direction if *sens*=−1). The 3D vector n is optional; it indicates a normal vector to the plane of the circle when the points *last*, p_1, and p_2 are collinear (and in this case, the vector n is mandatory).
- `p1,r1,r2,dir1,n,"e"` (ellipse): draws an ellipse centered at p_1, passing through the point *last*, and normal to the 3D vector n. The two radii are r_1 and r_2, and *dir1* is a direction vector for the axis containing radius r_1. If the point *last* does not lie exactly on the ellipse, a line segment is drawn between that point and the ellipse.
- `p1,p2,r1,r2,sens,dir1,n,"ea"` (elliptical arc): draws an elliptical arc centered at p_1, extending from point *last* to point p_2, and normal to the 3D vector n. The two radii are r_1 and r_2, and *dir1* is a direction vector for the axis containing radius r_1. The 3D vector n is optional; it specifies a vector normal to the ellipse's plane and determines its orientation. This vector n is mandatory when the points *last*, p_1, and p_2 are collinear.
- `"cl"` (closepath) is used alone; it closes the current component by drawing a line segment connecting the last point to the first point (of the current component).

Here, for example, is the code in figure 2.

```
\begin{luadraw}{name=viewdir}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK
local g = ld.graph3d:new{ size={8,8} }
local i, M = ld.cpx.I, ld.pt3d.M
local O, A = Origin, M(4,4,4)
local B, C, D, E = ld.pxy(A), ld.px(A), ld.py(A), ld.pz(A) --projected from A onto the xOy plane and onto the axes
g:Dpolyline3d( {{O,A},{-5*vecI,5*vecI},{-5*vecJ,5*vecJ},{-5*vecK,5*vecK}}, "->") -- axes
g:Dpolyline3d( {{E,A,B,O}, {C,B,D}}, "dashed")
g:Dpath3d( {C,O,B,2.5,1,"ca",O,"l","cl"}, "draw=none,fill=cyan,fill opacity=0.8") --angular sector
g:Darc3d(C,O,B,2.5,1,"->") -- arc de cercle pour theta
g:Dpath3d( {E,O,A,2.5,1,"ca",O,"l","cl"}, "draw=none,fill=cyan,fill opacity=0.8") --angular sector
g:Darc3d(E,O,A,2.5,1,"->") -- arc of a circle for phi
g:Dballdots3d(O) -- the point of origin in the form of a small sphere
g:Labelsize("footnotesize")
g:Dlabel3d(
  "$x$", 5.25*vecI,{}, "$y$", 5.25*vecJ,{}, "$z$", 5.25*vecK,{},
  "towards the observer", A, {pos="E"},
  "$O$", O, {pos="NW"},
  "$\\theta$", (B+C)/2, {pos="N", dist=0.15},
  "$\\varphi$", (A+E)/2, {pos="S",dist=0.25})
g:Dlabel('viewdir=\\{"ortho",$\\theta,\\varphi$\\} (in degrees)',-5*i,{pos="N"}) -- label 2D
g:Show()
\end{luadraw}
```

Functions for 3D paths

- The function **ld.path3d(path [, nbdots])** returns a 3D polyline containing the points that make up the ⟨*path*⟩. The optional argument ⟨*nbdots*⟩ specifies the minimum number of points calculated for each Bézier curve within the path; it defaults to the global variable `ld.bezier_nbdots`, which is initialized to 12.
- Conversely, the function **ld.polyline2path3d(L [, close])**, where ⟨*L*⟩ is a 3D polyline (a list of 3D points or a list of lists of 3D points), returns ⟨*L*⟩ as a path. The optional argument ⟨*close*⟩ determines whether or not each component of the path is closed (default is `false`).
- The method **g:Convpath3d(path)** converts the ⟨*path*⟩ into another path using only the "m", "l", and "b" instructions (all other instructions are converted into Bézier curves). The function returns the new path; the advantage of this new path is that a 3D transformation matrix can be applied to it (using the **ld.mtransform3d()** function).
- The method **g:Path3d2path2d(path)** converts the 3D path ⟨*path*⟩ into a 2D path (representing the projection onto the screen) and returns the result.

Plane: Dplane

- The method **g:Dplane(P, V, L1, L2 [, mode, draw_options])** draws the edges of the plane $\langle P\rangle=\{A,u\}$ where A is a point in the plane and u is a normal vector to the plane (⟨*P*⟩ is therefore a table of two 3D points). The argument ⟨*V*⟩ must be a non-zero vector in the plane ⟨*P*⟩. The arguments ⟨*L1*⟩ and ⟨*L2*⟩ are two lengths. The method constructs a parallelogram centered on A, with one side $L_1\frac{V}{\|V\|}$ and the other $L_2\frac{W}{\|W\|}$ where $W=u\wedge V$. The argument ⟨*mode*⟩ is a natural number indicating the edges to draw. To calculate this integer, we use the predefined variables: `ld.top` (=8), `ld.right` (=4), `ld.bottom` (=2), `ld.left` (=1), and `ld.all` (=15), which can be added together, for example:
  - `mode=ld.bottom+ld.left`: for the bottom and left sides
  - `mode=ld.top+ld.right+ld.bottom`: for the top, right, and bottom sides
  - etc.

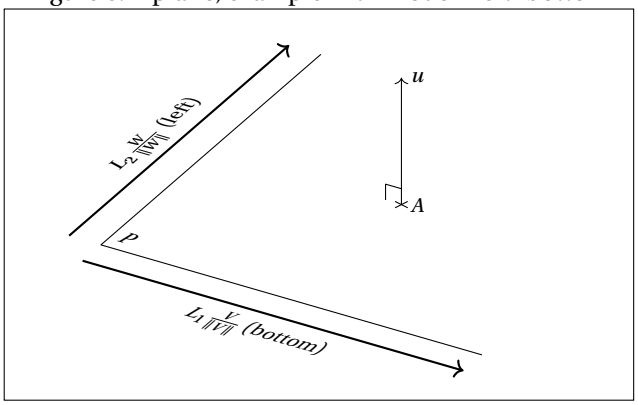
  By default, the mode is `ld.all`, which corresponds to `mode=ld.top+ld.right+ld.bottom+ld.left`.
- The function **ld.plane2rectangle(P [, V, L1, L2])** calculates and returns the rectangle (list of 3D points) drawn by the **g:Dplane()** method. The vector ⟨*V*⟩ is optional; it allows you to define an edge of the rectangle (it must lie on the plane). If it is omitted, the function will choose a vector V itself. The lengths ⟨*L1*⟩ and ⟨*L2*⟩ are optional and default to 5. When the argument ⟨*L2*⟩ is omitted, it implicitly has the same value as ⟨*L1*⟩. The result can be drawn using the **g:Dpolyline3d()** method, or drawn as a facet.

```
\begin{luadraw}{name=Dplane}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{size={8,8},window={-5.25,3,-2.5,2.5},margin={0,0,0,0},border=true}
local i = cpx.I
g:Labelsize("footnotesize")
local A = Origin
local P = {A, vecK}
g:Dplane(P, vecJ, 6, 6, ld.left + ld.bottom)
g:Dcrossdots3d({A,vecK},nil,0.75)
g:Dseg3d({A,A+2*vecK},"->")
g:Dangle3d(-vecJ,A,vecK,0.25)
g:Dpolyline3d({{M(3.5,-3,0),M(3.5,3,0)},{M(3,-3.5,0), M(-3,-3.5,0)}}, "->,line width=0.8pt")
g:Dlabel3d("$A$",A,{pos="E"},
    "$u$",2*vecK,{},
    "$P$", M(3,-3,0),{pos="NE", dir={vecJ,-vecI}},
    "$L_1\\frac{V}{\\|V\\|}$ (bottom)", M(3.5,0,0), {pos="S"},
    "$L_2\\frac{W}{\\|W\\|}$ (left)", M(0,-3.5,0), {pos="N",dir={-vecI,-vecJ}}
)
g:Show()
```

```
\end{luadraw}
```

Figure 6: Dplane, example with mode = left+bottom



Warning : The concepts of top, right, bottom, and left are relative! They depend on the direction of the vectors u (vector normal to the plane) and V (vector given in the plane). The third vector W is the cross product $u \wedge V$.

Parametric curve: Dparametric3d

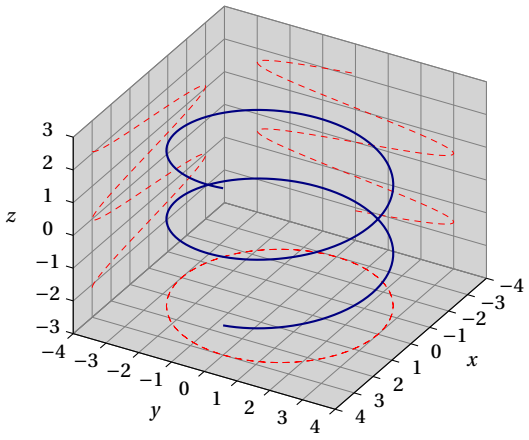
- The function **ld.parametric3d(p, t1, t2 [, nbdots, discont, nbdiv])** calculates the points of the curve and returns a 3D polygonal line (no drawing).
  - The argument ⟨*p*⟩ is the parameterization. It must be a function of a real variable t with values in $\mathbf{R}^3$ (the images are 3D points), for example: `local p=function(t) return Mc(3,t,t/3) end`.
  - The arguments ⟨*t1*⟩ and ⟨*t2*⟩ are mandatory with $t_1 < t_2$; they form the bounds of the interval for the parameter.
  - The argument ⟨*nbdots*⟩ is optional; it is the (minimum) number of points to calculate; it is 40 by default.
  - The argument ⟨*discont*⟩ is an optional boolean that indicates whether there are discontinuities or not. It is `false` by default.
  - The argument ⟨*nbdiv*⟩ is a positive integer equal to 5 by default and indicates the number of times the interval between two consecutive parameter values can be split in two (dichotomized) when the corresponding points are too far apart.
- The method **g:Dparametric3d(p, options)** calculates the points and draws the curve parameterized by ⟨*p*⟩. The argument ⟨*options*⟩ is a table specifying the possible options. Here are these options with their default values:
  - `t={g:Xinf(),g:Xsup()}`,
  - `nbdots=40`,
  - `discont=false`,
  - `nbdiv=5`,
  - `clip=false`, it indicates whether the curve should be clipped with the current 3D window.
  - `draw_options=""`, string that will be passed as is to the *\draw* instruction.

```
\begin{luadraw}{name=Dparametric3d}
local ld = luadraw
local pt3d = ld.pt3d
local vecI, vecJ, vecK, M, Mc = pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, pt3d.Mc

local g = ld.graph3d:new{window3d={-4,4,-4,4,-3,3}, window={-7.5,6.5,-7,6}, size={8,8}}
local pi = math.pi
g:Labelsize("footnotesize")
local p = function(t) return Mc(3,t,t/3) end
local L = ld.parametric3d(p,-2*pi,2*pi,25,false,2)
g:Dboxaxes3d({grid=true,gridcolor="gray",fillcolor="LightGray"})
g:Lineoptions("dashed","red",2)
```

```
-- projection onto the plane y=-4
g:Dpolyline3d(ld.proj3d(L,{M(0,-4,0),vecJ}))
-- projection onto the plane x=-4
g:Dpolyline3d(ld.proj3d(L,{M(-4,0,0),vecI}))
-- projection onto the plane z=-3
g:Dpolyline3d(ld.proj3d(L,{M(0,0,-3),vecK}))
-- curve drawing
g:Lineoptions("solid","Navy",8)
g:Dparametric3d(p,{t={-2*pi,2*pi}})
g:Show()
\end{luadraw}
```

Figure 7: A curve and its projections onto three planes



Parameterization of a Polygonal Line: *curvilinear_param3d*

Let L be a list of 3D points representing a continuous line. It is possible to obtain a parameterization of this line based on a parameter t between 0 and 1 (t is the curvilinear abscissa divided by the total length of L).

The function **ld.curvilinear_param3d(L [, close])** returns a function of one variable $t \in [0;1]$ and values on the line $\langle L \rangle$ (3D points). The value at $t = 0$ is the first point of $\langle L \rangle$, and the value at $t = 1$ is the last point; This function is followed by a number representing the total length of $\langle L \rangle$. The optional argument $\langle close \rangle$ indicates whether the line should be closed (`false` by default).

The reference: Dboxaxes3d

The **g:Dboxaxes3d(options)** method allows you to draw the three axes, with a number of options defined in the $\langle options \rangle$ table. These options are:

- `xaxe=true`, `yaxe=true`, and `zaxe=true`: Indicates whether the corresponding axes should be drawn or not.
- `drawbox=false`: Indicates whether a box should be drawn with the axes.
- `grid=false`: Indicates whether a grid should be drawn (one for x, one for y, and one for z). When this option is true, the following options can also be used:
  - `gridwidth=1`: Indicates the grid line thickness in tenths of a point,
  - `gridcolor="black"`: Indicates the grid color,
  - `fillcolor=""`: color to paint the grid background.
- `xlimits={x1,x2}`, `ylimits={y1,y2}`, `zlimits={z1,z2}`: Allows you to define the three intervals used for the axis lengths. By default, these are the values provided to the `window3d` argument when creating the graph.

- `xgradlimits={x1,x2}`, `ygradlimits={y1,y2}`, `zgradlimits={z1,z2}`: Allows you to define the three graduation intervals on the axes. By default, these options are set to `"auto"`, meaning they take the same values as `xlimits`, `ylimits`, and `zlimits`.
- `xyzstep=1`: Specifies the tick step on all three axes.
- `xstep=xyzstep`, `ystep=xyzstep`, `zstep=xyzstep`: Specifies the tick step on each axis (value of `xyzstep` by default).
- `xlabels={x1,…,xn}`, `ylabels={y1,…,yn}`, `zlabels={z1,…,zn}`: these options allow you to apply labels to the axes. By default, these options have the value `nil`, in which case the default labels (one per graduation mark) are displayed.
- `xyzticks=0.2`: Specifies the length of the tick marks.
- `labels=true`: Specifies whether or not to display the tick mark values.
- `xlabelsep=0.25`, `ylabelsep=0.25`, `zlabelsep=0.25`: Specifies the distance between the labels and the graduations.
- `xlabelstyle=<current style>`, `ylabelstyle=<current style>`, `zlabelstyle=<current style>`: Specifies the label style, i.e., the position relative to the anchor point. By default, the current style is applied.
- `xlegend="$x$"`, `ylegend="$y$"`, `zlegend="$z$"`: Allows you to define a legend for the axes.
- `xlegendsep=0.5`, `ylegendsep=0.5`, `zlegendsep=0.5`: Specifies the distance between the legends and the graduations.

Drawing on a Plane

It is possible to use 2D drawing methods on a plane P in space. To do this, you must first choose a Cartesian coordinate system (A,u,v) of the plane P.

- The method **g:BeginOnPlane(coordsystem [, options])** allows you to switch to "drawing on a plane" mode. The argument ⟨*coordsystem*⟩ is a table of the form {A,u,v} representing a Cartesian coordinate system of the chosen plane. Therefore, A is a 3D point belonging to the plane, and u and v are two 3D vectors forming a basis for the direction of the plane. The argument ⟨*options*⟩ is a table defining the options, which are (with their default values):
  - `labeldir=nil`, this option defines the writing direction. With the value `nil`, it's the current direction; with the value `"auto"`, the writing direction is determined by the vectors u and v. A different writing direction can also be imposed like this: `labeldir={dir1, dir2}`, where *dir1* and *dir2* are two independent 3D vectors.

    This option doesn't always give good results in central projection, because central projection is not an affine transformation.
  - `view=nil`, with the value `nil`, the viewport will be the same as the 2D viewport of the graph. With a value of the form `view={x1,x2,y1,y2}`, the drawing will be clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **in the coordinate system** (A,u,v).
  - `border=false` and `border_options=""`: if set to `true`, a frame is drawn around the area defined by the view; in this case, the `border_options` string is passed to the *\draw* command to draw the frame.
  - `tikzmatrix=true`, with the value `true` (default value), the method uses the *cm* option of TikZ (current matrix). With the value `false`, the matrix of *luadraw* is used, leaving the TikZ matrix unchanged.
  - `lowlevel=false`, with the value `true`, the `tikzmatrix` option also takes the value `true`, and the TikZ transformation matrix will be applied at the lowest level using the macro `\pgflowlevelsynccm`. This can be useful, for example, to draw arrows in the desired plane (and not the screen plane). However, this has a consequence: all graphic elements will be transformed: arrows, text, line thickness, etc.
  - `out=nil`, if the user assigns this option a variable of type table (`out=<var>`), then they can retrieve a 2D matrix in this variable that redefines the axes to correspond to those of the chosen plane. This matrix is particularly useful for using the **g:Dimage()** method on this plane.

  Once this method is executed, any complex number $z=Z(a,b)$ will actually correspond to the 3D point $A+a\cdot u+b\cdot v$.

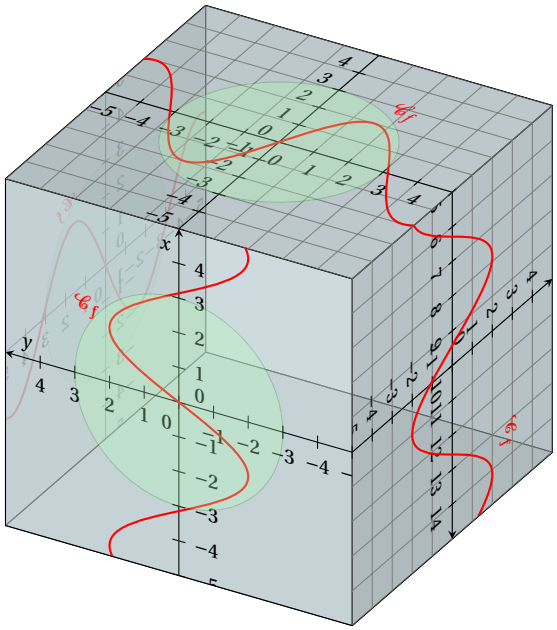
  Note: This method will not yield the expected result in central projection mode, as this is not an affine transformation.

- The method **g:EndOnPlane()** terminates the ”drawing on a plane” mode.

```
\begin{luadraw}{name=draw_on_plane}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Z, M = cpx.Z, pt3d.M
local Origin, vecI, vecJ, vecK = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK

local g = ld.graph3d:new{ window={-8,8,-9,8}, size={10,10}}
g:Labelsize("footnotesize")
local function f(x) return 2*math.sin(x) end
-- left
g:BeginOnPlane( {M(0,-5,0), vecI, vecK}, {labeldir="auto", view={-5,5,-5,5}})
    g:Daxes(nil, {limits={{-5,5},{-5,5}}})
    g:Dcartesian(f, {x={-5,5}, draw_options="red,thick"})
    g:Dcircle(0,3, "ForestGreen, fill=green!30,opacity=0.3")
    g:Dlabel("$\\mathcal{C}_{f}$", Z(2,2),{pos="N",node_options="red"})
g:EndOnPlane()
-- cube
g:Dpoly( g:Box3d(), {color="LightBlue!50!white", opacity=0.6})
-- top
g:BeginOnPlane( {M(0,0,5), vecJ, -vecI}, {labeldir="auto", view={-5,5,-5,5}})
    g:Daxes(nil, {limits={{-5,5},{-5,5}}, grid=true})
    g:Dcartesian(f, {x={-5,5}, draw_options="red,thick"})
    g:Dcircle(0,3, "ForestGreen, fill=green!30,opacity=0.3")
    g:Dlabel("$\\mathcal{C}_{f}$", Z(2,2),{pos="N",node_options="red"})
g:EndOnPlane()
-- front
g:BeginOnPlane( {M(5,0,0), vecK, -vecJ}, {labeldir={vecJ,vecK}, view={-5,5,-5,5}})
    g:Daxes(nil, {limits={{-5,5},{-5,5}}, labelstyle={"E","S"}, arrows="-stealth",
        legend={"$x$","$y$"}, legendpos={0.9,0.98}})
    g:Dcartesian(f, {x={-5,5}, draw_options="red,thick"})
    g:Dcircle(0,3, "ForestGreen, fill=green!30,opacity=0.3")
    g:Dlabel("$\\mathcal{C}_{f}$", Z(2,2),{pos="W",node_options="red"})
g:EndOnPlane()
-- right
g:BeginOnPlane( {M(0,5,10), -vecK, -vecI}, {labeldir="auto", view={5,15,-5,5}})
    g:Daxes({10,1,1}, {grid=true, arrows="-stealth", labelshift={0,0}})
    g:Dcartesian(f, {x={5,15}, draw_options="red,thick"})
    g:Dlabel("$\\mathcal{C}_{f}$", Z(14,2),{pos="NW",node_options="red"})
g:EndOnPlane()
g:Show()
\end{luadraw}
```

Figure 8: Draw on a plane



2) Points and Labels

3D Points: Ddots3d, Dballdots3d, Dcrossdots3d

There are three ways to draw 3D points. For the first two, the argument ⟨*L*⟩ can be either a single 3D point, a list (a table) of 3D points, or a list of lists of 3D points:

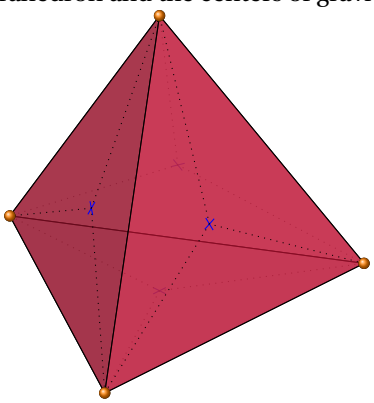
- The **g:Ddots3d(L [, mark_options, clip])** method. The principle is the same as in the 2D version; the points are drawn in the current line color with the current style. The ⟨*mark_options*⟩ argument is an optional string that will be passed as is to the *\draw* instruction (local modifications). The ⟨*clip*⟩ argument is set to `false` by default; it indicates whether the plot should be clipped to the current 3D window.
- The **g:Dballdots3d(L [, color, scale, clip])** method draws the points of ⟨*L*⟩ as a sphere. The optional ⟨*color*⟩ argument specifies the color of the sphere (`"black"` by default), and the optional ⟨*scale*⟩ argument allows you to change the size of the sphere (1 by default).
- The **g:Dcrossdots3d(L [, color, scale, clip])** method draws the points of ⟨*L*⟩ as a plane cross. The argument ⟨*L*⟩ is a list of the form {3D point, normal vector} or { {3D point, normal vector}, {3D point, normal vector}, ...}. For each 3D point, the associated normal vector is used to determine the plane containing the cross. The optional ⟨*color*⟩ argument specifies the color of the cross (`"black"` by default), and the optional ⟨*scale*⟩ argument allows you to change the size of the cross (1 by default).

```
\begin{luadraw}{name=Ddots3d}
local ld = luadraw
local pt3d = ld.pt3d
local vecI, vecJ, vecK, M = pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{viewdir={15,60},bbox=false,size={8,8}}
local A, B, C, D = 4*M(1,0,-0.5), 4*M(-1/2,math.sqrt(3)/2,-0.5), 4*M(-1/2,-math.sqrt(3)/2,-0.5), 4*M(0,0,1)
local u, v, w = B-A, C-A, D-A
-- drawing of the center of gravity of hidden faces
for _, F in ipairs({{A,B,C},{B,C,D}}) do
local G, u = pt3d.isobar3d(F), pt3d.prod(F[2]-F[1],F[3]-F[1])
g:Dcrossdots3d({G,u}, "blue",0.75)
g:Dpolyline3d({{F[1],G,F[2]},{G,F[3]}},"dotted")
end
```

```
-- drawing of the tetrahedron constructed on A, B, C and D
g:Dpoly( ld.tetra(A,u,v,w),{mode=ld.mShaded,opacity=0.7,color="Crimson"})
-- drawing of the center of gravity of visible faces
for _, F in ipairs({{A,B,D},{A,C,D}}) do
    local G, u = pt3d.isobar3d(F), pt3d.prod(F[2]-F[1],F[3]-F[1])
    g:Dcrossdots3d({G,u}, "blue",0.75)
    g:Dpolyline3d({{F[1],G,F[2]},{G,F[3]}},"dotted")
end
g:Dballdots3d({A,B,C,D}, "orange") --vertices
g:Show()
\end{luadraw}
```

Figure 9: A tetrahedron and the centers of gravity of each face



3D Labels: Dlabel3d

The method for placing a label in space is:

g:Dlabel3d(text1, anchor1, options1, text2, anchor2, options2, ...).

- The arguments ⟨*text1*⟩, ⟨*text2*⟩, ..., are strings; they are the labels.
- The arguments arguanchor1, ⟨*anchor2*⟩, ..., are 3D points representing the anchor points of the labels.
- The arguments ⟨*options1*⟩, ⟨*options2*⟩, ..., allow you to locally define the label options. These options are (with their default value)!
  - `pos="center"`: indicates the label's position in the screen plane relative to the anchor point. It can be `"N"` for north, `"NE"` for northeast, `"NW"` for northwest, or `"S"`, `"SE"`, `"SW"`. By default, it is `"center"`, and in this case, the label is centered on the anchor point.
  - `dist=0`: distance in cm between the label and its anchor point when `pos` is not equal to `"center"`.
  - `dir={}`: indicates the direction of writing in space, the empty list means the default direction. In general, `dir={dirX,dirY,dep}`, the three values *dirX*, *dirY* and *dep* are three 3D points representing three vectors. The first two indicate the direction of writing, the third a displacement (translation) of the label relative to the anchor point.
  - `node_options=""`: string intended to receive options that will be passed directly to TikZ in the *\node[]* instruction.
  - The labels are drawn in the current color of the document text, but the color can be changed with the `node_options` argument, for example, by setting: `node_options="color=blue"`.

    Warning: The options chosen for a label also apply to subsequent labels if they are unchanged.

3) Basic Solids (Without Facets)

The four methods described below are sensitive to the global variable `ld.Hiddenlines` which is `false` by default.

Cylinder: Dcylinder

Draw a cylinder with a circular base (right or tilted). Several possible syntaxes:

- Old syntax: **g:Dcylinder(A, V, r [, options])** draws a right cylinder, where $\langle A \rangle$ is a 3D point representing the center of one of the circular faces, $\langle V \rangle$ is a 3D point, a vector representing the axis of the cylinder, the center of the opposite circular face is the point $A + V$ (this face is orthogonal to V), and $\langle r \rangle$ is the radius of the circular base.
- Syntax: **g:Dcylinder(A, r, B [, options])** draws a right cylinder, where $\langle A \rangle$ is a 3D point representing the center of one of the circular faces, $\langle B \rangle$ is the center of the opposite face, and $\langle r \rangle$ is the radius. The cylinder is right, meaning that the circular faces are orthogonal to the axis (AB).
- For a tilted cylinder: **g:Dcylinder(A, r, V, B [, options])**, where $\langle A \rangle$ is a 3D point representing the center of one of the circular faces, $\langle B \rangle$ is the center of the opposite circular face, $\langle r \rangle$ is the radius, and $\langle V \rangle$ is a non-zero 3D vector orthogonal to the plane of the circular faces.

For all three syntaxes, $\langle options \rangle$ is a table whose fields define the options. These options are (with their default values):

- `mode=ld.mWireframe`, two possible values, `ld.mGrid` or `ld.mWireframe`. In `ld.mWireframe` mode, this is a wireframe drawing; in `ld.mGrid` mode, this is a grid drawing (as if there were facets).
- `hiddenstyle=ld.Hiddenlinestyle`, sets the line style for hidden areas (set it to `"noline"` to hide them). By default, this option has the value of the global variable `ld.Hiddenlinestyle`, which is itself initialized with the value `"dotted"`.
- `hiddencolor=edgecolor`, sets the color of hidden lines. By default, it is the same value as the `edgecolor` option.
- `edgecolor=<current color>`, sets the color of the lines.
- `edgestyle=<current style>`, defines the line style for visible edges.
- `edgewidth=<current width>`, sets the thickness of the visible edges in tenths of a point.
- `color=""`, when this option is an empty string (the default value), there is no fill; when it is a color (as a string), there is a fill with or without a linear gradient.
- `gradient=true`, a boolean indicating whether a gradient fill should be used when a color has been specified. With the value `false`, a continuous fill is used.
- Gradient parameters: when there is a color fill and `gradient=true`, a linear gradient is used for the side of the cylinder and another for the cross-section; in both cases, the gradient is defined as follows::

  `left color=<color>!<l>, right color=<color>!<r>, middlecolor=<color>!<m>`

  where *<color>* denotes the color used, and *<l>*, *<m>*, *<r>* are three numbers between 0 and 100, these three numbers constitute the parameters of the gradient in the form of a table *{<l>,<m>,<r>}*:

  - For the side of the cylinder, it's the `gradside` option, by default: `gradside={50,10,100}`.
  - For the cylinder section, it's the `gradsection` option, by defaut : `gradsection={25,18,50}`.
- `opacity=1`, sets the transparency of the drawing.

Cone: Dcone

Draw a circular cone (right or inclined). Several possible syntaxes:

- Old syntax: **g:Dcone(A, V, r [, options])** draws a right cone, where $\langle A \rangle$ is a 3D point representing the cone's vertex, $\langle V \rangle$ is a 3D point, a vector representing the cone's axis, the center of the circular face is point $A + V$ (this face is orthogonal to $\langle V \rangle$), and $\langle r \rangle$ is the radius of the circular base.
- Syntax: **g:Dcone(C, r, A [, options])** draws a right cone, where $\langle A \rangle$ is a 3D point representing the cone's vertex, $\langle C \rangle$ is the center of the circular face, and $\langle r \rangle$ is the radius. The cone is right, meaning that the circular face is orthogonal to the axis (AC).

- For a tilted cone: **g:Dcone(C, r, V, A [, options])**, where $\langle A \rangle$ is a 3D point representing the cone's vertex, $\langle C \rangle$ is the center of the circular face, $\langle r \rangle$ is the radius, and $\langle V \rangle$ is a non-zero 3D vector orthogonal to the plane of the circular face.

For all three syntaxes, the $\langle options \rangle$ are the same as for the cylinder, plus the following option:

- `apex=true` : with the value `true`, the apex is marked by a point when the cone is viewed from above.

Frustum: Dfrustum

Draw a truncated cone with a circular base (right or slanted). Two possible syntaxes:

- The syntax: **g:Dfrustum(A, R, r, B [, options])** for a right truncated cone, $\langle A \rangle$ is a 3D point representing the center of the face with radius $\langle R \rangle$, $\langle B \rangle$ is the center of the second circular face, and its radius is $\langle r \rangle$, the faces are orthogonal to the vector $\vec{AB}$. When $R = r$ we simply have a cylinder.
- Syntax: **g:Dfrustum(A, R, r, V, B [, options])** for a tilted cone frustum, $\langle A \rangle$ is a 3D point representing the center of the face with radius $\langle R \rangle$, $\langle V \rangle$ is a 3D vector representing a normal vector to the circular faces, the center of the second circular face is point $\langle B \rangle$, and its radius is $\langle r \rangle$. When $R = r$ we have a tilted cylinder.

For all two syntaxes, the $\langle options \rangle$ are the same as for the cylinder, plus the option:

- `old=false`: with the value `true`, the old syntax **g:Dfrustum(A, R, r, V [, options])** can be used; in this case, the center of the second circular face is $A + V$.

Sphere: Dsphere

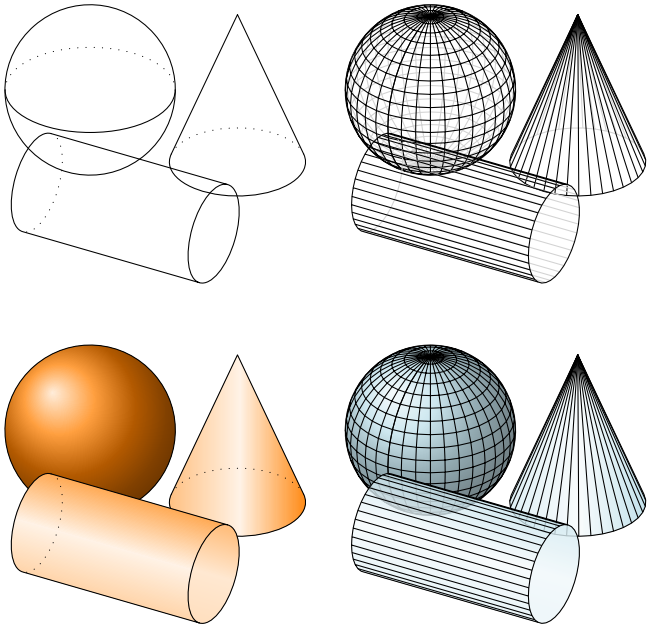
The **g:Dsphere(A, r [, options])** method draws a sphere.

- $\langle A \rangle$ is a 3D point representing the center of the sphere.
- $\langle r \rangle$ is the radius of the sphere.
- $\langle options \rangle$ is a table whose fields define the options. These options are (with their default values):
  - `mode=ld.mWireframe`, three possible values, `ld.mGrid` or `ld.mWireframe` or `ld.mBorder`. In `ld.mWireframe` mode, the outline (circle) and the equator are drawn; in `ld.mGrid` mode, the outline with meridians and spindles (grid) is drawn; and in `ld.mBorder` mode, the outline only is drawn.
  - `hiddenstyle=ld.Hiddenlinestyle`, sets the line style for hidden areas (set it to `"noline"` to hide them). By default, this option has the value of the global variable `ld.Hiddenlinestyle`, which is itself initialized with the value `"dotted"`.
  - `hiddencolor=edgecolor`, sets the color of hidden lines. By default, it is the same value as the `edgecolor` option.
  - `color=""`, when this option is an empty string (the default value), there is no fill; when it is a color (as a string), there is a fill with a "ball color".
  - `opacity=1`, defines the transparency of the drawing.
  - `edgecolor=<current color>`, sets the color of the lines.
  - `edgestyle=<current style>`, defines the line style for visible edges.
  - `edgewidth=<current width>`, sets the thickness of the visible edges in tenths of a point.

```
\begin{luadraw}{name=cylindre_cone_sphere}
local ld = luadraw
local pt3d = ld.pt3d
local vecI, vecJ, vecK, M = pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M
local g = ld.graph3d:new{ size={10,10} }
ld.Hiddenlines = true
local dessin = function(args)
    g:Dsphere(M(-1,-2.5,1),2.5, args)
    g:Dcone(M(-1,2.5,5),-5*vecK,2, args)
    g:Dcylinder(M(3,-2,0),6*vecJ,1.5, args)
```

```
end
-- top left, default options
g:Saveattr(); g:Viewport(-5,0,0,5); g:Coordsystem(-5,5,-5,5,true); dessin(); g:Restoreattr()
g:Saveattr(); g:Viewport(0,5,0,5); g:Coordsystem(-5,5,-5,5,true)
-- top right
dessin({mode=ld.mGrid, hiddenstyle="solid", hiddencolor="LightGray"}); g:Restoreattr()
g:Saveattr(); g:Viewport(-5,0,-5,0); g:Coordsystem(-5,5,-5,5,true)
-- bottom left
dessin({mode=ld.mBorder, color="orange"}); g:Restoreattr()
g:Saveattr(); g:Viewport(0,5,-5,0); g:Coordsystem(-5,5,-5,5,true)
-- bottom right
dessin({mode=ld.mGrid,opacity=0.8,hiddenstyle="noline",color="LightBlue"}); g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 10: Cylinders, Cones, and Spheres



Cut sphere: Dcut_sphere

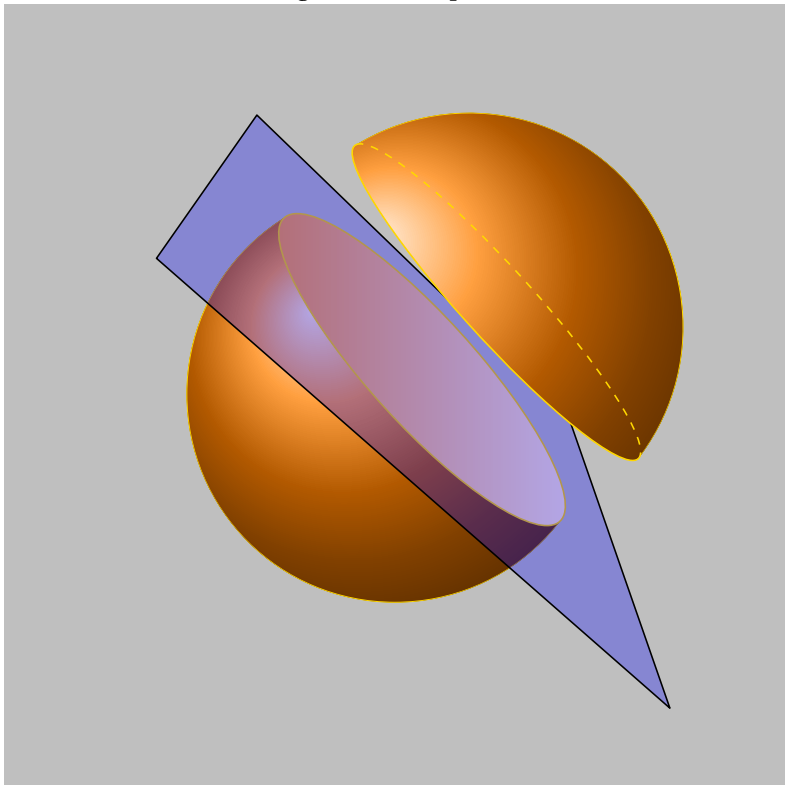
The method **g:Dcut_sphere(A, r, P, [, options])** draws the portion of the sphere centered at $\langle A\rangle$ with radius $\langle r\rangle$, intersected by the plane $\langle P\rangle = \{B, n\}$ (the plane passing through the 3D point b and with normal vector n). It is the portion of the sphere that lies within the half-plane containing the vector n that is drawn; only the outer part of the sphere is drawn, not the interior.

The $\langle options\rangle$ are the same as for the method **g:Dsphere()**, plus the following options:

- `visibletrace=""`, a string defining the options (passed to the *\draw* command) for drawing the portion on the visible face (if it exists) of the circle of intersection between the plane and the sphere. When this string is empty (default), this arc is not drawn.
- `hiddentrace=""`, a string defining the options (passed to the *\draw* command) for drawing the portion on the hidden face (if it exists) of the circle at the intersection of the plane and the sphere. When this string is empty (default), this arc is not drawn.
- `outline=false`, with the value `true`, the outline of the spherical part is drawn using the options contained in the string `visibletrace` (or the current options if the string is empty).
- `section=""`, a character string defining the options (passed to the *\draw* command) for drawing the circular section when it is visible. If this string is empty (the default value), the section is not drawn.

```
\begin{luadraw}{name=cut_sphere}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M
local g = ld.graph3d:new{ viewdir={"central",0,65}, size={10,10}, bg="lightgray"}
g:Linewidth(6)
local O, P = Origin, {vecK, -vecK-vecJ}
local c, r, n = ld.interPS(P, {O, 3})
g:Dcut_sphere(O, 3, P, {color="orange", mode=ld.mBorder, edgecolor="Gold", section=true,
    section_options="Gold,left color=orange!80, right color=orange!10"})
g:Dplane({c,P[2]}, vecI,8,8,"fill=blue, fill opacity=0.3")
g:Shift3d(-P[2])
g:Dcut_sphere(O, 3, {P[1],-P[2]},
    {color="orange", mode=ld.mBorder, edgecolor="Gold", visibletrace="Gold,line width=0.6",
     hiddentrace="dashed,Gold,line width=0.6"})
g:Show()
\end{luadraw}
```

Figure 11: Cut sphere



Cone_outline, Cylinder_outline, Frustum_outline, Sphere_outline

The following methods calculate and return a table containing characteristic elements relating to the cone, cylinder, truncated cone, or sphere. These methods take into account the projection mode as well as the graph's 3D transformation matrix. The first three functions return a table with the following fields:

- `side`: 3D path representing the outline of the visible lateral part of the solid.
- `hiddenside`: 3D path representing the outline of the non-visible lateral part of the solid.
- `section`: list of 3D paths, where each path represents the outline of one of the visible bases of the solid.
- `hiddensection`: list of 3D paths, where each path represents the outline of one of the non-visible bases of the solid.
- `visible`: 3D path representing the visible outline of the solid.
- `hidden`: 3D path representing the hidden outline of the solid.

- `tangency`: list of 3D points representing the tangency points of the solid—points where the plane tangent to the solid contains the observer's direction (the vector **g.Normal**); these points lie on the circular bases.
- `angle`: the angle, in degrees, between the axis of the solid and the horizontal.

Description of the four methods

- The method **g:Cone_outline(C, r, [V,] A)** calculates and returns the characteristic elements of the cone with vertex $\langle A \rangle$ and a circular base centered at $\langle C \rangle$ with radius $\langle r \rangle$. The cone is a right cone if $\langle V \rangle$ is omitted; otherwise, the base is orthogonal to vector $\langle V \rangle$ (oblique cone).
- The method **g:Cylinder_outline(C, r, [V,] A)** calculates and returns the characteristic elements of the cylinder with radius $\langle r \rangle$ and circular bases centered at $\langle C \rangle$ and $\langle A \rangle$. The cylinder is a right cylinder if $\langle V \rangle$ is omitted; otherwise, the circular bases are orthogonal to vector $\langle V \rangle$ (oblique cylinder).
- The method **g:Frustum_outline(C, R, r, [V,] A)** calculates the characteristic elements of the truncated cone with circular bases centered at $\langle C \rangle$ and $\langle A \rangle$, and radii $\langle R \rangle$ (for the first base) and $\langle r \rangle$. The truncated cone is a right truncated cone if $\langle V \rangle$ is omitted; otherwise, the circular bases are orthogonal to vector $\langle V \rangle$ (oblique truncated cone).
- The method **g:Sphere_outline(C, R)** calculates the characteristic elements of the sphere, returning a table with the following fields:
  - `visible`: 3D path representing the apparent outline of the sphere.
  - `data`: list of the form $\{O, r, n\}$, where O is the center of the apparent circle, r is its radius, and n is a vector normal to the plane of the apparent circle.

Cone_tangency, Cylinder_tangency, Frustum_tangency, Sphere_tangency

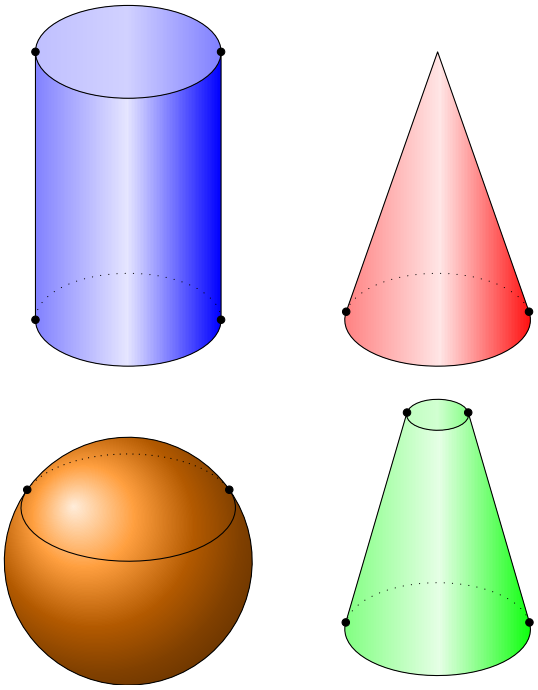
The following methods calculate and return, as a list, the tangency points—if they exist—for the cone, cylinder, truncated cone, or sphere.

- The method **g:Cone_tangency(C, r, [V,] A)** calculates and returns the two tangency points on the cone with vertex $\langle A \rangle$ and a circular base centered at $\langle C \rangle$ with radius $\langle r \rangle$. The cone is right if $\langle V \rangle$ is omitted; otherwise, the base is orthogonal to vector $\langle V \rangle$ (oblique cone). The two tangency points lie on the circular base; they are points where the plane tangent to the cone contains the observer's direction (vector **g.Normal**).
- The method **g:Cylinder_tangency(C, r, [V,] A)** calculates and returns the four tangency points on the cylinder of radius $\langle r \rangle$, with circular bases centered at $\langle C \rangle$ and $\langle A \rangle$. The cylinder is right if $\langle V \rangle$ is omitted; otherwise, the circular bases are orthogonal to vector $\langle V \rangle$ (oblique cylinder). The first two tangency points lie on the circular base centered at $\langle C \rangle$, and the next two on the other circular base.
- The method **g:Frustum_tangency(C, R, r, [V,] A)** calculates and returns the four tangency points on the truncated cone with circular bases centered at $\langle C \rangle$ and $\langle A \rangle$, and radii $\langle R \rangle$ (for the first) and $\langle r \rangle$. The truncated cone is right if $\langle V \rangle$ is omitted; otherwise, the circular bases are orthogonal to vector $\langle V \rangle$ (oblique truncated cone). The first two points of tangency lie on the circular base centered at $\langle C \rangle$, and the next two on the other circular base.
- The method **g:Sphere_tangency(C, R, P)** calculates and returns the two points of tangency to the sphere with center $\langle C \rangle$ and radius $\langle R \rangle$, and to the circle formed by the intersection of this sphere with the plane $\langle P \rangle$ (which is a list of the form $\{A, n\}$, where A is a point on the plane and n is a normal vector to the plane).

```
\begin{luadraw}{name=tangency}
local ld = luadraw
local pt3d, cpx = ld.pt3d, ld.cpx
local M, Z = pt3d.M, cpx.Z
local O, I, J, K = M(0,0,0), M(1,0,0), M(0,1,0), M(0,0,1)
local g = ld.graph3d:new{ size={10,10}, window={-5,5,-5,6} }
ld.Hiddenlines = true
g:Shift( Z(-2.5,3.5) ) -- cylinder
local A, B, C, D = table.unpack( g:Cylinder_tangency(-3*K, 1.5, 2*K) )
```

```
g:Dcylinder(-3*K, 1.5, 2*K, {color="blue"})
g:Ddots3d({A,B,C,D})
g:Shift( Z(5,0) ) -- cone
local A, B = table.unpack( g:Cone_tangency(-3*K, 1.5, 2*K) )
g:Dcone(-3*K, 1.5, 2*K, {color="red"})
g:Ddots3d({A,B})
g:Shift( Z(0,-5) ) -- frustum
local A, B, C, D = table.unpack( g:Frustum_tangency(-3*K, 1.5,0.5, K) )
g:Dfrustum(-3*K, 1.5, 0.5, K, {color="green"})
g:Ddots3d({A,B,C,D})
g:Shift(Z(-5,-1.5)) -- sphere
local R, P = 2, {K, K}
local I, r, n = ld.interPS(P, {O,R})
local A, B = table.unpack( g:Sphere_tangency(O, R, P) )
g:Dsphere(O, R, {color="orange", mode=ld.mBorder})
g:Ddots3d({A,B})
g:Darc3d(A,I,B,r,1,n); g:Darc3d(A,I,B,r,-1,n,"dotted")
g:Show()
\end{luadraw}
```

Figure 12: Points of tangency



IV Faceted Solids

1) Definition of a Solid

There are two ways to define a solid:

1. As a list (table) of facets. A facet is itself a list of 3D points (at least 3) that are coplanar and unaligned, which are the vertices. Facets are assumed to be convex and are oriented by the order of appearance of the vertices. That is, if A, B, and C are the first three vertices of a facet F, then the facet is oriented with the normal vector $\vec{AB} \wedge \vec{AC}$. If this normal vector is directed toward the observer, then the facet is considered visible. The method **g:Cosine_incidence(N [, A])** returns the cosine of the angle at point $\langle A \rangle$ between the vector $\langle N \rangle$ (which must be unit) and the unit vector directed towards the observer; if this cosine is positive then $\langle N \rangle$ is directed towards the observer (point $\langle A \rangle$ is optional EXCEPT in central projection).

   In the definition of a solid, the normal vectors to the facets must be directed **outside** the solid for the orientation to be correct.

2. In the form of a **polyhedron**, that is to say a table with two fields, a first field called *vertices* which is the list of vertices of the polyhedron (3D points), and a second field called *facets* which is the list of facets, but here, in the definition of the facets, the vertices are replaced by their index in the *vertices* list. The facets are oriented in the same way as before. This definition corresponds to the *obj* format but without normal vectors.

For example, let's consider the four points $A = M(-2,-2,0)$, $B = M(3,0,0)$, $C = M(-2,2,0)$, and $D = M(0,0,4)$. We can then define the tetrahedron constructed on these four points:

- either as a list of facets: `T={{A,B,D},{B,C,D},{C,A,D},{A,C,B}}` (pay attention to the orientation),
- or as a polyhedron: `T={vertices={A,B,C,D},facets={{1,2,4},{2,3,4},{3,1,4},{1,3,2}}}`.

Functions for converting between the two definitions

- The function **ld.poly2facet(P)** where ⟨*P*⟩ is a polyhedron, returns this solid as a list of facets.
- The function **ld.facet2poly(L [, epsilon])** returns the list of facets ⟨*L*⟩ as a polyhedron. The optional argument ⟨*epsilon*⟩ defaults to 10^{-8}, specifying the accuracy of the comparisons between 3D points. If there are many facets, the computation time can become significant.

2) Drawing a Polyhedron: Dpoly

The function **g:Dpoly(P, options)** allows you to represent the polyhedron ⟨*P*⟩ (using the naive painter's algorithm). The argument ⟨*options*⟩ is a table containing the options, these are, with their default value:

- `mode=ld.mShaded`: Sets the representation mode. There are six possible values:
  - `ld.mWireframe`: Wireframe mode, draws both visible and hidden edges.
  - `ld.mFlat`: Draws solid-color faces, as well as visible edges.
  - `ld.mFlatHidden`: Draws solid-color faces, visible edges, and hidden edges.
  - `ld.mShaded`: Draws the faces in shaded color based on their inclination, as well as the visible edges. This is the default mode.
  - `ld.mShadedHidden`: Draws the faces in shaded color based on their inclination, with both visible and hidden edges.
  - `ld.mShadedOnly`: Draws the faces in shaded color based on their inclination, but not the edges.
- `contrast=1`: This number allows you to accentuate or diminish the shade of the facet colors in the `ld.mShaded`, `ld.mShadedHidden`, and `ld.mShadedOnly` modes.
- `edgestyle=<current style>`: This is a string that defines the line style of the edges.
- `edgecolor=<current line color>`: This is a string that defines the edge color.
- `hiddenstyle=ld.Hiddenlinestyle`: This is a string that defines the line style of hidden edges. By default, this is the value contained in the global variable `ld.Hiddenlinestyle` (which itself is `"dotted"` by default).
- `hiddencolor=<current line color>`: This is a string that defines the color of hidden edges.
- `edgewidth=<current line width>`: This is the line thickness of edges in tenths of a point.
- `opacity=1` : This is a number between 0 and 1 that allows you to set transparency or not on the facets.
- `backcull=false`: When this boolean is `true`, facets considered invisible (normal vectors not directed towards the observer) are not displayed. This option is useful for convex polyhedra because it reduces the number of facets to draw.
- `twoside=true`: Boolean that defaults to true, meaning that both sides of the facets (inner and outer) are distinguished; the two sides will not have exactly the same color.

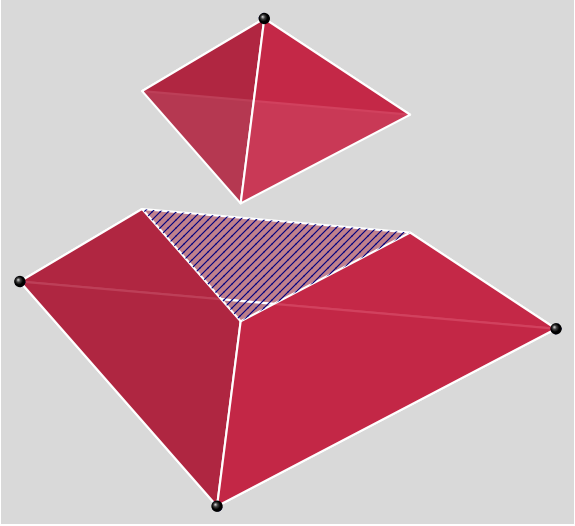
- `reverse=false` : with the value `true` the orientation of the facets is reversed.
- `color="white"`: String defining the fill color of the facets.
- `usepalette=nil`: this option allows you to specify a color palette for painting the facets as well as a calculation mode, the syntax is: `usepalette={palette,mode[,minmax]}`, where ⟨*palette*⟩ designates a table of colors which are themselves tables of the form $\{r, g, b\}$ where r, g and b are numbers between 0 and 1. The argument ⟨*mode*⟩ can be :
  - one of the strings: `"x"`, `"y"`, `"z"`. In the first case, for example, facets with the minimum x-coordinate at their centroid have the first color in the palette, facets with the maximum x-coordinate at their centroid have the last color in the palette, and for the others, the color is calculated based on the x-coordinate of the centroid by linear interpolation.
  - a function: ⟨*mode*⟩ : $f \mapsto \text{mode}(f) \in \mathbb{R}$, where f denotes a facet (a list of 3D points). Facets with the minimum value have the first color in the palette, those with the maximum value have the last color in the palette, and for the others, the color is calculated by linear interpolation.

  The ⟨*minmax*⟩ argument is a table of two numbers; these two numbers are used as the minimum and maximum values for the ⟨*mode*⟩ argument. If ⟨*minmax*⟩ is omitted, these two values are calculated automatically.

```
\begin{luadraw}{name=tetra_coupe}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{viewdir={10,60},bbox=false, size={10,10}, bg="gray!30"}
local A,B,C,D = M(-2,-4,-2),M(4,0,-2),M(-2,4,-2),M(0,0,2)
local T = ld.tetra(A,B-A,C-A,D-A) -- tetrahedron with vertices A, B, C, D
local plan = {Origin, -vecK} -- sectional plane
local T1, T2, section = ld.cutpoly(T,plan) -- we cut the tetrahedron
-- T1 is the resulting polyhedron in the half-space containing -vecK
-- T2 is the resulting polyhedron in the other half-space
-- section is a facet (it's the cut)
g:Dpoly(T1,{color="Crimson", edgecolor="white", opacity=0.8, edgewidth=8})
g:Filloptions("bdiag","Navy"); g:Dpolyline3d(section,true,"draw=none")
g:Dpoly(ld.shift3d(T2,2*vecK), {color="Crimson", edgecolor="white", opacity=0.8, edgewidth=8})
g:Dballdots3d({A,B,C,D+2*vecK}) -- we drew T2 translated with the vector 2*vecK
g:Show()
\end{luadraw}
```

Figure 13: Section of a tetrahedron by a plane



3) Displaying the Face and/or Vertex Numbers of a Polyhedron

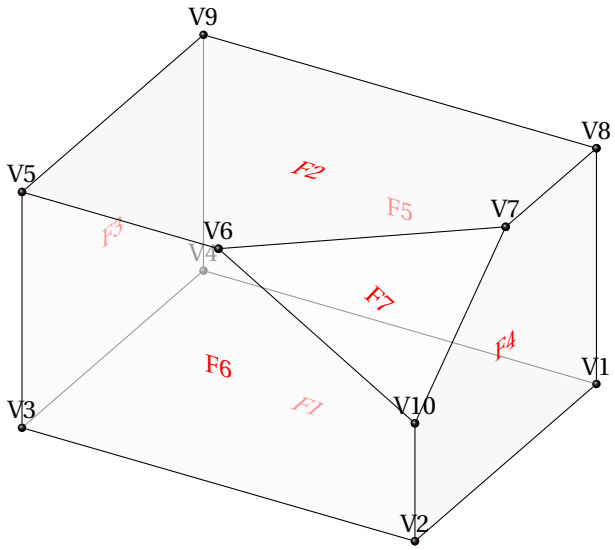
The method **g:Dpolynames(P,option, opacity)** displays the polyhedron P with the option to add to each face its number (which is its position in the *P.facets* list) preceded by the letter F, and optionally the number of each vertex (which is its

position in the *P.vertices* list) preceded by the letter *V*. The argument *option* can take the values: *"facet"*, *"vertex"*, or *"both"* (which is the default value). The argument *opacity* is a number between 0 and 1 which defaults to 0.6.

```
\begin{luadraw}{name=show_facet_number}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{viewdir={30,60}, window={-2.5,5,-3,3},size={10,10}}
P = ld.parallelep(Origin, 4*vecI,5*vecJ,3*vecK)
local A, B, C = M(4,2.5,3), M(2,5,3), M(4,5,1.5)
P = ld.cutpoly(P, ld.plane(A,B,C), true) -- we cut P with the plane, and add a facet in place of the section
g:Dpolynames(P) -- we want to see facets and vertices numbers of P
g:Show()
\end{luadraw}
```

Figure 14: Visualize the faces and vertices of a polyhedron



4) Polyhedron Construction Functions

The following functions return a polyhedron, that is, a table with two fields: a first field called *vertices*, which is the list of the polyhedron's vertices (3D points), and a second field called *facets*, which is the list of facets. However, in the definition of facets, the vertices are replaced by their index in the *vertices* list.

- **ld.tetra(S, v1, v2, v3)** returns the tetrahedron constructed from vertex ⟨*S*⟩ (3D point) and the three vectors ⟨*v1*⟩, ⟨*v2*⟩, ⟨*v3*⟩ (3D points) assumed to be in the forward direction. The vertices of this tetrahedron are S, $S+v_1$, $S+v_2$, and $S+v_3$.
- **ld.parallelep(S, v1, v2, v3)** returns the parallelepiped constructed from vertex ⟨*S*⟩ (3D point) and the three vectors ⟨*v1*⟩, ⟨*v2*⟩, ⟨*v3*⟩ (3D points) assumed to be in the forward direction.
- **ld.prism(base, vector [, open])** returns a prism. The argument ⟨*base*⟩ is a list of 3D points (one of the two bases of the prism). ⟨*vector*⟩ is the translation vector (3D point) used to obtain the second base. The optional argument ⟨*open*⟩ is a Boolean indicating whether the prism is open or not (`false` by default). If it is open, only the lateral facets are returned. The ⟨*base*⟩ must be oriented by the ⟨*vector*⟩.
- **ld.pyramid(base, vertex [, open])** returns a pyramid. The argument ⟨*base*⟩ is a list of 3D points, and ⟨*vertex*⟩ is the apex of the pyramid (3D point). The optional argument ⟨*open*⟩ is a Boolean indicating whether the pyramid is open or not (`false` by default). If it is open, only the side facets are returned. The base must be oriented with the apex.
- **ld.regular_pyramid(n, side, height [, open, center, axis])** returns a regular pyramid. ⟨*n*⟩ is the number of sides of the base, the argument ⟨*side*⟩ is the length of a side, and ⟨*height*⟩ is the height of the pyramid. The optional argument

⟨*open*⟩ is a Boolean indicating whether the pyramid is open or not (`false` by default). If it is open, only the lateral facets are returned. The optional argument ⟨*center*⟩ is the center of the base (`Origin` by default), and the optional argument ⟨*axis*⟩ is a direction vector of the pyramid axis (`vecK` by default).

- **ld.truncated_pyramid(base, vertex, height [, open])** returns a truncated pyramid; the argument ⟨*base*⟩ is a list of 3D points; ⟨*vertex*⟩ is the apex of the pyramid (3D point). The argument ⟨*height*⟩ is a number indicating the height from the base where the truncation occurs; this is parallel to the plane of the base. The optional argument ⟨*open*⟩ is a boolean indicating whether the pyramid is open or not (`false` by default). If it is open, only the lateral facets are returned. The base must be oriented by the apex.

- **ld.cylinder(A, V, R [, nbfacet, open])** returns a right cylinder of radius ⟨*R*⟩, ⟨*A*⟩ (3D point) is the center of one of the circular bases, and ⟨*V*⟩ is a non-zero 3D vector such that the center of the second base is the point $A+V$. The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a Boolean indicating whether the cylinder is open or not (`false` by default). If it is open, only the lateral facets are returned.

- **ld.cylinder(A, R, B [, nbfacet, open])** returns a right cylinder of radius ⟨*R*⟩, ⟨*A*⟩ (3D point) is the center of one of the circular bases, and ⟨*B*⟩ the center of the second base. The cylinder is right. The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a Boolean indicating whether the cylinder is open or not (`false` by default). If it is open, only the lateral facets are returned.

- **ld.cylinder(A, R, V, B [, nbfacet, open])** returns a cylinder of radius ⟨*R*⟩, ⟨*A*⟩ (3D point) is the center of one of the circular bases, and ⟨*B*⟩ the center of the second base, and ⟨*V*⟩ is a 3D vector normal to the plane of the circular bases (the cylinder can therefore be tilted). The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a boolean indicating whether the cylinder is open or not (`false` by default). If it is open, only the lateral facets are returned.

- **ld.cone(A, V, R [, nbfacet, open])** returns a cone with vertex ⟨*A*⟩ (3D point), axis directed by ⟨*V*⟩ (3D point), and base the circle with center $A+V$ and radius ⟨*R*⟩ (in a plane orthogonal to ⟨*V*⟩). The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a boolean indicating whether the cone is open or not (`false` by default). If it is open, only the lateral facets are returned.

- **ld.cone(C, R, A [, nbfacet, open])** returns a cone with vertex ⟨*A*⟩ (3D point), ⟨*C*⟩ is the circular base center, and ⟨*R*⟩ is its radius (in a plane orthogonal to the (AC) axis). The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a Boolean indicating whether the cone is open or not (`false` by default). If it is open, only the lateral facets are returned.

- **ld.cone(C, R, V, A [, nbfacet, open])** returns a cone with vertex ⟨*A*⟩ (3D point), ⟨*C*⟩ is the circular base center, ⟨*R*⟩ is its radius, and the base is in a plane orthogonal to ⟨*V*⟩ (3D vector). The (AC) axis is therefore not necessarily orthogonal to the circular face (tilted cone). The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a Boolean indicating whether the cone is open or not (`false` by default). If it is open, only the lateral facets are returned.

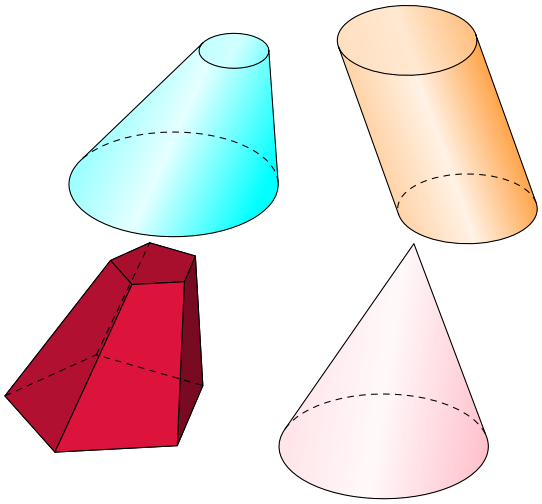
- **ld.frustum(C, R, r, A [, nbfacet, open])** returns a right frustum. ⟨*C*⟩ (3D point) is the center of the circular base of radius ⟨*R*⟩, the center of the other circular base is point ⟨*A*⟩, and its radius is ⟨*r*⟩, the bases are orthogonal to the vector $\vec{CA}$. The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a Boolean indicating whether the frustum is open or not (`false` by default). If it is open, only the lateral facets are returned.

- **ld.frustum(C, R, r, V, A [, nbfacet, open])** returns a frustum of a cone. ⟨*C*⟩ (3D point) is the center of the circular base of radius ⟨*R*⟩, the center of the other circular base is point ⟨*A*⟩, and its radius is ⟨*r*⟩. The bases are orthogonal to vector ⟨*V*⟩, but not necessarily orthogonal to axis (AC). The optional argument ⟨*nbfacet*⟩ is 35 by default (number of lateral facets). The optional argument ⟨*open*⟩ is a Boolean indicating whether the frustum is open or not (`false` by default). If it is open, only the lateral facets are returned.

- **ld.sphere(A, R [, nbu, nbv])** returns the sphere with center ⟨*A*⟩ (3D point) and radius ⟨*R*⟩. The optional argument ⟨*nbu*⟩ represents the number of spindles (36 by default) and the optional argument ⟨*nbv*⟩ the number of parallels (20 by default).

```
\begin{luadraw}{name=frustum_pyramid}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M
local g = ld.graph3d:new{adjust2d=true,bbox=false, size={10,10} }
ld.Hiddenlines = true; ld.Hiddenlinestyle = "dashed"
g:Dfrustum(M(-1,-4,0),3,1,vecK, M(-1,-2,5), {color="cyan"})
g:Dcylinder(M(-4,4,0),2,vecK,M(-4,2,5), {color="orange"})
local base = ld.map(pt3d.toPoint3d, ld.polyreg(0,3,5))
g:Dpoly(ld.truncated_pyramid( ld.shift3d(base,8*vecI-vecJ-2*vecK), M(5,0,5),4),
    {mode=ld.mShadedHidden,color="Crimson"})
g:Dcone(M(6,7,-2),3,vecK,M(6,8,5),{color="Pink"})
g:Show()
\end{luadraw}
```

Figure 15: Truncated cone, truncated pyramid, oblique cylinder

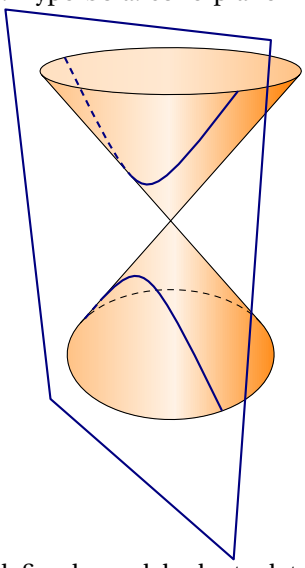


Note : We already have primitives for drawing cylinders, cones, and spheres without using facets. One of the advantages of defining these objects as polyhedra is that we can perform certain calculations on them, such as plane sections.

```
\begin{luadraw}{name=hyperbole}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-8,6,-9,9},bbox=false,
    viewdir={"central",45,65}, size={10,10}}
ld.Hiddenlinestyle = "dashed"; ld.Hiddenlines = true
local C1 = ld.cone(Origin,4*vecK,3,35,true)
local C2 = ld.cone(Origin, -4*vecK,3,35,true)
local P = {M(1,-1,-2),vecI} -- sectional plane
local I1 = g:Intersection3d(C1,P) -- intersection between cone C1 and plane P
local I2 = g:Intersection3d(C2,P) -- intersection between cone C2 and plane P
-- I1 and I2 are of the Edges type
g:Dcone(Origin,4*vecK,3,{color="orange"}); g:Dcone(Origin,-4*vecK,3,{color="orange"})
g:Lineoptions("solid","Navy",8)
g:Dedges(I1); g:Dedges(I2) -- drawing of edges I1 and I2
g:Dplane(P, vecK,14,9)
g:Show()
\end{luadraw}
```

Figure 16: Hyperbola: cone-plane intersection

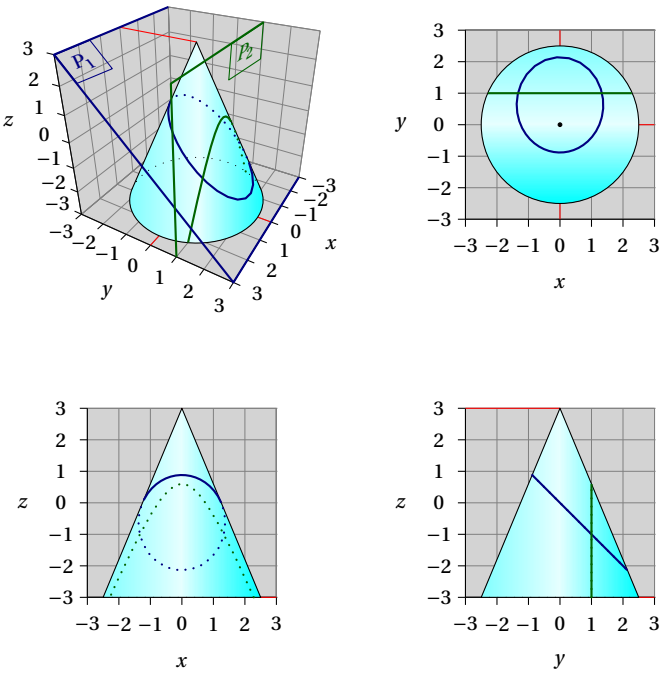


In this example, cones C_1 and C_2 are defined as polyhedra to determine their intersection with plane P, but not to draw them. The method **g:Intersection3d(C1, P)** returns the intersection of polyhedron C_1 with plane P as a two-field table: one field named *visible* that contains a 3D polygonal line representing the visible edges (segments) of the intersection (i.e., those that are on a visible facet of C_1), and another field named *hidden* that contains a 3D polygonal line representing the hidden edges of the intersection (i.e., those that are on a non-visible facet of C_1). The method **g:Dedges()** can be used to draw these types of objects.

```
\begin{luadraw}{name=several_views}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M
local g = ld.graph3d:new{window3d={-3,3,-3,3,-3,3}, size={10,10}, margin={0,0,0,0}}
ld.Hiddenlines = true
g:Labelsize("footnotesize")
local y0, R = 1, 2.5
local C = ld.cone(M(0,0,3),-6*vecK,R,35,true) -- open cone
local P1 = {M(0,0,0),vecK+vecJ} -- first plane
local P2 = {M(0,y0,0),vecJ} -- second plane
local I, I2
local dessin = function() -- one drawing per view
    g:Dboxaxes3d({grid=true,gridcolor="gray",fillcolor="LightGray"})
    I1 = g:Intersection3d(C,P1) -- intersection entre le cône C et les plans P1 et P2
    I2 = g:Intersection3d(C,P2) -- I1 and I2 are the Edges type
    g:Dpolyline3d( {{M(0,-3,3),M(0,0,3),M(0,0,-3),M(3,0,-3)}, {M(0,0,-3),M(0,3,-3)}},"red,line width=0.4pt" )
    g:Dcone( M(0,0,3),-6*vecK,R, {color="cyan"})
    g:Dedges(I1, {hidden=true,color="Navy", width=8})
    g:Dedges(I2, {hidden=true,color="DarkGreen", width=8})
end
-- In the upper left corner, the view in space, we add the plans to the drawing
g:Saveattr(); g:Viewport(-5,0,0,5); g:Coordsystem(-7,6,-6,5,1); g:Setviewdir("central"); dessin()
g:Dpolyline3d( {M(-3,-3,3),M(3,-3,3),M(3,3,-3),M(-3,3,-3)},"Navy,line width=0.8pt")
g:Dpolyline3d( {M(-3,y0,3),M(3,y0,3),M(3,y0,-3)},"DarkGreen,line width=0.8pt")
g:Dlabel3d( "$P_1$",M(3,-3,3),{pos="SE",dir={-vecI,-vecJ+vecK},node_options="Navy, draw"})
g:Dlabel3d( "$P_2$",M(-3,y0,3),{pos="SW",dir={-vecI,vecK},node_options="DarkGreen,draw"})
g:Restoreattr()
-- top right, projection onto the xy-plane
g:Saveattr(); g:Viewport(0,5,0,5); g:Coordsystem(-6,6,-6,5,1); g:Setviewdir("xOy"); dessin()
g:Restoreattr()
-- bottom left, projection onto the xz-plane
g:Saveattr(); g:Viewport(-5,0,-5,0); g:Coordsystem(-6,6,-6,5,1); g:Setviewdir("xOz"); dessin()
g:Restoreattr()
-- bottom right, projection onto the yz-plane
g:Saveattr(); g:Viewport(0,5,-5,0); g:Coordsystem(-6,6,-6,5,1); g:Setviewdir("yOz"); dessin()
g:Restoreattr()
g:Show()
```

```
\end{luadraw}
```

Figure 17: Cone section with multiple views



5) Converting a Polyhedron to a Graph

The function **ld.poly2graph(P [, mode])** converts the polyhedron ⟨*P*⟩ into a graph and returns the result. The argument ⟨*mode*⟩ selects the graph type; two values are possible:

- `mode="facet"` (default value): the vertices of the graph are the facets, and the neighboring vertices in the graph are the adjacent facets. The result is a list of lists; the element number i in the result is the list of numbers of the facets adjacent to facet number i. The number of a facet is its index in the list ⟨*P*⟩.facets.
- `mode="vertex"`: The vertices of the graph are the vertices of the polyhedron ⟨*P*⟩, and the neighboring vertices in the graph are the neighboring vertices in the polyhedron. The result is a list of lists; the element i of the result is the list of vertex numbers neighboring vertex number i. The number of a vertex is its index in the list ⟨*P*⟩.vertices.

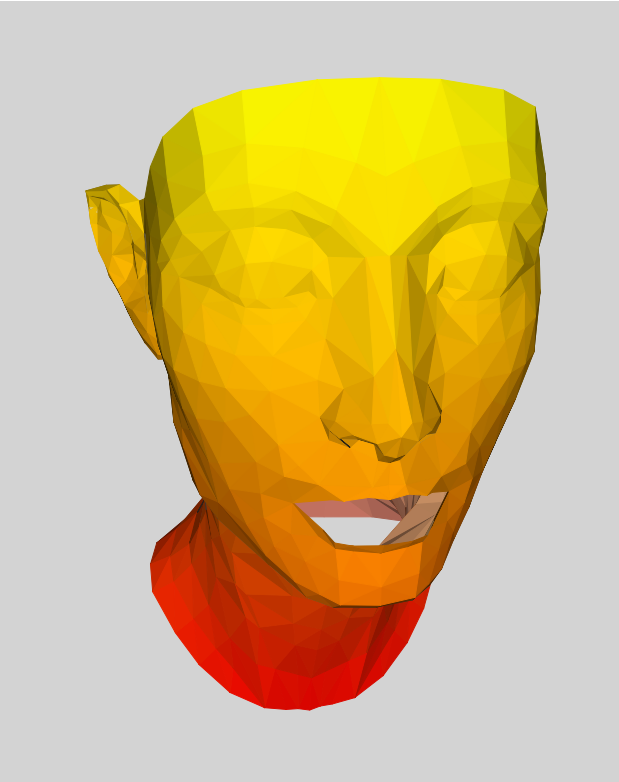
6) Reading from an obj file

The function **ld.read_obj_file(file)**[1] allows you to read the contents of the file *obj* designated by the string ⟨*file*⟩. The function reads the vertex definitions (lines beginning with `v`), and the lines defining the facets (lines beginning with `f`). The other lines are ignored. The function returns a sequence consisting of the polyhedron, followed by a list of six real numbers `{x1,x2,y1,y2,z1,z2}` representing the 3D bounding box of the polyhedron.

```
\begin{luadraw}{name=lecture_obj}
local ld = luadraw
local P,bbox = ld.read_obj_file("obj/nefertiti.obj")
local g = ld.graph3d:new{window3d=bbox,window={-6,5,-7,7},viewdir={"central",35,65,20},
margin={0,0,0,0}, size={10,10}, bg="LightGray"}
g:Dpoly(P, {usepalette={ld.palAutumn,"z"},mode=ld.mShadedOnly})
g:Show()
\end{luadraw}
```

[1] This function is a contribution by Christophe BAL.

Figure 18: Mask of Nefertiti



7) Surface in obj format

Two possible syntaxes:

1. **ld.obj_surface(f, u1, u2, v1, v2 [, grid])** returns, in the format *obj*, the surface parameterized by the function $\langle f \rangle : (u,v) \mapsto f(u,v) \in \mathbf{R}^3$. The interval for the parameter u is given by $\langle u1 \rangle$ and $\langle u2 \rangle$. The interval for the parameter v is given by $\langle v1 \rangle$ and $\langle v2 \rangle$. The optional parameter $\langle grid \rangle$ defaults to $\{25,25\}$, and defines the number of points to calculate for the parameter u followed by the number of points to calculate for the parameter v (the values of u and v are equally distributed).

2. **ld.obj_surface(f, mesh)** with $\langle mesh \rangle = \{\{u_1,\ldots,u_n\},\{v_1,\ldots,v_m\}\}$ returns the surface parameterized by the function $\langle f \rangle : (u,v) \mapsto f(u,v) \in \mathbf{R}^3$. The values of the parameters u and v are given by the argument $\langle mesh \rangle$; they must be in strictly ascending order, but they are not necessarily equally distributed.

In both cases, the result is not a list of facets but a table with three fields:

`{vertices={…}, normals={…}, facets={{…},{…},…} }.`

- The *vertices* and *facets* fields are identical to the case of polyhedra: lists of vertices (3D points) and a list of facets with vertex numbers, except that here the facets are all **triangular**.
- The *normals* field is a list of unit 3D vectors representing normal vectors to the surface, one per vertex.

8) Drawing a List of Facets: Dfacet and Dmixfacet

There are two possible methods:

1. For a solid S in the form of a list of facets (with 3D points), the method is:

   g:Dfacet(S, options)

   where $\langle S \rangle$ is the list of facets and $\langle options \rangle$ is a table defining the options. These are:

   - `mode=ld.mShaded`: Sets the representation mode. The possible values are:
     - `ld.mWireframe`: Wireframe mode, draws only the edges. When the `usepalette` option is not `nil`, the color of each edge is calculated using the palette (in the same way as for facets when they are painted).

  - `ld.mFlat` or `ld.mFlatHidden`: Draws the faces in a solid color, as well as the edges.
  - `ld.mShaded` or `ld.mShadedHidden`: The faces are drawn in shaded color based on their inclination, as well as the edges.
  - `ld.mShadedOnly`: The faces are drawn in shaded color based on their inclination, but not the edges.

- `contrast=1`: This number allows you to accentuate or diminish the shade of the facet colors in the `ld.mShaded`, `ld.mShadedHidden`, and `ld.mShadedOnly` modes.
- `edgestyle=<current style>`: This is a string that defines the line style of the edges.
- `edgecolor=<current line color>`: This is a string that defines the edge color.
- `edgewidth=<current line width>`: This is the line thickness of edges in tenths of a point.
- `opacity=1` : This is a number between 0 and 1 that allows you to set transparency or not on the facets.
- `backcull=false`: When this boolean is `true`, facets considered invisible (normal vectors not directed towards the observer) are not displayed. This option is useful for convex polyhedra because it reduces the number of facets to draw.
- `twoside=true`: Boolean that defaults to true, meaning that both sides of the facets (inner and outer) are distinguished; the two sides will not have exactly the same color.
- `reverse=false` : with the value `true` the orientation of the facets is reversed.
- `color="white"`: String defining the fill color of the facets.
- `usepalette=nil`: this option allows you to specify a color palette for painting the facets as well as a calculation mode, the syntax is: `usepalette={palette,mode[,minmax]}`, where ⟨*palette*⟩ designates a table of colors which are themselves tables of the form $\{r, g, b\}$ where r, g and b are numbers between 0 and 1. The argument ⟨*mode*⟩ can be :
  - one of the strings: `"x"`, `"y"`, `"z"`. In the first case, for example, facets with the minimum x-coordinate at their centroid have the first color in the palette, facets with the maximum x-coordinate at their centroid have the last color in the palette, and for the others, the color is calculated based on the x-coordinate of the centroid by linear interpolation.
  - a function: ⟨*mode*⟩ : $f \mapsto \text{mode}(f) \in \mathbb{R}$, where f denotes a facet (a list of 3D points). Facets with the minimum value have the first color in the palette, those with the maximum value have the last color in the palette, and for the others, the color is calculated by linear interpolation.

  The ⟨*minmax*⟩ argument is a table of two numbers; these two numbers are used as the minimum and maximum values for the ⟨*mode*⟩ argument. If ⟨*minmax*⟩ is omitted, these two values are calculated automatically.

2. For multiple facet lists in the same drawing, the method is:

g:Dmixfacet(S1, options1, S2, options2, ...)

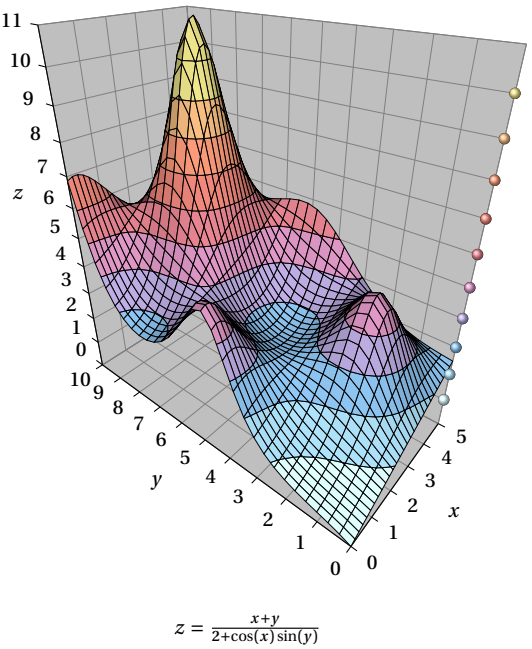
where ⟨*S1*⟩, ⟨*S2*⟩, ..., are facet lists, and ⟨*options1*⟩, ⟨*options2*⟩, ..., are the corresponding options. The options in one facet list also apply to the following ones if they are not changed. These options are identical to the previous method.

This method is useful for drawing multiple solids together, provided there are no intersections between the objects, as these are not handled here.

```
\begin{luadraw}{name=courbes_niv}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M, Z = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, ld.cpx.Z
local cos, sin = math.cos, math.sin
local g = ld.graph3d:new{window3d={0,5,0,10,0,11}, adjust2d=true, size={10,10},
    viewdir={"central",220,60,15,M(2.5,5,5.5)}}
g:Labelsize("footnotesize")
local S = ld.cartesian3d(function(u,v) return (u+v)/(2+cos(u)*sin(v)) end,0,5,0,10,{30,30})
local n = 10 -- number of levels
local Colors = ld.getpalette(ld.palGasFlame,n,true) -- list of 10 colors in table format
local niv, S1 = {}
for k = 1, n do
```

```
    S1, S = ld.cutfacet(S,{M(0,0,k),-vecK}) -- section of S with the plane z=k
    ld.insert(niv,{S1, {color=Colors[k],mode=ld.mShaded,edgewidth=0.5}}) -- S1 is the part below the plane and S is
    ↪ above it
end
ld.insert(niv,{S, {color=Colors[n+1]}}) -- inserting the last level
-- niv is a list of the type {facets1, options1, facets2, options2, ...}
g:Dboxaxes3d({grid=true, gridcolor="gray",fillcolor="lightgray"})
g:Dmixfacet(table.unpack(niv))
for k = 1, n do
    g:Dballdots3d( M(5,0,k), ld.rgb(Colors[k]))
end
g:Dlabel("$z=\\frac{x+y}{2+\\cos(x)\\sin(y)}$", Z((g:Xinf()+g:Xsup())/2, g:Yinf()), {pos="N"})
g:Show()
\end{luadraw}
```

Figure 19: Example of contour lines on a surface



9) Functions for Constructing Facet Lists

The following functions return a solid as a list of facets (with 3D points).

surface()

Two possible syntaxes:

1. **ld.surface(f, u1, u2, v1, v2 [, grid])** returns the surface parameterized by the function $\langle f\rangle\colon (u,v)\mapsto f(u,v)\in\mathbf{R}^3$. The interval for the parameter u is given by $\langle u1\rangle$ and $\langle u2\rangle$. The interval for the parameter v is given by $\langle v1\rangle$ and $\langle v2\rangle$. The optional parameter $\langle grid\rangle$ defaults to $\{25,25\}$, and defines the number of points to calculate for the parameter u followed by the number of points to calculate for the parameter v (the values of u and v are equally distributed).

2. **ld.surface(f, mesh)** with $\langle mesh\rangle = \{\{u_1,\ldots,u_n\},\{v_1,\ldots,v_m\}\}$, returns the surface parameterized by the function $\langle f\rangle\colon (u,v)\mapsto f(u,v)\in\mathbf{R}^3$. The values of the parameters u and v are given by the argument $\langle mesh\rangle$; they must be in strictly ascending order, but they are not necessarily equally distributed.

There are two variants for surfaces:

cartesian3d()

The function **ld.cartesian3d(f, x1, x2, y1, y2 [, grid, addwall])** returns the Cartesian surface with equation $z = f(x,y)$ where $\langle f\rangle : (x,y) \mapsto f(x,y) \in \mathbb{R}$. The interval for x is given by $\langle x1\rangle$ and $\langle x2\rangle$. The interval for y is given by $\langle y1\rangle$ and $\langle y2\rangle$. The optional parameter $\langle grid\rangle$ is {25,25} by default; it defines the number of points to calculate for x followed by the number of points to calculate for y. The parameter $\langle addwall\rangle$ is `0` or `"x"`, or `"y"`, or `"xy"` (`0` by default). When this option is set to `"x"` (or `"xy"`), the function returns, after the list of facets composing the surface, a list of separating facets (walls or partitions) between each "layer" of facets. A layer corresponds to two consecutive values of the parameter x[2]. With the value `"y"` (or `"xy"`), it is a list of separating facets (walls) between each "layer" corresponding to two consecutive values of the parameter "y"[3]. This option can be useful with the **g:Dscene3d()** method (only), because the separating partitions form a partition of space isolating the facets of the surface, which avoids unnecessary intersection calculations between them. This is particularly the case with non-convex surfaces.

For example, here is the code for figure 1:

```
\begin{luadraw}{name=point_col}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window3d={-2,2,-2,2,-4,4}, window={-3.5,3,-5,5}, size={8,9,0}, viewdir={120,60}}
local S = ld.cartesian3d(function(u,v) return u^2-v^2 end, -2,2,-2,2,{20,20}) -- surface of equation z=x^2-y^2
local Tx = g:Intersection3d(S, {Origin,vecI}) --intersection of S with the yOz plane
local Ty = g:Intersection3d(S, {Origin,vecJ}) --intersection of S with the xOz plane
g:Dboxaxes3d({grid=true,gridcolor="gray",fillcolor="LightGray",drawbox=true})
g:Dfacet(S,{mode=ld.mShadedOnly,color="ForestGreen"}) -- surface drawing
g:Dedges(Tx, {color="Crimson", hidden=true, width=8}) -- intersection with yOz
g:Dedges(Ty, {color="Navy",hidden=true, width=8}) -- intersection with xOz
g:Dpolyline3d( {M(2,0,4),M(-2,0,4),M(-2,0,-4)}, "Navy,line width=.8pt")
g:Dpolyline3d( {M(0,-2,4),M(0,2,4),M(0,2,-4)}, "Crimson,line width=.8pt")
g:Show()
\end{luadraw}
```

cylindrical_surface()

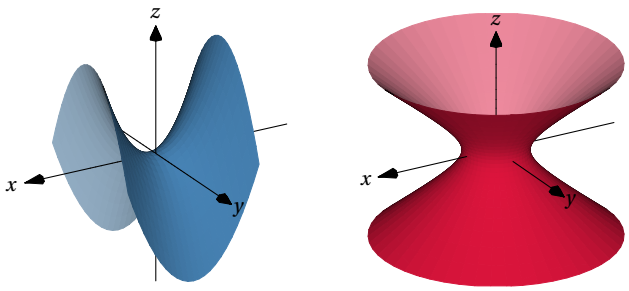
The function **ld.cylindrical_surface(r, z, u1, u2, theta1, theta2 [, grid, addwall])** returns the surface parameterized as cylindrical by *r(u,theta), theta, z(u,theta)*. The arguments $\langle r\rangle$ and $\langle z\rangle$ are therefore two real-valued functions of u and θ. The interval for u is given by $\langle u1\rangle$ and $\langle u2\rangle$. The interval for θ is given by $\langle theta1\rangle$ and $\langle theta2\rangle$ (in radians). The optional parameter $\langle grid\rangle$ is {25,25} by default; it defines the number of points to calculate for u followed by the number of points to calculate for θ. The parameter $\langle addwall\rangle$ is 0 or `"v"` or `"z"` or `"vz"` (0 by default). When this option is `"v"` or `"vz"`, the function returns, after the list of facets composing the surface, a list of separating facets (walls or partitions) between each "layer" of facets, a layer corresponds to two consecutive values of the angle θ[4]. When this option is set to `"z"` or `"vz"`, the function returns, after the list of facets composing the surface, a list of separating facets (walls or partitions) between each "layer" of facets. A layer corresponds to two consecutive values of the dimension z[5], the values of z are calculated from the values of the parameter u and with the value $\langle theta1\rangle$. This is useful when z only depends on u (and therefore not on θ). This option can be useful with the **g:Dscene3d()** method (only), because the separating partitions form a partition of space isolating the surface facets, which avoids unnecessary intersection calculations between them. This is particularly the case with non-convex surfaces.

```
\begin{luadraw}{name=surface_with_addWall}
local ld = luadraw
local pi, ch, sh = math.pi, math.cosh, math.sinh
local O = ld.pt3d.Origin
local g = ld.graph3d:new{window3d={-4,4,-4,4,-5,5}, window={-10,10,-4,4}, size={10,10}, viewdir={60,60}}
g:Labelsize("footnotesize")
local S,wall = ld.cartesian3d(function(x,y) return x^2-y^2 end,-2,2,-2,2,nil,"xy")
g:Saveattr(); g:Viewport(-10,0,-4,4); g:Coordsystem(-4.5,4.5,-4.5,4.75)
```

[2] These partitions are actually planes with the equation x =constant
[3] These partitions are actually planes with the equation y =constant
[4] These partitions are in fact planes of equation θ =constant
[5] These partitions are actually planes with the equation z =constant

```
g:Dscene3d(
    g:addWall(wall), -- 2 facet cutouts with this instruction, and 529 facet cutouts without it
    g:addFacet(S,{color="SteelBlue"}),
    g:addAxes(O,{arrows=1}) )
g:Restoreattr()
g:Saveattr(); g:Viewport(0,10,-4,4); g:Coordsystem(-5,5,-5,5)
local r = function(u,v) return ch(u) end
local z = function(u,v) return sh(u) end
S,wall = ld.cylindrical_surface(r,z,-2,2,-pi,pi,{25,51},"zv")
g:Dscene3d(
    g:addWall(wall), -- 13 facet cutouts with this instruction, and more than 17000 facet cutouts without it ...
    g:addFacet(S,{color="Crimson"}),
    g:addAxes(O,{arrows=1}) )
g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 20: Surfaces using the *addwall* option



curve2cone()

The function **ld.curve2cone(f, t1, t2, S, options)** constructs a cone with vertex $\langle S\rangle$ (3D point) and the base curve parametrized by $\langle f\rangle$: $t \mapsto f(t) \in \mathbf{R}^3$ on the interval defined by $\langle t1\rangle$ et $\langle t2\rangle$. The argument $\langle options\rangle$ is an table whose fields define the options, which are (with their default value):

- `nbdots=15`: minimum number of points on the curve to calculate.
- `ratio=0`: number representing the homothety ratio (centered at vertex $\langle S\rangle$) to construct the other part of the cone, with the value 0 there is no second part.
- `nbdiv=0`: positive integer indicating the number of times the interval between two consecutive values of the parameter t can be bisected (dichotomized) when the corresponding points are too far apart.
- `obj=false` : with the value `false` this function builds a list of facets, with the value `true` it builds a table with three fields: `{vertices={3D points}, facets={{index1,…},…}, normals={3D vectors}}`.

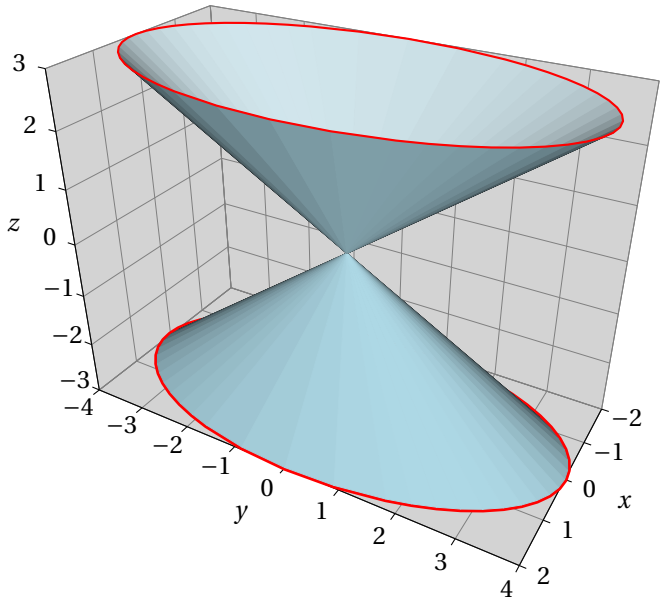
The function returns a sequence:

1. the list of facets or the table in *obj* format, followed by
2. a 3D polygonal line representing the cone's edges.

```
\begin{luadraw}{name=curve2cone}
local ld = luadraw
local O, M = ld.pt3d.Origin, ld.pt3d.M
local cos, sin, pi = math.cos, math.sin, math.pi
local g = ld.graph3d:new{ window3d={-2,2,-4,4,-3,3}, window={-5.5,5.5,-5.5,5},
    size={10,10}, viewdir="central"}
local f = function(t) return M(2*cos(t),4*sin(t),-3) end -- ellipse in the plane z=-3
local C, bord = ld.curve2cone(f,-pi,pi,O,{nbdiv=2, ratio=-1})
g:Dboxaxes3d({grid=true,gridcolor="gray",fillcolor="LightGray"})
g:Dpolyline3d(bord[1],"red,line width=2.4pt") -- bottom edge
```

```
g:Dfacet(C, {mode=ld.mShadedOnly,color="LightBlue"}) -- cone
g:Dpolyline3d(bord[2],"red,line width=0.8pt") -- upper edge
g:Show()
\end{luadraw}
```

Figure 21: Elliptical Cone Example



curve2cylinder()

The function **ld.curve2cylinder(f, t1, t2, V [, options])** constructs a cylinder with axis directed by the vector $\langle V \rangle$ (3D point) and with a base parameterized by $\langle f \rangle$: $t \mapsto f(t) \in \mathbf{R}^3$ on the interval defined by $\langle t1 \rangle$ and $\langle t2 \rangle$. The second base is the translation of the first with the vector V. The argument $\langle options \rangle$ is an table whose fields define the options, which are (with their default value):

- `nbdots=15`: minimum number of points on the curve to calculate.
- `nbdiv=0`: positive integer indicating the number of times the interval between two consecutive values of the parameter t can be bisected (dichotomized) when the corresponding points are too far apart.
- `obj=false` : with the value `false` this function builds a list of facets, with the value `true` it builds a table with three fields: `{vertices={3D points}, facets={{index1,…},…}, normals={3D vectors}}`.

The function returns a sequence:

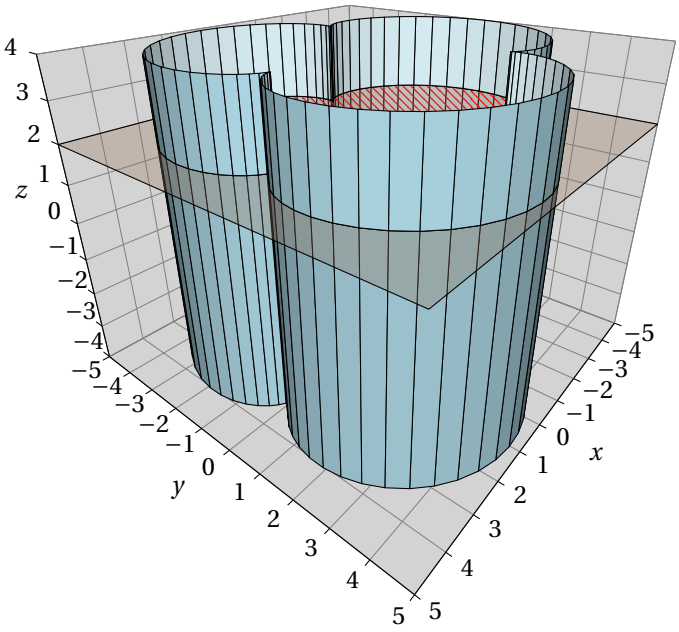
1. the list of facets or the table in *obj* format, followed by
2. a 3D polygonal line representing the cylinder's edges.

```
\begin{luadraw}{name=curve2cylinder}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M
local cos, sin, pi = math.cos, math.sin, math.pi

local g = ld.graph3d:new{ window3d={-5,5,-5,5,-4,4},window={-9,8,-10.5,5.5},
    viewdir={"central",39,64}, size={10,10}}
local f = function(t) return M(4*cos(t)-cos(4*t),4*sin(t)-sin(4*t),-4) end -- curve in the plane z=-3
local V = 8*vecK
local C = ld.curve2cylinder(f,-pi,pi,V,{nbdots=25,nbdiv=2})
local plan = {M(0,0,2), -vecK} -- cutting plane z=2
```

```
local C1, C2, section = ld.cutfacet(C,plan)
g:Dboxaxes3d({grid=true,gridcolor="gray",fillcolor="LightGray"})
g:Dfacet(C1, {mode=ld.mShaded,color="LightBlue"}) -- part below the plane
g:Dfacet(g:Plane2facet(plan), {opacity=0.3,color="Chocolate"}) -- draw the plane as a facet
g:Filloptions("fdiag","red"); g:Dpolyline3d(section) -- drawing of the section
g:Dfacet(C2, {mode=ld.mShaded,color="LightBlue"}) -- part of the cylinder above the plane
g:Show()
\end{luadraw}
```

Figure 22: Section of a non-circular cylinder



line2tube(); section2tube()

The function **ld.line2tube(L, r [, options])** constructs (as a list of facets) a tube centered on ⟨*L*⟩, which must be a 3D polygonal line (list of 3D points or list of lists of 3D points). The argument ⟨*r*⟩ represents the radius of this tube. The argument ⟨*options*⟩ is an table whose fields define the options, which are (with their default value):

- `nbfacet=3`: number of lateral facets of the tube.
- `close=false`: boolean indicating whether the polygonal line should be closed.
- `hollow=false`: boolean indicating whether both ends of the tube should be open or not. When the `close` option is set to `true`, the `hollow` option is automatically set to `true`.
- `angle={0,360}` : by default, the tube is circular; this option allows you to modify the start and end angles (in degrees) so that the cross-section forms only part of a circle.
- `opensection=false` : if set to `true`, the cross-section will not be closed—meaning there will be no facets connecting the last and first points of the section—resulting in a tube with a lateral opening.
- `addwall=0`: number that is 0 or 1. When this option is 1, the function returns, after the tube, a list of separating facets (walls) between each "section" of the tube, which can be useful with the **g:Dscene3d()** method (only).
- `obj=false` : with the value `false` this function returns a list of facets, with the value `true` it returns a table with three fields: `{vertices={3D points}, facets={{index1,…},…}, normals={3D vectors}}`.

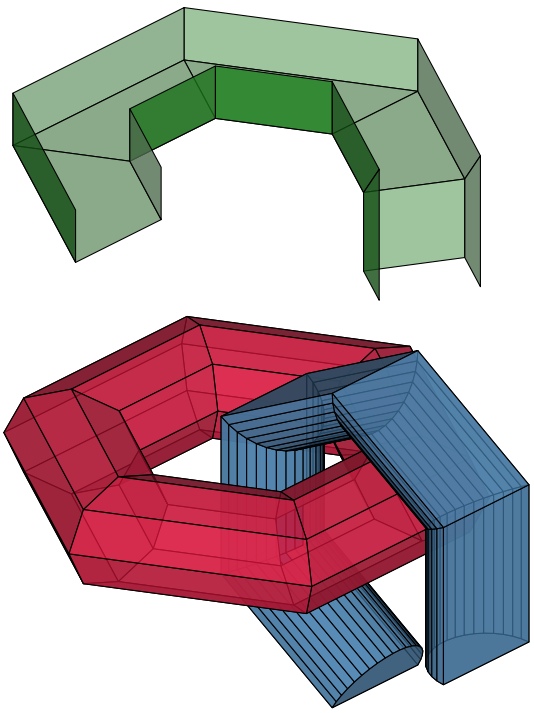
The function **ld.section2tube(section, L [, options])** also constructs a tube centered on ⟨*L*⟩, which must be a list of 3D points. The argument ⟨*section*⟩ must be a facet centered on the first point of ⟨*L*⟩; it represents a section of the tube to be constructed. The argument ⟨*options*⟩ is an table whose fields define the options, which are (with their default value):

- `nbfacet=3`: number of lateral facets of the tube.
- `close=false`: boolean indicating whether the polygonal line should be closed.
- `hollow=false`: boolean indicating whether both ends of the tube should be open or not. When the `close` option is set to `true`, the `hollow` option is automatically set to `true`.
- `opensection=false` : if set to `true`, the cross-section will not be closed—meaning there will be no facets connecting the last and first points of the section—resulting in a tube with a lateral opening.
- `addwall=0`: number that is 0 or 1. When this option is 1, the function returns, after the tube, a list of separating facets (walls) between each "section" of the tube, which can be useful with the **g:Dscene3d()** method (only).
- `obj=false` : with the value `false` this function returns a list of facets, with the value `true` it returns a table with three fields: `{vertices={3D points}, facets={{index1,…},…}, normals={3D vectors}}`.

```
\begin{luadraw}{name=line2tube_section2tube}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-5,6,-4.5,8}, viewdir={45,60}, margin={0,0,0,0}, size={10,10}}
local L1 = ld.map(pt3d.toPoint3d,ld.polyreg(0,3,6)) -- regular hexagon in the xOy plane, with center O and vertex
 ↪ M(3,0,0)
local L2 = ld.shift3d(ld.rotate3d(L1,90,{Origin,vecJ}),3*vecJ)
local L3 = ld.shift3d(ld.reverse(L1),6*vecK)
L3[6] = L3[5]-2*vecK -- modification of the last point
local section = ld.shift3d({M(4,0,0.5),M(4,0,-0.5),M(2,0,-0.5),M(2,0,0.5)},6*vecK)
local T1 = ld.line2tube(L1,1,{nbfacet=8,close=true}) -- tube 1 closed
local T2 = ld.line2tube(L2,1,{nbfacet=20, angle={-90,90}}) -- open semi-circular tube 2
local T3 = ld.section2tube(section, L3,{hollow=true, opensection=true}) -- side opening
g:Dmixfacet( T1, {color="Crimson",opacity=0.8}, T2, {color="SteelBlue"}, T3, {color="ForestGreen"} )
g:Show()
\end{luadraw}
```

Figure 23: Example with line2tube and section2tube



read_table3d()

The function **ld.read_table3d(table, options)** interprets the contents of the argument ⟨*table*⟩. This function returns a table whose fields contain, depending on the options, facets, vertices, edges, or functions.

- By default, the argument ⟨*table*⟩ represents the values of one (or more) function(s) $f_j : (x,y) \mapsto f_j(x,y) \in \mathbb{R}$ $(1 \leqslant j \leqslant n)$ on a tile $[x_1;x_2] \times [y_1;y_2]$, with a certain number of values for x in $[x_1;x_2]$, and a certain number of values for y in $[y_1;y_2]$. The ⟨*table*⟩ is a list of lists of the type:

$$\{x_i, y_i, f_1(x_i,y_i), \ldots, f_n(x_i,y_i)\}$$

These lists must be arranged in lexicographical order according to x and y (or according to y and x). For example, here is the table representing the function $(x,y) \mapsto x^2+y^2$ on $[-1;1] \times [-1;1]$ with 5 values per interval (therefore 25 lists):

```
{ { -1.0, -1.0, 2.0 },
{ -1.0, -0.5, 1.25 },
{ -1.0, 0.0, 1.0 },
{ -1.0, 0.5, 1.25 },
{ -1.0, 1.0, 2.0 },
{ -0.5, -1.0, 1.25 },
{ -0.5, -0.5, 0.5 },
{ -0.5, 0.0, 0.25 },
{ -0.5, 0.5, 0.5 },
{ -0.5, 1.0, 1.25 },
{ 0.0, -1.0, 1.0 },
{ 0.0, -0.5, 0.25 },
{ 0.0, 0.0, 0.0 },
{ 0.0, 0.5, 0.25 },
{ 0.0, 1.0, 1.0 },
{ 0.5, -1.0, 1.25 },
{ 0.5, -0.5, 0.5 },
{ 0.5, 0.0, 0.25 },
{ 0.5, 0.5, 0.5 },
{ 0.5, 1.0, 1.25 },
{ 1.0, -1.0, 2.0 },
{ 1.0, -0.5, 1.25 },
{ 1.0, 0.0, 1.0 },
{ 1.0, 0.5, 1.25 },
{ 1.0, 1.0, 2.0 } }
```

Such tables can also be read from a file using the **ld.read_csv_file()** function, one list per line.

Note: All values in the table must be numeric.

- The argument ⟨*options*⟩ is a table whose fields are the options, which are (with their default values):
  - `mode="xyz"` : a character string defining the table type. The `"xyz"` mode (default) corresponds to the type described above. There are two other possible modes:
    - `mode="y/x"`, in which case the table has the following format:

```
      x1  x2  ... xn
y1  z11 z12 ... z1n
y2  z21 z22 ... z2n
...
yp  zp1 zp2 ... zpn
```

    - `mode="x/y"`, in which case the table has the following format:

```
      y1  y2  ... yp
x1  z11 z12 ... z1p
```

```
x2  z21 z22 ... z2p
...
xn  zn1 zn2 ... znp
```

- `header=nil`, a list of strings representing the column names (none by default).
- `x=1`, `y=2`, `z=3`: column numbers representing the values of x, y, and z, respectively. When an `header` option has been specified, column names can be used instead of numbers.
- `func={}`: list of column numbers (or column names if the `header` option has been specified). Each of the listed columns will be transformed according to x and y (by linear interpolation). The returned table will have a field called *func* containing the list of these functions in order (if there is only one function, then the *func* field will contain that function).
- `facets=true`: With the value `true` (default), the returned table will have a field called *facets* containing the list of facets (a facet is a list of 3D points).
- `edges=false`: With the value `true`, the returned table will have a field called *edges* containing the outlines of the rectangular facets (3D polygonal lines).
- `vertices=false`: With the value `true`, the returned table will have a field called *vertices* containing the 3D points $M(x,y,z)$ of the table.
- `triangle=false`: With the value `true`, the facets will be triangular, not rectangular.
- `bbox=false`: With the value `true`, the returned table will have a field called *bbox* containing a table of the form $\{x_1,x_2,y_1,y_1,z_1,z_2\}$ representing the bounding box.

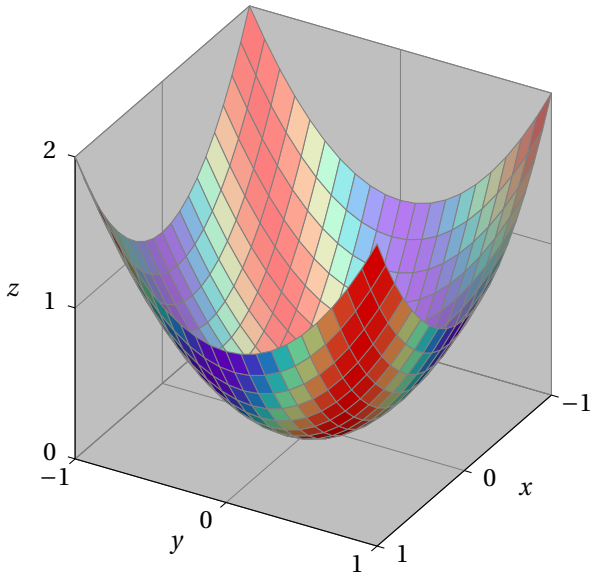
For an example, we create the file *data3d.csv* with the following code:

```
local ld = luadraw
local f = io.open(ld.cachedir.."data3d.csv", "w")
local h = function(x,y) return x^2+y^2 end
local color = function(x,y) return math.cos(math.pi*(x-y)) end
local xvalues, yvalues = ld.linspace(-1,1,21), ld.linspace(-1,1,21)
for _,x in ipairs(xvalues) do
    for _,y in ipairs(yvalues) do
        f:write( x.." "..y.." "..h(x,y).." "..color(x,y).."\n")
    end
end
f:close()
```

Reading and using the file:

```
\begin{luadraw}{name=read_table3d}
local ld = luadraw
local pt3d  = ld.pt3d
local data3d = ld.read_csv_file( ld.cachedir.."data3d.csv", {header=false, sep=" "})
local S = ld.read_table3d(data3d, {bbox=true, func=4})
local color = function(f) -- f is a facet
    local G = pt3d.isobar3d(f)
    return S.func(G.x,G.y)
end
local g = ld.graph3d:new{ window3d=S.bbox, adjust2d=true, size={10,10} }
g:Dboxaxes3d({grid=true, gridcolor="gray", fillcolor="lightgray"})
g:Dfacet( S.facets, {usepalette={ld.palRainbow,color}, edgecolor=ld.Gray, reverse=true})
g:Show()
\end{luadraw}
```

Figure 24: The function *ld.read_table3d()*



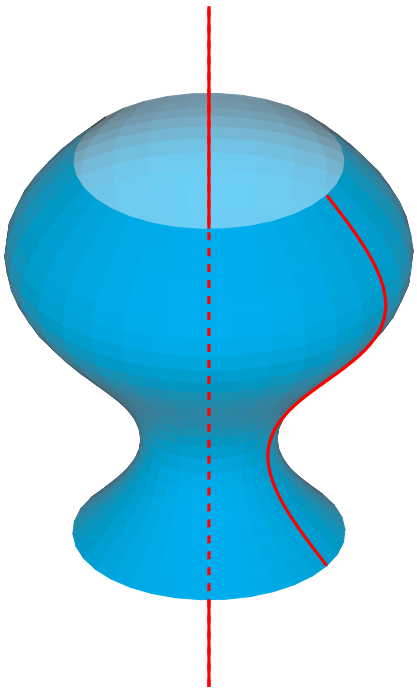
rotcurve()

The function **ld.rotcurve(p, t1, t2, axe, angle1, angle2 [, options])** constructs, as a list of facets, the surface swept by the curve parameterized by ⟨*p*⟩: $t \mapsto p(t) \in \mathbf{R}^3$ over the interval defined by ⟨*t1*⟩ and ⟨*t2*⟩, by rotating it around ⟨*axe*⟩ (which is a table of the form {3D point, 3D vector} representing an oriented line in space), by an angle ranging from ⟨*angle1*⟩ (in degrees) to ⟨*angle2*⟩. The argument ⟨*options*⟩ is an table whose fields define the options, which are (with their default value):

- `grid={25,25}`: table consisting of two numbers, the first being the number of points calculated for the t parameter, and the second being the number of points calculated for the angular parameter.
- `addwall=0`: number equal to 0, 1, or 2. When this option is set to 1, the function returns, after the surface, a list of separating facets (walls) between each "layer" of facets (a layer corresponds to two consecutive values of the *t* parameter), and with a value of 2, it is a list of separating facets (walls) between each rotation "slice" (a layer corresponds to two consecutive values of the angular parameter; this is useful when the curve is in the same plane as the rotation axis). This option can be useful with the **g:Dscene3d()** method (only).
- `obj=false` : with the value `false` this function returns a list of facets, with the value `true` it returns a table with three fields: `{vertices={3D points}, facets={{index1,…},…}, normals={3D vectors}}`.

```
\begin{luadraw}{name=rotcurve}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M
local cos, sin, pi = math.cos, math.sin, math.pi
local g = ld.graph3d:new{viewdir={30,60},size={10,10}}
local p = function(t) return M(0,sin(t)+2,t) end -- curve in the plane yOz
local axe = {Origin,vecK}
local S = ld.rotcurve(p,pi,-pi,axe,0,360,{grid={25,35}})
local visible, hidden = g:Classifyfacet(S)
g:Dfacet(hidden, {mode=ld.mShadedOnly,color="cyan"})
g:Dline3d(axe,"red,line width=1.2pt")
g:Dfacet(visible, {mode=ld.mShadedOnly,color="cyan"})
g:Dline3d(axe,"red,line width=1.2pt,dashed")
g:Dparametric3d(p,{t={-pi,pi},draw_options="red,line width=1.2pt"})
g:Show()
\end{luadraw}
```

Figure 25: Example with rotcurve



Note : If the surface orientation does not seem correct, simply swap the parameters ⟨*t1*⟩ and ⟨*t2*⟩, or ⟨*angle1*⟩ and ⟨*angle2*⟩.

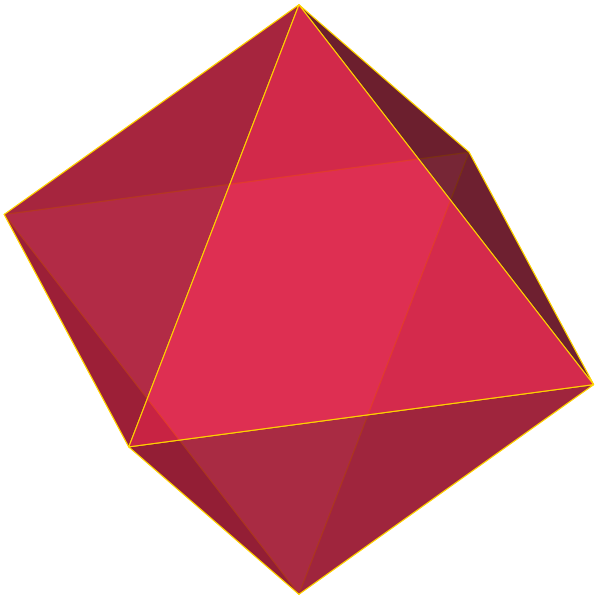
rotline()

The function **ld.rotline(L, axe, angle1, angle2 [, options])** constructs, as a list of facets, the surface swept by the list of 3D points ⟨*L*⟩ by rotating it around ⟨*axe*⟩ (which is a table of the form {3D point, 3D vector} representing an oriented line in space), through an angle ranging from ⟨*angle1*⟩ (in degrees) to ⟨*angle2*⟩. The argument ⟨*options*⟩ is an table whose fields define the options, which are (with their default value):

- `nbdots=25`: number of points calculated for the angular parameter.
- `close=false`: boolean indicating whether ⟨*L*⟩ should be closed.
- `addwall=0`: number equal to 0, 1, or 2. When this option is set to 1, the function returns, after the surface, a list of separating facets (walls) between each "layer" of facets (a layer corresponds to two consecutive values of the *t* parameter), and with a value of 2, it is a list of separating facets (walls) between each rotation "slice" (a layer corresponds to two consecutive values of the angular parameter; this is useful when the curve is in the same plane as the rotation axis). This option can be useful with the **g:Dscene3d()** method (only).
- `obj=false` : with the value `false` this function returns a list of facets, with the value `true` it returns a table with three fields: `{vertices={3D points}, facets={{index1,…},…}, normals={3D vectors}}`.

```
\begin{luadraw}{name=rotline}
local ld = luadraw
local pt3d = ld.pt3d
local M = pt3d.M
local g = ld.graph3d:new{window={-4,4,-4,4},size={10,10}}
local L = {M(0,0,4),M(0,4,0),M(0,0,-4)} -- list of points in the yz plane
local axe = {pt3d.Origin, pt3d.vecK}
local S = ld.rotline(L,axe,0,360,{nbdots=5}) -- point 1 and point 5 are confused
g:Dfacet(S,{color="Crimson",edgecolor="Gold",opacity=0.8})
g:Show()
\end{luadraw}
```

Figure 26: Example with rotline



10) Edges of a solid

An "edge" object is a table with two fields: one field named *visible* that contains a 3D polygonal line corresponding to the visible edges, and another field named *hidden* that contains a 3D polygonal line corresponding to the hidden edges.

- The method **g:Edges(P)**, where ⟨*P*⟩ is a polyhedron, returns the edges of ⟨*P*⟩ as an "edge" object. An edge of ⟨*P*⟩ is visible when it belongs to at least one visible face.
- The method **g:Intersection3d(P, plane)**, where ⟨*P*⟩ is a polyhedron or a list of facets, returns as an "edge" object the intersection between ⟨*P*⟩ and the plane represented by ⟨*plane*⟩ (it is a table of the form {A,u} where *A* is a point on the plane and *u* is a normal vector, so they are two 3D points).
- The method **g:Dedges(edges, options)** allows you to draw ⟨*edges*⟩, which must be an "edge" object. The argument ⟨*options*⟩ is an table whose fields define the options, which are (with their default value):
  - `hidden=false`: Boolean indicating whether hidden edges should be drawn.
  - `visible=true`: Boolean indicating whether visible edges should be drawn.
  - `clip=false`: Boolean indicating whether edges should be clipped by the 3D window.
  - `hiddenstyle=ld.Hiddenlinestyle`: String defining the line style of hidden edges. By default, this option contains the value of the global variable `ld.Hiddenlinestyle` (which defaults to `"dotted"`).
  - `hiddencolor=color`: String defining the color of hidden edges. By default, this option contains the same color as the `color` option.
  - `style=<current line style>`: String defining the line style of visible edges.
  - `color=<current line color>`: String defining the color of the visible edges.
  - `width=<current width>`: Number representing the line thickness of the edges (in tenths of a point).
- **Complement**:
  - The function **ld.facetedges(F)**, where ⟨*F*⟩ is a list of facets or a polyhedron, returns a list of 3D segments representing all the edges of ⟨*F*⟩. The result is not an "edge" object, and is drawn with the **g:Dpolyline3d()** method. Of course, each edge only appears once in the list.
  - The function **ld.facetvertices(F)**, where ⟨*F*⟩ is a list of facets or a polyhedron, returns the list of all vertices of ⟨*F*⟩ (3D points).

11) Methods and functions applying to facets or polyhedra

- The method **g:Isvisible(F)**, where ⟨*F*⟩ denotes **a** facet (list of at least 3 coplanar and non-aligned 3D points), returns true if facet ⟨*F*⟩ is visible (normal vector directed towards the observer). If *A*, *B*, and *C* are the first three points of ⟨*F*⟩, the normal vector is calculated by performing the vector product $\vec{AB} \wedge \vec{AC}$.

- The method **g:Classifyfacet(F)**, where ⟨*F*⟩ is a list of facets or a polyhedron, returns **two** lists of facets: the first is the list of visible facets, and the second, the list of invisible facets.

- The method **g:Sortfacet(F [, backcull])**, where ⟨*F*⟩ is a list of facets, returns this list of facets sorted from furthest to closest to the observer. The optional argument ⟨*backcull*⟩ is a boolean that defaults to `false`; when it is `true`, non-visible facets are excluded from the result (only visible facets are then returned after being sorted). The calculation of a facet's distance is based on its center of gravity. The so-called "painter" technique consists of displaying the facets from furthest to closest, therefore in the order of the list returned by this function (the displayed result, however, is not always correct depending on the size and shape of the facets).

- The method **g:Sortpolyfacet(P [, backcull])**, where ⟨*P*⟩ is a polyhedron, returns the list of facets of ⟨*P*⟩ (facets with 3D points) sorted from furthest to closest to the observer. The optional argument ⟨*backcull*⟩ is a boolean that defaults to `false`; when it is `true`, invisible facets are excluded from the result, as in the previous method. These two sorting methods are used by the methods for drawing polyhedrons or facets (**g:Dpoly()**, **g:Dfacet()** and **g:Dmixfacet()**).

- The method **g:Outline(P)**, where ⟨*P*⟩ is a polyhedron, returns the "outline" of ⟨*P*⟩ as a two-field table. One field, named *visible*, contains a 3D polygonal line representing the "edges" (segments) belonging to a single facet, which is visible, or to two facets, one visible and one hidden; the other field, named *hidden*, contains a 3D polygonal line representing the "edges" belonging to a single facet, which is hidden.

- The function **ld.border(P)**, where ⟨*P*⟩ is a polyhedron or a list of facets, returns a 3D polygonal line corresponding to the edges belonging to a single facet of ⟨*P*⟩ (these edges are placed "end to end" to form a polygonal line).

- The function **ld.getfacet(P, list)**, where ⟨*P*⟩ is a polyhedron, returns the list of facets of ⟨*P*⟩ (with 3D points) whose number appears in the table ⟨*list*⟩. If the argument ⟨*list*⟩ is not specified, the list of all facets of ⟨*P*⟩ is returned (in this case, it is the same as **ld.poly2facet(P)**).

- The function **ld.facet2plane(L)**, where ⟨*L*⟩ is either a facet or a list of facets, returns either the plane containing the facet or the list of planes containing each of the facets of ⟨*L*⟩. A plane is a table of the type {A,u} where *A* is a point on the plane and *u* is a normal vector to the plane (i.e., two 3D points).

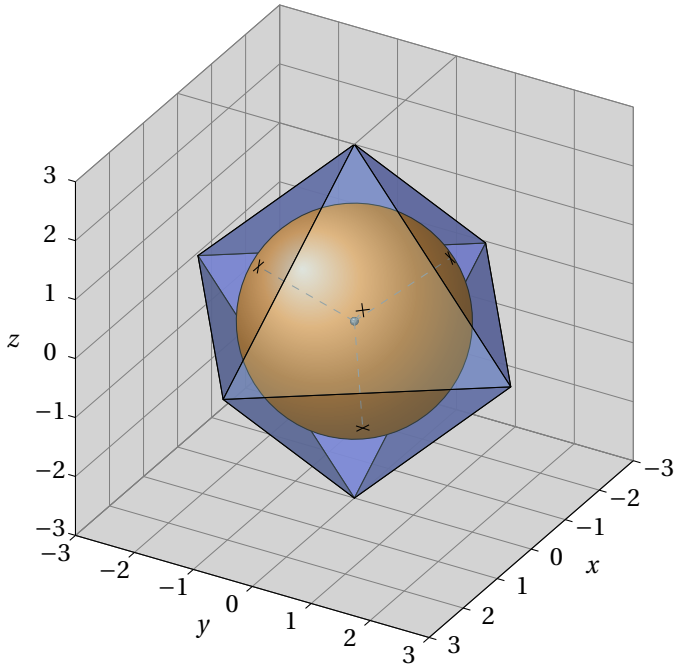
- The function **ld.reverse_face_orientation(F)** where ⟨*F*⟩ is either a facet, a list of facets, or a polyhedron, returns a result of the same nature as ⟨*F*⟩ but in which the order of the vertices of each facet has been reversed. This can be useful when the orientation of space has been changed.

```
\begin{luadraw}{name=sphere_octaedre}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local poly = require "luadraw_polyhedrons"
local g = ld.graph3d:new{ window3d={-3,3,-3,3,-3,3}, size={10,10}}
local P = poly.octahedron(Origin,M(0,0,3)) -- polyhedron defined in the luadraw_polyhedrons module
P = ld.rotate3d(P,-10,{Origin,vecK}) -- rotate3d on a polyhedron returns a polyhedron
local V, H = g:Classifyfacet(P) -- V for visible facets, H for hidden
local S = ld.map(function(p) return {ld.proj3d(Origin,p),p[2]} end, ld.facet2plane(V) )
-- S contains the list of: {projected, normal vector} (projected from Origin onto the visible faces)
local R = pt3d.abs(S[1][1]) -- sphere radius
g:Dboxaxes3d({grid=true, gridcolor="gray", fillcolor="LightGray"})
g:Dfacet(H, {color="blue",opacity=0.9}) -- drawing of non-visible facets
g:Dsphere(Origin,R,{mode=ld.mBorder,color="orange"}) -- drawing of the sphere
g:Dballdots3d(Origin,"gray",0.75) -- center of the sphere
for _,D in ipairs(S) do -- segments connecting the origin to the projected
    g:Dpolyline3d( {Origin,D[1]},"dashed,gray")
```

```
end
g:Dfacet(V,{opacity=0.4, color="LightBlue"}) -- visible facets of the octahedron
g:Dcrossdots3d(S,nil,0.75) -- drawing of the projections on the faces
g:Dpolyline3d( {M(0,-3,3), M(0,0,3), M(-3,0,3)},"gray")
g:Show()
\end{luadraw}
```

Figure 27: Sphere inscribed in an octahedron with the center projected onto the faces



12) Cutting a solid: cutpoly, cutfacet and cutfacet2

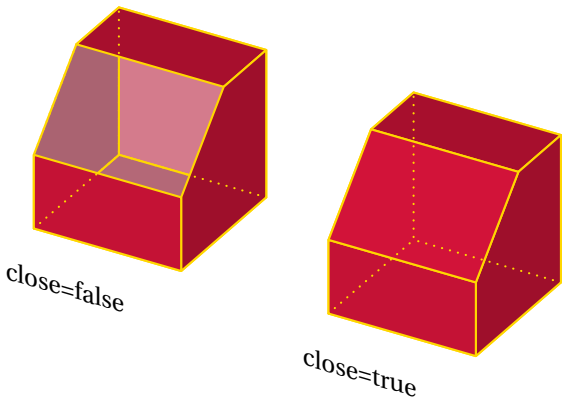
- The function **ld.cutpoly(P, plane [, close])** cuts the polyhedron ⟨*P*⟩ with the plane ⟨*plane*⟩ (a table of type {A,n} where *A* is a point on the plane and *n* is a vector normal to the plane). The function returns three things: the part located in the half-space containing the vector *n* (in the form of a polyhedron), followed by the part located in the other half-space (still in the form of a polyhedron), followed by the section in the form of a facet oriented by $-n$. When the optional argument ⟨*close*⟩ is `true`, the section is added to both resulting polyhedra, which closes them (`false` by default).

  Note: When the polyhedron ⟨*P*⟩ is not convex, the section result is not always correct.

```
\begin{luadraw}{name=cutpoly}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window3d={-3,3,-3,3,-3,3}, window={-4,4,-3,3},size={10,10}}
local P = ld.parallelep(M(-1,-1,-1),2*vecI,2*vecJ,2*vecK)
local A, B, C = M(0,-1,1), M(0,1,1), M(1,-1,0)
local plane = {A, pt3d.prod(B-A,C-A)}
local P1 = ld.cutpoly(P,plane)
local P2 = ld.cutpoly(P,plane,true)
g:Lineoptions(nil,"Gold",8)
g:Dpoly( ld.shift3d(P1,-2*vecJ), {color="Crimson",mode=ld.mShadedHidden} )
g:Dpoly( ld.shift3d(P2,2*vecJ), {color="Crimson",mode=ld.mShadedHidden} )
g:Dlabel3d(
    "close=false", M(2,-2,-1), {dir={vecJ,vecK}},
    "close=true", M(2,2,-1), {} )
g:Show()
\end{luadraw}
```

Figure 28: Cube cut by a plane (cutpoly), with *close*=false and with *close*=true



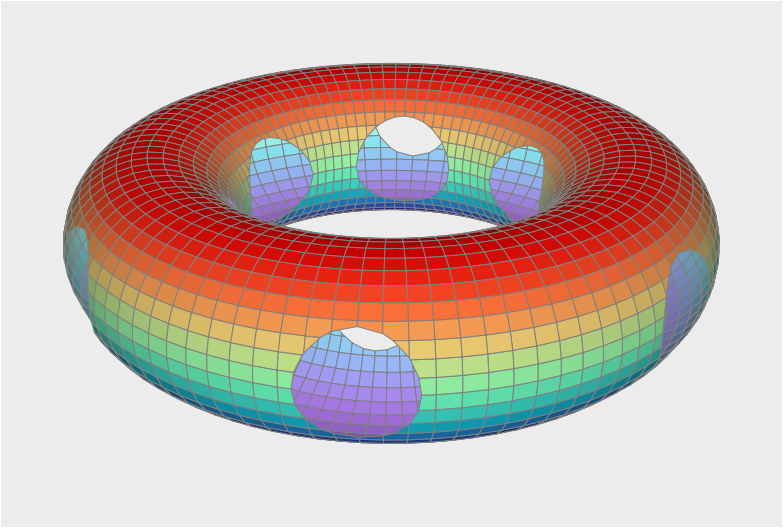
- The function **ld.cutfacet(F, plane [, close])**, where ⟨*F*⟩ is a facet, a list of facets, or a polyhedron, does the same thing as the previous function except that this function returns lists of facets and not polyhedra. This function was used in the contour line example in Figure 19.

- The function **ld.cutfacet2(F, f, sign)** also allows for cutting/subdivision. The argument ⟨*F*⟩ can be a facet, a list of facets, or a polyhedron. The argument ⟨*f*⟩ is a function ⟨*f*⟩: $\mathbf{R}^3 \to \mathbf{R}$, and the argument ⟨*sign*⟩ is either the string `">"` or the string `"<"`. The facets are cut by the surface defined by the equation $f(x,y,z)=0$; the function returns two lists of facets. The first list contains the facets that satisfy $f(x,y,z)>0$ when ⟨*sign*⟩=`">"` (or those that satisfy $f(x,y,z)<0$ when ⟨*sign*⟩=`"<"`).

  Note: The function f must be continuous and must change sign **at most once** along each edge.

```
\begin{luadraw}{name=torus_with_holes}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, pt3d.Mc
local g = ld.graph3d:new{ window={-4,4,-3,2}, bg="lightgray!30", viewdir={"central",5,65}, size={10,10} }
local cos, sin, pi = math.cos, math.sin, math.pi
local R, r, rc = 2.75, 1, 0.6
local f = function(t) return M(R+r*cos(t),0,r*sin(t)) end
local C  = ld.rotcurve(f,-pi,pi,{Origin,vecK}, 30,-30, {grid={37,17}}) -- one-sixth of the torus
local f = function(x,y,z) return y^2+z^2-rc^2 end -- f(x,y,z)=0 is the cylinder equation
C = ld.cutfacet2(C,f,">") -- C is cut out using the cylinder.
local torus = table.copy(C)
for k = 1, 5 do -- the entire torus is reconstructed
    torus = ld.concat(torus, ld.rotate3d(C,k*60,{Origin,vecK}))
end
g:Dfacet( torus, {usepalette={ld.palRainbow,"z"}, edgecolor="gray", edgewidth=1})
g:Show()
\end{luadraw}
```

Figure 29: The function *ld.cutfacet2*



13) Clipping Facets with a Convex Polyhedron: clip3d

The function **ld.clip3d(S, P [, exterior])** clips the solid ⟨*S*⟩ (which is a list of facets or a polyhedron) with the convex solid ⟨*P*⟩ (which is a list of facets or a polyhedron) and returns the resulting list of facets. The optional argument ⟨*exterior*⟩ is a boolean that defaults to `false`. In this case, the part of ⟨*S*⟩ that is interior to ⟨*P*⟩ is returned; otherwise, the part of ⟨*S*⟩ that is exterior to ⟨*P*⟩ is returned.

Note: The result is not always satisfactory for the exterior part.

Special case : Clipping a list of facets *S* (or polyhedron) with the current 3D window can be done with this function as follows:

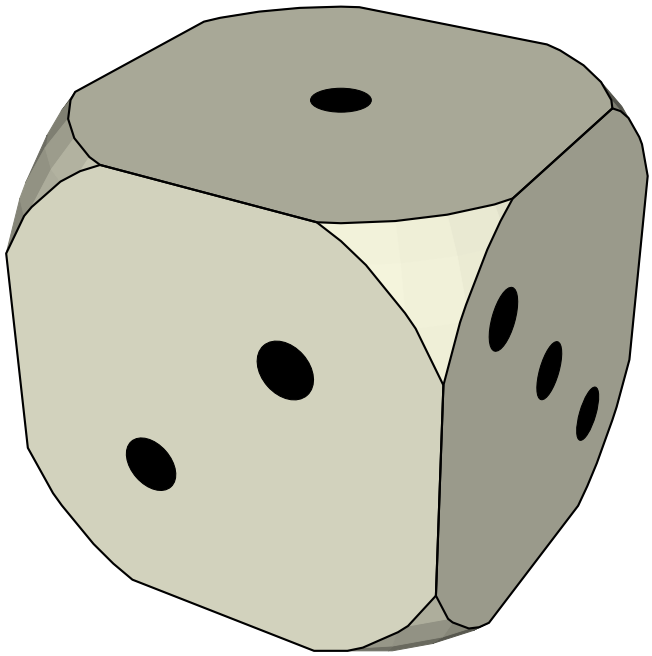
S = ld.clip3d(S, g:Box3d())

Indeed, the **g:Box3d()** method returns the current 3D window as a parallelepiped.

```
\begin{luadraw}{name=clip3d}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-3,3,-3,3},size={10,10}, viewdir="central"}
local S = ld.sphere(Origin,3)
local C = ld.parallelep(M(-2,-2,-2),4*vecI,4*vecJ,4*vecK)
local C1 = ld.clip3d(S,C) -- sphere clipped by the cube
local C2 = ld.clip3d(C,S) -- cube clipped by the sphere
local V = g:Classifyfacet(C2) -- visible facets of C2
g:Dfacet( ld.concat(C1,C2), {color="Beige",mode=ld.mShadedOnly,backcull=true} ) -- only visible faces
g:Dpolyline3d(V,true,"line width=0.8pt") -- outline of the visible faces of C2
local A, B, C, D = M(2,-2,-2), M(2,2,2), M(-2,2,-2), M(0,0,2) -- drawing black dots
g:Filloptions("full","black")
g:Dcircle3d( D,0.25,vecK); g:Dcircle3d( (2*A+B)/3,0.25,vecI)
g:Dcircle3d( (A+2*B)/3,0.25,vecI); g:Dcircle3d( (3*B+C)/4,0.25,vecJ)
g:Dcircle3d( (B+C)/2,0.25,vecJ); g:Dcircle3d( (B+3*C)/4,0.25,vecJ)
g:Show()
\end{luadraw}
```

Figure 30: Example with clip3d: constructing a die from a cube and a sphere



14) Clip a plane with a convex polyhedron: clipplane

The function **ld.clipplane(plane, P)**, where the argument ⟨*plane*⟩ is a table of the form *{A,n}* representing the plane passing through A (3D point) and normal vector n (non-zero 3D point), and ⟨P⟩ is a convex polyhedron, returns the section of the polyhedron through the plane, if it exists, in the form of a facet (list of 3D points) oriented by n.

V The Dscene3d Method

1) The Principle, the Limitations

The major flaw of the **g:Dpoly()**, **g:Dfacet()**, and **g:Dmixfacet()** methods is that they do not handle possible intersections between facets of different solids. Not to mention that sometimes, even for a simple convex polyhedron, the painter's algorithm does not always produce the correct result (because the facets are sorted only by their center of gravity). Furthermore, these methods only allow you to draw facets.

The principle of the **g:Dscene3d()** method is to classify the 3D objects to be drawn (facets, polygonal lines, points, labels, etc.) in a tree (which represents the scene). At each node of the tree, there is a 3D object, let's call it A, and two descendants. One of the descendants will contain the 3D objects that are in front of object A (i.e., closer to the observer than A), and the other descendant will contain the 3D objects that are behind object A (i.e., further from the observer than A).

In particular, to classify a facet B with respect to a facet A that is already in the tree, we proceed as follows: we split facet B with the plane containing facet A, which generally results in two "half" facets, one that will be in front of A (the one in the half-space "containing" the observer), and the other that will therefore be behind A.

This method is effective but has limitations because it can cause the number of facets in the tree to explode, thus increasing its size exponentially. This can make it prohibitive to use this method when there are many facets (long computation time[6], excessively large **.tkz* file size, excessively long drawing time per TikZ). However, it is very effective when there are few facets, and therefore few facet intersections (convex objects with few facets). Furthermore, it is possible to draw below and above the 3D scene, i.e., before using the **g:Dscene3d()** method, and after its use.

This method should therefore be reserved for very simple scenes. For complex 3D scenes, the vector format is not suitable, so it is better to turn to other tools like POV-Ray, Blender, or WebGL.

[6]Lua is an interpreted language, so execution is generally slower than with a compiled language.

2) Construction of a 3D scene

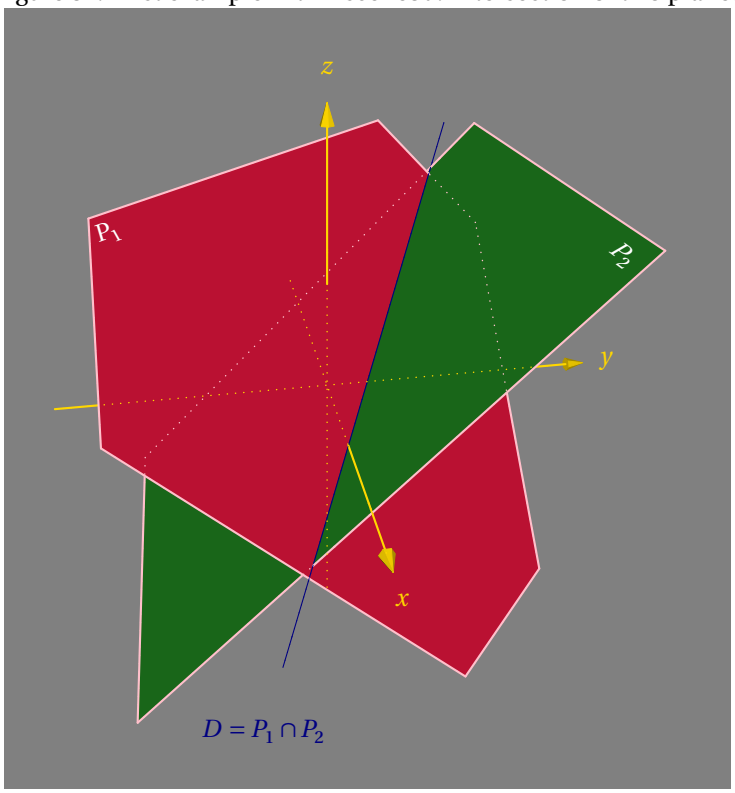
The **g:Dscene3d(...)** method allows this construction. It takes as arguments the 3D objects that will constitute this scene one after the other. These 3D objects are themselves created using dedicated methods that will be detailed later. In the current version, these 3D objects can be:

- polyhedra,
- lists of facets (with 3D points),
- 3D polygonal lines,
- 3D points,
- labels,
- axes,
- planes, lines,
- angles,
- circles, and arcs.

```
\begin{luadraw}{name=intersection_plans}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{viewdir={"central",-10,60}, window={-5,6.5,-6.5,6},bg="gray", size={10,10}}
local P1 = ld.planeEq(1,1,1,-2) -- plane of equation x+y+z-2=0
local P2 = {Origin, vecK-vecJ} -- plane passing through O and normal to (0,-1,1)
local D = ld.interPP(P1,P2) -- line of intersection between P1 and P2 (D = {A,u})
local posD = D[1]+1.85*D[2] -- to place the label
ld.Hiddenlines = true; ld.Hiddenlinestyle = "dotted" -- display of hidden lines as dotted lines
g:Dscene3d(
    g:addPlane(P1, {color="Crimson",edge=true,edgecolor="Pink",edgewidth=8}), --addition of plane P1
    g:addPlane(P2, {color="ForestGreen",edge=true,edgecolor="Pink",edgewidth=8}), --addition of plane P2
    g:addLine(D, {color="Navy",edgewidth=12}), --addition of line D
    g:addAxes(Origin, {arrows=1, color="Gold",width=8}), -- addition of arrowed axes
    g:addLabel( -- adding labels; these could have been added on top of the scene.
        "$D=P_1\\cap P_2$",posD,{color="Navy"},
        "$P_2$", M(3,0,0)+3.5*M(0,1,1)+0.2*vecI,{color="white",dir={vecI,vecJ+vecK}},
        "$P_1$",M(2,0,0)+1.85*M(-1,-1,2)-1.5*M(-1,1,0), {dir={M(-1,1,0),M(-1,-1,2)}} )
)
g:Show()
\end{luadraw}
```

Figure 31: First example with Dscene3d: intersection of two planes



3) Methods for adding an object to the 3D scene

These methods are to be used as arguments to the **g:Dscene3d(...)** method, as in the example above.

Adding facets: g:addFacet and g:addPoly

The **g:addFacet(F, options)** method, where ⟨*F*⟩ is a facet or a list of facets (with 3D points), allows you to add these facets to the scene.

The **g:addPoly(P, options)** method allows you to add the polyhedron ⟨*P*⟩ to the scene.

In both cases, the ⟨*options*⟩ argument is a table whose fields define these options (with their default values) are:

- `color="white"`: Sets the fill color of the facets. This color will be shaded depending on their inclination. By default, the edges of the facets are not drawn (only the fill).
- `usepalette=nil`, this option allows you to specify a color palette for painting the facets as well as a calculation mode, the syntax is: `usepalette={palette,mode[,minmax]}`, where ⟨*palette*⟩ designates a table of colors which are themselves tables of the form $\{r, g, b\}$ where r, g and b are numbers between 0 and 1. The ⟨*mode*⟩ argument can be :
  - one of the strings: `"x"`, `"y"`, `"z"`. In the first case, for example, facets with the minimum x-coordinate at their centroid have the first color in the palette, facets with the maximum x-coordinate at their centroid have the last color in the palette, and for the others, the color is calculated based on the x-coordinate of the centroid by linear interpolation.
  - a function: ⟨*mode*⟩ : $f \mapsto \text{mode}(f) \in \mathbb{R}$, where f denotes a facet (a list of 3D points). Facets with the minimum value have the first color in the palette, those with the maximum value have the last color in the palette, and for the others, the color is calculated by linear interpolation.

  The ⟨*minmax*⟩ argument is a table of two numbers; these two numbers are used as the minimum and maximum values for the ⟨*mode*⟩ argument. If ⟨*minmax*⟩ is omitted, these two values are calculated automatically.
- `opacity=1`: Number between 0 and 1 to define the opacity of the facets (1 means no transparency).
- `backcull=false`: Boolean indicating whether invisible facets should be excluded from the scene. By default, they are present.

- `clip=false`: Boolean indicating whether the facets should be clipped by the 3D window.
- `contrast=1`: Numerical value to increase or decrease the color contrast between the facets. With a value of 0, all facets have the same color.
- `twoside=true`: Boolean indicating whether the inner and outer sides of the facets are distinguished. The color of the inner side is slightly lighter than that of the outer side.
- `reverse=false` : with the value `true` the orientation of the facets is reversed.
- `edge=false`: Boolean indicating whether edges should be added to the scene.
- `edgecolor=<current line color>`: Indicates the color of the edges when they are drawn.
- `edgewidth=<current line width>`: Indicates the line thickness (in tenths of a point) of the edges.
- `hidden=ld.Hiddenlines`: Boolean indicating whether hidden edges should be drawn. `ld.Hiddenlines` is a global variable that defaults to `false`.
- `hiddenstyle=ld.Hiddenlinestyle`: String defining the line style of the hidden edges. `ld.Hiddenlinestyle` is a global variable that defaults to `"dotted"`.
- `hiddenscale=ld.Hiddenlinescale`: a number representing a percentage; the thickness of hidden lines is equal to that of visible lines multiplied by this number. `ld.Hiddenlinescale` is a global variable that defaults to `2/3`.
- `matrix=ld.ID3d`: 3D facet transformation matrix, by default this is the 3D identity matrix, i.e. the table:

```
{M(0,0,0),vecI,vecJ,vecK}
```

Adding a plane: g:addPlane and g:addPlaneEq

The **g:addPlane(P, options)** method adds the plane $\langle P \rangle$ to the 3D scene. This plane is defined as a table $\{A, u\}$ where A is a point on the plane (a 3D point) and u is a normal vector to the plane (a non-zero 3D point). This function determines the intersection between this plane and the parallelepiped given by the `window3d` argument (itself defined when the graph is created), which results in a facet, which is added to the scene. This method uses **g:addFacet()**.

The method **g:addPlaneEq(coef, options)**, where $\langle coef \rangle$ is a table consisting of four real numbers $\{a, b, c, d\}$, allows you to add the plane with the equation $ax + by + cz + d = 0$ to the scene (this method uses the previous one).

In both cases, the optional argument $\langle options \rangle$ is a table whose fields define the options. These options are those of the **g:addFacet()** method, plus the options:

- `rectangle=nil`: With the value `nil`, the plane is intersected with the 3D window, and the resulting facet will be drawn. It is not necessarily rectangular. With `rectangle={V,L1,L2}`, the drawn facet will be rectangular with sides of lengths $\langle L1 \rangle$ and $\langle L2 \rangle$, and the vector $\langle V \rangle$ (which must belong to the plane) will determine the direction of one side of the rectangle. This vector $\langle V \rangle$ is optional; if it is omitted, the function will choose it itself. The argument $\langle L2 \rangle$ is also optional; if omitted, it implicitly has the same value as $\langle L1 \rangle$.
- `scale=1`: This number is a scaling ratio; it is only taken into account when `rectangle=nil`. In this case, the facet is scaled with a scale ratio of `scale`. This allows you to adjust the size of the plane in its representation.

Add a polygonal line: g:addPolyline

The method **g:addPolyline(L, options)**, where $\langle L \rangle$ is a list of 3D points, or a list of lists of 3D points, adds $\langle L \rangle$ to the scene. The $\langle options \rangle$ argument is a table whose fields define these options (with their default values) are:

- `style=<current line style>`: to set the line style; this is the current style by default.
- `color=<current line color>`: line color.
- `close=false`: indicates whether the line $\langle L \rangle$ (or each component of $\langle L \rangle$) should be closed.
- `clip=false`: Indicates whether the line $\langle L \rangle$ (or each component of $\langle L \rangle$) should be clipped by the 3D window.

- `width=<current line width>`: line thickness in tenths of a point.
- `opacity=1`: opacity of the line drawing (1 means no transparency).
- `hidden=ld.Hiddenlines`: boolean indicating whether the hidden parts of the line should be represented. `ld.Hiddenlines` is a global variable that defaults to `false`.
- `hiddenstyle=ld.Hiddenlinestyle`: string defining the line style of the hidden parts. `ld.Hiddenlinestyle` is a global variable that defaults to `"dotted"`.
- `hiddenscale=ld.Hiddenlinescale`: a number representing a percentage; the thickness of hidden lines is equal to that of visible lines multiplied by this number. `ld.Hiddenlinescale` is a global variable that defaults to `2/3`.
- `arrows=0`: this option can be 0 (no arrow added to the line), 1 (an arrow added at the end of the line), or 2 (an arrow at the beginning and end of the line). The arrows are small cones.
- `arrowscale={1,1}`: table of two numbers, the first is a scale factor for the radius of the base of the arrows (these are cones), the second is a scale factor for the height of the arrows.
- `matrix=ld.ID3d`: 3D transformation matrix (of the line). By default, this is the 3D identity matrix, i.e., the table `{M(0,0,0),vecI,vecJ,vecK}`.
- `double=nil`: this experimental option adds a border to each side of the line; the syntax is `double={border color, border width}`. With the default value (`nil`), there is no border.

Add Axes: g:addAxes

The **g:addAxes(O, options)** method adds the axes $(O,vecI)$, $(O,vecJ)$, and $(O,vecK)$ to the 3D scene, where the argument $\langle O\rangle$ is a 3D point. The options are those of the **g:addPolyline()** method, plus the `legend=true` option, which allows you to automatically add a legend to the end of each axis; these legends are managed by the option `labels={"$x$", "$y$","$z$"}`. The axes are not graduated.

Add a line: g:addLine

The **g:addLine(d, options)** method adds the line $\langle d\rangle$ to the scene. This line is a table of the form $\{A,u\}$ where A is a point on the line (3D point) and u is a direction vector (non-zero 3D point). The optional argument $\langle options\rangle$ is a table whose fields define the options, these are those of the **g:addPolyline()** method, plus the `scale=1` option: this number is a homothety ratio. The homothety is applied, with the center being the midpoint of the segment representing the line, and the ratio `scale`. This allows you to adjust the size of the segment in its representation. This segment is the line clipped by the polyhedron given by the `window3d` option (itself defined when the graph is created), which results in a segment (possibly empty).

Adding a "right" angle: g:addAngle

The **g:addAngle(B, A, C [, r, options])** method allows you to add the angle $\widehat{BAC}$ in the form of a parallelogram with side $\langle r\rangle$ (0.25 by default). Only two sides are represented. The arguments $\langle B\rangle$, $\langle A\rangle$ and $\langle C\rangle$ are 3D points. The options are those of the **g:addPolyline()** method.

Add a circular arc: g:addArc

The **g:addArc(B, A, C, r, direction [, normal, options])** method adds the arc of a circle centered at $\langle A\rangle$ (3D point), with radius $\langle r\rangle$, extending from $\langle B\rangle$ to $\langle C\rangle$ (3D points) in the direct direction if $\langle direction\rangle$ is 1 (indirect otherwise). The arc is drawn in the plane passing through $\langle A\rangle$ and orthogonal to the $\langle normal\rangle$ vector (non-zero 3D point); this same vector orients the plane. If the vector $\langle normal\rangle$ is not specified, then by default it will be the cross product $\vec{AB}\wedge\vec{AC}$. The options are those of the **g:addPolyline()** method.

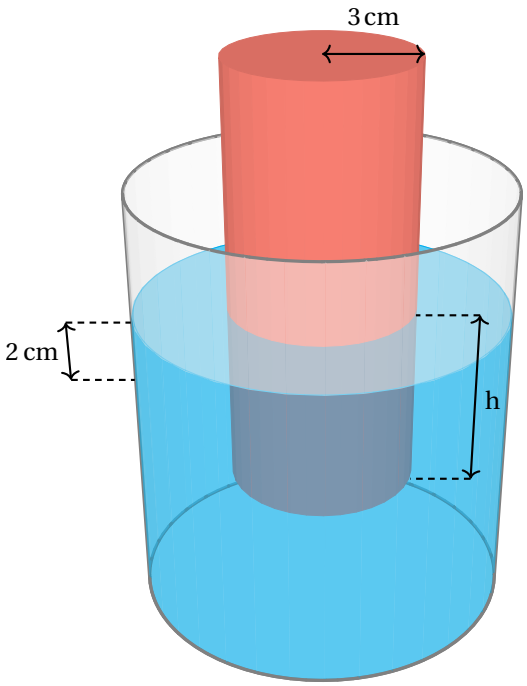
Add a circle: g:addCircle

The **g:addCircle(A,r,normal,options)** method adds the circle with center *A* (3D point) and radius *r* in the plane passing through *A* and orthogonal to the *normal* vector (non-zero 3D point). The options are those of the **g:addPolyline** method.

```
\begin{luadraw}{name=cylindres_imbriques}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-5,5,-7,5}, viewdir={"central",30,65,20},
    size={10,10},margin={0,0,0,0}}
ld.Hiddenlines = false
local R, r, A, B = 3, 1.5
local C1 = ld.cylinder(M(0,0,-5),5*vecK,R)  -- to model water
local C2 = ld.cylinder(Origin,2*vecK,R,35,true) -- part of the container above the water (open cylinder)
local C3 = ld.cylinder(M(0,0,-3),7*vecK,r) -- small cylinder submerged in water
-- under the 3D scene
g:Lineoptions(nil,"gray",12)
g:Dcylinder(M(0,0,-5),7*vecK,R,{hiddenstyle="noline"}) -- outline of the container (large cylinder)
-- scène 3D
g:Dscene3d(
        g:addPoly(C1,{contrast=0.125,color="cyan",opacity=0.5}), -- water
        g:addPoly(C2,{contrast=0.125,color="WhiteSmoke", opacity=0.5}), -- part of the container above the water
        g:addPoly(C3,{contrast=0.25,color="Salmon",backcull=true}), -- small cylinder in the water
        g:addCircle(M(0,0,2),R,vecK,{color="gray"}), -- upper edge of the container
        g:addCircle(M(0,0,-5),R,vecK,{color="gray"}), -- lower edge of the container
        g:addCircle(Origin,R-0.025,vecK, {width=2,color="cyan"}) -- upper edge water
        )
-- over the 3D scene
g:Lineoptions(nil,"black",8); A = 4*vecK; B = A+r*g:ScreenX()
g:Dpolyline3d( {A,B}, "<->"); g:Dlabel3d("$3\\,$cm",(A+B)/2,{pos="N",dist=0.25})
A = Origin+(r+1)*g:ScreenX(); A = ld.rotate3d(A,-10,{Origin,vecK})
B = A-3*vecK
g:Dpolyline3d( {A,B}, "<->"); g:Dlabel3d("h",(A+B)/2,{pos="E"})
g:Lineoptions("dashed")
g:Dpolyline3d({{A,A-g:ScreenX()},{B,B-g:ScreenX()}})
A = Origin-(R+1)*g:ScreenX(); A = ld.rotate3d(A,10,{Origin,vecK})
B = A-vecK
g:Dpolyline3d({{A,A+g:ScreenX()},{B,B+g:ScreenX()}})
g:Linestyle("solid")
g:Dpolyline3d( {A,B}, "<->"); g:Dlabel3d("$2$\\,cm",(A+B)/2,{pos="W"})
g:Show()
\end{luadraw}
```

Figure 32: Solid cylinder immersed in water



Notes :

- The **g:ScreenX()** method returns the space vector (3D point) corresponding to the vector with affix 1 in the screen plane, and the **g:ScreenY()** method returns the space vector (3D point) corresponding to the vector with affix $\imath$ in the screen plane.
- For the small cylinder (C3), we use the `backcull=true` option to reduce the number of facets; however, we do not do this for the other two cylinders (C1 and C2) because they are transparent.

Adding points: g:addDots

The **g:addDots(dots, options)** method allows you to add 3D points to the scene. The argument ⟨*dots*⟩ is either a 3D point or a list of 3D points. The optional argument ⟨*options*⟩ is a table whose fields define the options, these options are (with the default value):

- `style="ball"`: String defining the dot style. These are all 2D point styles, plus the `"ball"` (sphere) style, which is the default.
- `color="black"`: String defining the dot color.
- `scale=1`: Number allowing you to adjust the size of the points.
- `matrix=ld.ID3d`: 3D transformation matrix. By default, this is the 3D identity matrix, i.e., the table:

```
{M(0,0,0),vecI,vecJ,vecK}
```

Adding Labels: g:addLabel

The **g:addLabel(text1, anchor1, options1, text2, anchor2, options2, ...)** method allows you to add the labels ⟨*text1*⟩, ⟨*text2*⟩, etc. The (required) arguments ⟨*anchor1*⟩, ⟨*anchor2*⟩, etc., are 3D points representing the anchor points of the labels. The (required) arguments ⟨*options1*⟩, ⟨*options2*⟩, etc., tables whose fields define the options. These options are (with the default value):

- `color=<current color>`: String defining the label color.
- `pos=<current style>`: A string defining the position of the label relative to the anchor point (as in 2D: `"N"`, `"NW"`, `"W"`, ...).

- `dist=0`: Expresses the distance between the label and its anchor point (in the screen plane).
- `size=<current size>`: String defining the label size.
- `dir={}`: Table defining the writing direction in space (usual direction by default). In general, `dir={dirX,dirY,dep}`, and the three values ⟨*dirX*⟩, ⟨*dirY*⟩, and ⟨*dep*⟩ are three 3D points representing three vectors: the first two indicate the writing direction, the third a displacement (translation) of the label relative to the anchor point.
- `showdot=false`: Boolean indicating whether a (2D) point should be drawn at the anchor point.
- `matrix=ld.ID3d`: 3D transformation matrix; by default, this is the 3D identity matrix, i.e., the table:

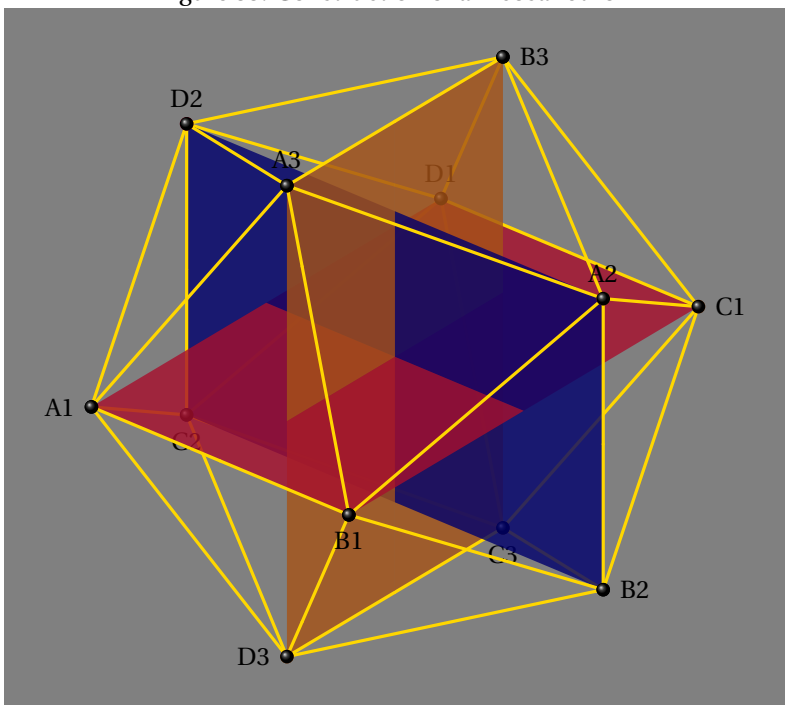
```
{M(0,0,0),vecI,vecJ,vecK}
```

Note: As in 2D, the options of a label apply to subsequent labels as long as they are not modified.

```
\begin{luadraw}{name=icosaedre}
local ld = luadraw
local M = ld.pt3d.M

local g = ld.graph3d:new{window={-2.25,2.25,-2,2}, viewdir={40,60},bg="gray",size={10,10},margin={0,0,0,0}}
ld.Hiddenlines = false
local phi = (1+math.sqrt(5))/2 -- golden ratio
local A1, B1, C1, D1 = M(phi,-1,0), M(phi,1,0), M(-phi,1,0), M(-phi,-1,0) -- in the plane z=0
local A2, B2, C2, D2 = M(0,phi,1), M(0,phi,-1), M(0,-phi,-1), M(0,-phi,1) -- in the plane x=0
local A3, B3, C3, D3 = M(1,0,phi), M(-1,0,phi), M(-1,0,-phi), M(1,0,-phi) -- in the plane y=0
local ico = {
{A1,B1,A3}, {B1,A1,D3}, {D1,C1,C3}, {C1,D1,B3},
{B2,A2,B1}, {A2,B2,C1}, {D2,C2,A1}, {C2,D2,D1},
{B3,A3,A2}, {A3,B3,D2}, {D3,C3,B2}, {C3,D3,C2},
{A1,A3,D2}, {B1,A2,A3}, {A2,C1,B3}, {D1,D2,B3},
{B2,B1,D3}, {A1,C2,D3}, {B2,C3,C1}, {C2,D1,C3} }
g:Dscene3d(
    g:addFacet({A2,B2,C2,D2},{color="Navy",twoside=false,opacity=0.8}),
    g:addFacet({A1,B1,C1,D1},{color="Crimson",twoside=false,opacity=0.8}),
    g:addFacet({A3,B3,C3,D3},{color="Chocolate",twoside=false,opacity=0.8}),
    g:addPolyline(ld.facetedges(ico), {color="Gold",width=12}), -- drawing edges only
    g:addDots({A1,B1,C1,D1,A2,B2,C2,D2,A3,B3,C3,D3}, {color="black",scale=1.2}),
    g:addLabel("A1",A1,{style="W",dist=0.1}, "B1",B1,{style="S"}, "C2",C2,{}, "C3",C3,{},
        "A3",A3,{style="N"}, "D1",D1,{}, "A2",A2,{}, "D2",D2,{}, "B3",B3,{style="E"},
        "C1",C1,{}, "B2",B2,{}, "D3",D3,{style="W"} )
)
g:Show()
\end{luadraw}
```

Figure 33: Construction of an icosahedron



Adding dividing walls: g:addWall

Dividing walls are 3D objects that are inserted first into the scene tree. These objects are not drawn (therefore invisible); their role is to partition the space because a facet on one side of a dividing wall cannot be cut by the plane of a facet on the other side of the wall. This allows, in some cases, to significantly reduce the number of facet (or polygonal line) cuts during scene construction. A dividing wall can be an entire plane (i.e., a table of two 3D points of the form $\{A, n\}$, i.e., a point and a normal vector), or just a facet.

The syntax is: **g:addWall(C, options)** where $\langle C \rangle$ is either a plane, a list of planes, a facet, or a list of facets. The $\langle options \rangle$ argument is a table with only one option:

- `matrix=ld.ID3d`: 3D transformation matrix. By default, this is the 3D identity matrix, i.e., the table:

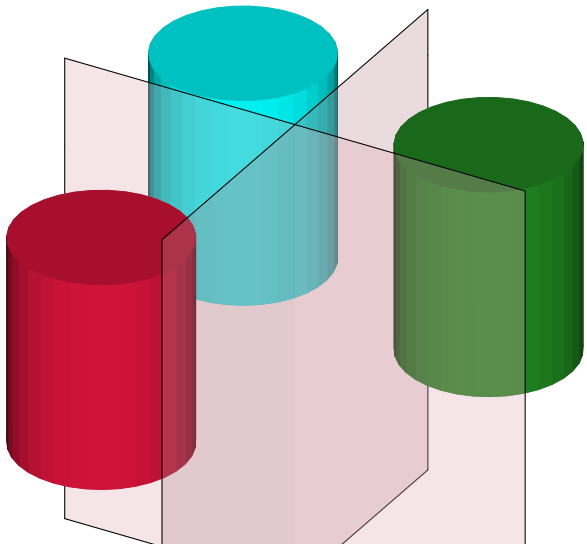
  `{M(0,0,0),vecI,vecJ,vecK}`

In the following example, the two dividing walls have been drawn for visualization, but they are normally invisible:

```
\begin{luadraw}{name=addWall}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{size={10,10},window={-8,8,-4,8}, margin={0,0,0,0}}
local C = ld.cylinder(M(0,0,-1),5*vecK,2)
g:Dscene3d(
    g:addWall( {{Origin,vecI}, {Origin,vecJ}}),
    g:addPlane({Origin,vecI}, {color="Pink",opacity=0.3,scale=1.125,edge=true}), -- to show the first wall
    g:addPlane({Origin,vecJ}, {color="Pink",opacity=0.3,scale=1.125,edge=true}), -- to show the second wall
    g:addPoly( ld.shift3d(C,M(-3,-3,1)), {color="Cyan"} ),
    g:addPoly( ld.shift3d(C,M(-3,3,0.5)), {color="ForestGreen"} ),
    g:addPoly( ld.shift3d(C,M(3,-3,-0.5)), {color="Crimson"} )
)
g:Show()
\end{luadraw}
```

Figure 34: Example with addWall (the two transparent pink facets are normally invisible)



Notes on this example :

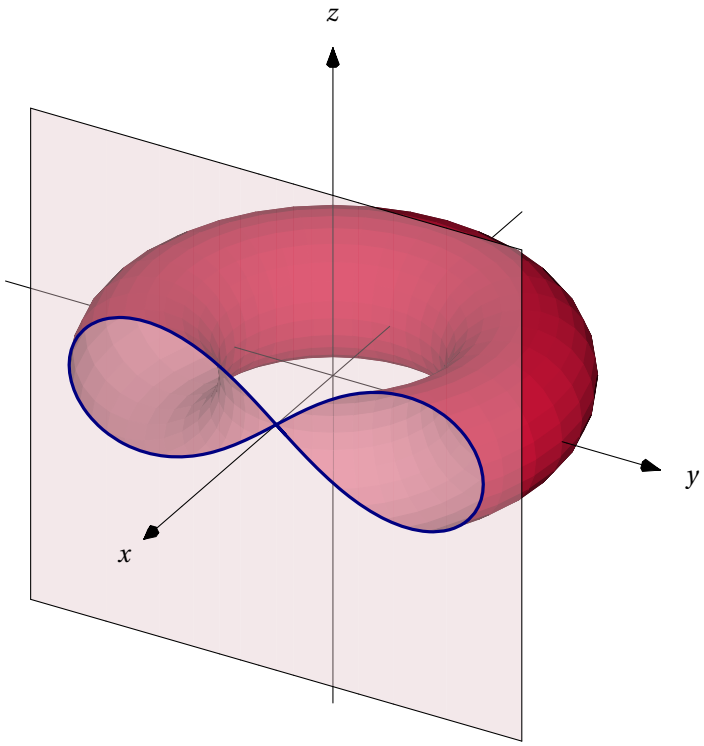
- with the two dividing walls, there are no cut facets, and the scene contains exactly 111 (37 per cylinder).
- without the dividing walls, there are 117 (useless) facet cuts, bringing their number to 228 in the scene.
- with the two dividing walls, and the `backcull=true` option for each cylinder, there are no cut facets, and the scene contains only 57.

Here is another, much more convincing example where the use of dividing walls is essential to have a drawing of reasonable size. It involves obtaining a lemniscate as the intersection of a torus with a certain plane. Since the torus is non-convex, the number of unnecessary facet cuts can be very high.

```
\begin{luadraw}{name=torus}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{size={10,10}, margin={0,0,0,0}}
local cos, sin, pi = math.cos, math.sin, math.pi
local R, r = 2.5, 1
local x0 = R-r
local f = function(t) return M(0,R+r*cos(t),r*sin(t)) end
local plan = {M(x0,0,0),-vecI} -- plane whose section with the torus gives the lemniscate
local C, wall = ld.rotcurve(f,-pi,pi,{Origin,vecK},360,0,{grid={25,37},addwall=2})
local C1 = ld.cutfacet(C,plan) -- part of the torus in the half-space containing -vecI
g:Dscene3d(
    g:addWall(plan), g:addWall(wall), -- addition of partition walls
    g:addFacet( C1, {color="Crimson", backcull=false}),
    g:addPlane(plan, {color="Pink",opacity=0.4,edge=true}), -- sectional plane
    g:addAxes( Origin, {arrows=1})
)
-- Cartesian equation of the torus : (x^2+y^2+z^2+R^2-r^2)^2-4*R^2*(x^2+y^2) = 0
-- the lemniscate therefore has the equation (x0^2+y^2+z^2+R^2-r^2)^2-4*R^2*(x0^2+y^2)=0 (implicit curve)
local h = function(y,z) return (x0^2+y^2+z^2+R^2-r^2)^2-4*R^2*(x0^2+y^2) end
local I = ld.implicit(h,-4,4,-3,3,{50,50}) -- 2D polygonal line (list of lists of complex numbers)
local lemniscate = ld.map(function(z) return M(x0,z.re,z.im) end, I[1]) -- conversion to 3D coordinates
g:Dpolyline3d(lemniscate,"Navy,line width=1.2pt")
g:Show()
\end{luadraw}
```

Figure 35: Torus and lemniscate



Notes on this example :

- With the dividing walls, we have 30 facets that are cut and a tkz file of approximately 140 KB.
- Without the dividing walls, we have 2068 facet cuts (!) and a tkz file of approximately 550 KB.
- We could have used the cut section returned by the **ld.cutfacet()** function, but the result is not very satisfactory (this is because the torus is non-convex).
- If we hadn't wanted the axes passing through the torus and the cutting plane, we could have drawn the drawing with the **g:Dfacet()** method, replacing the **g:Dscene3d()** instruction with:

```
g:Dfacet(C1, {mode=ld.mShadedOnly,color="Crimson"} )
g:Dfacet( g:Plane2facet(plan,0.75), {color="Pink",opacity=0.4})
```

We get exactly the same thing but without the axes (and without facet cutting, of course).

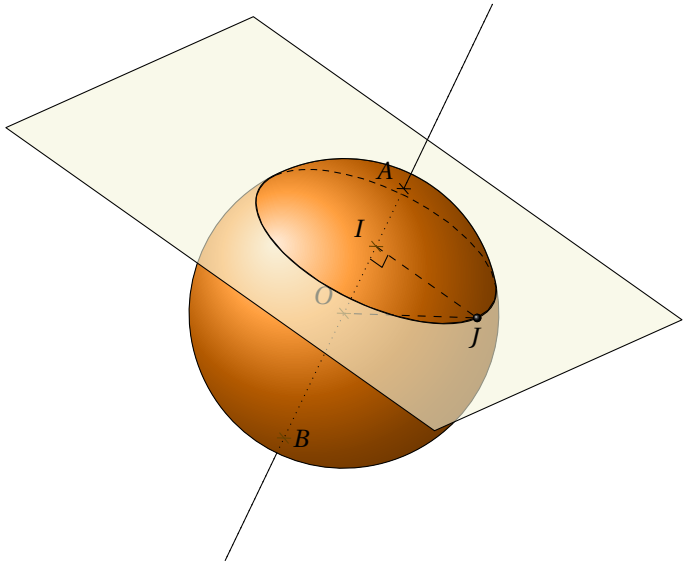
To conclude this section : we use the **g:Dscene3d()** method when it is not possible to do otherwise, for example when there are intersections (few) that cannot be handled "by hand". But this isn't the case for all intersections! In the following example, we represent a section of a sphere using a plane, but without using the **g:Dscene3d()** method, as this would require drawing a faceted sphere, which isn't very attractive. The trick here is to draw the sphere using the **g:Dsphere()** method, then draw a previously perforated facet over the plane, the hole corresponding to the outline (3D path) of the part of the sphere located above the plane:

```
\begin{luadraw}{name=section_sphere}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{ window3d={-4,4,-4,4,-4,4}, window={-5.5,5.5,-4,5}, viewdir={30,75}, size={10,10}}
local O, R = Origin, 2.5 -- center and radius
local S, P = ld.sphere(O,R), {M(0,0,1.5),vecK+vecJ/2} -- the sphere and the section plane
local w, n = pt3d.normalize(P[2]), g.Normal -- unit vectors normal to P for w and to the screen for n
local I, r = ld.interPS(P,{O,R}) -- center and radius of the small circle (intersection between the plane and the
 ↪ sphere)
local C = g:Intersection3d(S,P) -- It is a list of edges
```

```
local N = I-O
local J = I+r*pt3d.normalize(vecJ-vecK/2) -- a point on the small circle
local a = R/pt3d.abs(N)
local A, B = O+a*N, O-a*N -- points of intersection of the axis (O,I) with the sphere
local c1, alpha = ld.Orange, 0.4
local coul = {c1[1]*alpha, c1[2]*alpha,c1[3]*alpha} --to simulate transparency
g:Dhline( g:Proj3d({B,-N})) -- half-line (point B is not visible)
g:Dsphere(O,R,{mode=ld.mBorder,color="orange"})
g:Dline3d(A,B,"dotted") -- dotted line (A,B)
g:Dedges(C, {hidden=true,hiddenstyle="dashed"}) -- drawing of the intersection
g:Dpolyline3d({I,J,O},"dashed")
g:Dangle3d(O,I,J) -- right angle
g:Dcrossdots3d({{B,N},{I,N},{O,N}},ld.rgb(coul),0.75) -- points in the sphere
g:Dlabel3d("$O$", O, {pos="NW"})
local L = C.visible[1] -- visible part of the intersection (arc of a circle)
A1 = L[1]; A2 = L[#L] -- ends of L
local F = g:Plane2facet(P) -- plane converted to facet
-- hole plane as 3D path, the hole is the outline of the part of the sphere above the plane
ld.insert(F,{"l","cl",A1,"m",I,A2,r,-1,w,"ca",Origin,A1,R,-1,n,"ca"})
g:Dpath3d( F,"fill=Beige,fill opacity=0.6") -- drawing of the perforated plan
g:Dhline( g:Proj3d({A,N})) -- half-line, upper part of the axis (AB)
g:Dcrossdots3d({A,N},"black",0.75); g:Dballdots3d(J,"black",0.75)
g:Dlabel3d("$A$", A, {pos="NW"}, "$I$", I, {}, "$B$", B, {pos="E"}, "$J$", J, {pos="S"})
g:Show()
\end{luadraw}
```

Figure 36: Section of a sphere without Dscene3d()



VI Geometric Constructions

This section groups together functions that construct geometric figures without dedicated graphics methods.

1) Circumscribed circle, incircle: circumcircle3d(), incircle3d()

- The function **ld.circumcircle3d(A, B, C)**, where $\langle A\rangle$, $\langle B\rangle$ and $\langle C\rangle$ are three non-aligned 3D points, returns the circumcircle of the triangle formed by these three points, in the form of a sequence: O, R, n, where O is the center of the circle, R its radius, and n a normal vector to the plane of the circle.
- The function **ld.incircle3d(A, B, C)**, where $\langle A\rangle$, $\langle B\rangle$ and $\langle C\rangle$ are three non-aligned 3D points, returns the circle inscribed in the triangle formed by these three points, as a sequence: I, R, n, where I is the center of the circle, R its radius, and n a normal vector to the plane of the circle.

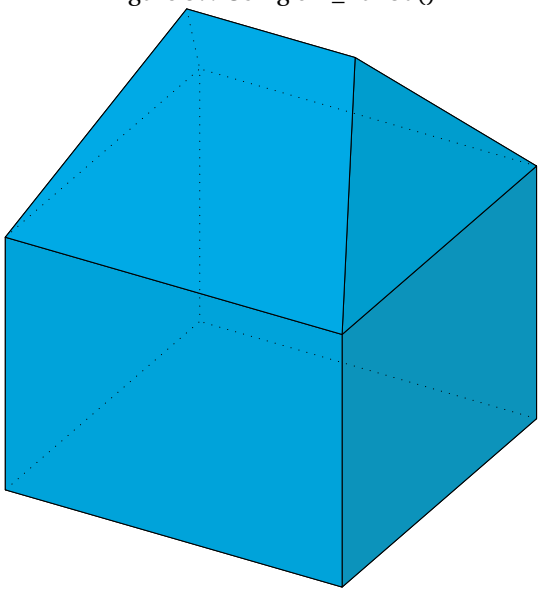
2) Convex Hull: cvx_hull3d()

The function **ld.cvx_hull3d(L)**, where ⟨*L*⟩ is a list of **distinct** 3D points, calculates and returns the convex hull of ⟨*L*⟩ as a list of facets.

```
\begin{luadraw}{name=cvx_hull3d}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-2,4,-6,1},bbox=false,size={10,10}}
local L = {Origin, 4*vecI, M(4,4,0), 4*vecJ}
ld.insert(L, ld.shift3d(L,-3*vecK))
ld.insert(L, {M(2,1,2), M(2,3,2)})
local V = ld.cvx_hull3d(L)
local P = ld.facet2poly(V)
g:Dpoly(P , {color="cyan",mode=ld.mShadedHidden})
g:Show()
\end{luadraw}
```

Figure 37: Using cvx_hull3d()



Special case : when all points of ⟨*L*⟩ are in the same plane, we can use the function **ld.cvx_hull3dcoplanar(L, n)** where ⟨*n*⟩ is a vector orthogonal to the plane. This function returns a facet (list of 3D points).

3) Planes: plane(), planeEq(), orthoframe(), plane2ABC()

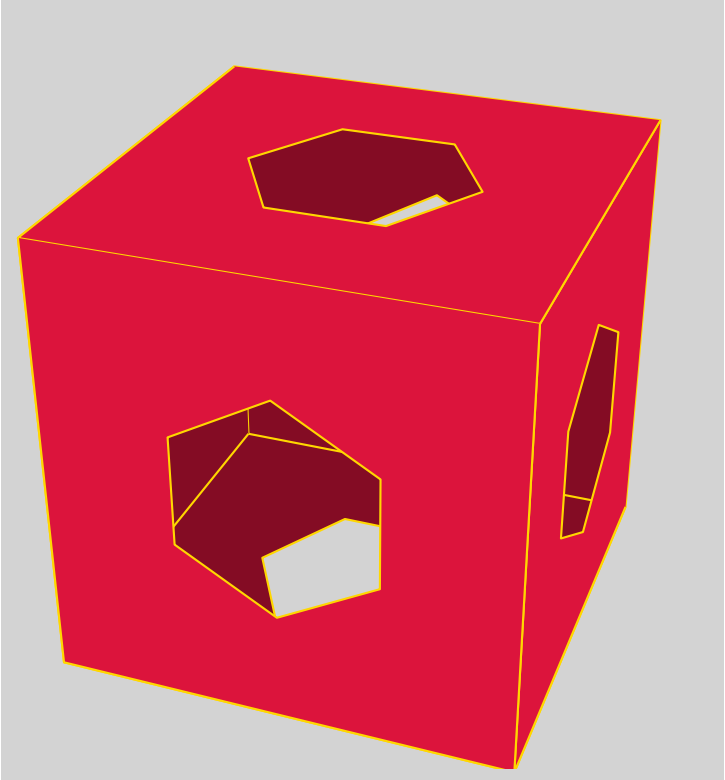
A plane in space is a table of the form $\{A, n\}$ where A is a point in the plane (3D point) and n is a normal vector to the plane (non-zero 3D point).

- The function **ld.plane(A, B, C)** returns the plane passing through the three 3D points ⟨*A*⟩, ⟨*B*⟩ and ⟨*C*⟩ (if they are not aligned, otherwise the result is `nil`).
- The function **ld.planeEq(a, b, c, d [, inequality])** returns the plane whose Cartesian equation is $ax + by + cz + d = 0$ (if the coefficients ⟨*a*⟩, ⟨*a*⟩ and ⟨*b*⟩ are not all zero, otherwise the result is `nil`). The optional argument ⟨*inequality*⟩ (`nil` by default) can also take the value `'<'` or `'>'`, the normal vector to the plane is then chosen so that it is in the half-space satisfying the inequality $ax + by + c < 0$ (or $ax + by + c > 0$ in the other case).
- The function **ld.plane2ABC(P)**, where ⟨*P*⟩= $\{A, n\}$ denotes a plane, returns a sequence of three 3D points A, B, C, belonging to the plane, and such that $(A, \vec{AB}, \vec{AC})$ is a direct orthonormal frame of this plane.
- The function **ld.orthoframe(P [, u])** where ⟨*P*⟩= $\{A, n\}$ denotes a plane, returns a sequence of three 3D points A, u, v, such that (A, u, v) is a direct orthonormal coordinate system of this plane. The optional argument ⟨*u*⟩, if present, must be a non-zero vector of the plane; this will be the first basis vector after normalization.

```
\begin{luadraw}{name=plans}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-3,3,-3.25,3.25}, margin={0,0,0,0},
    viewdir={"central",20,60}, bg="LightGray", size={10,10}}
ld.Hiddenlines = true; ld.Hiddenlinestyle = "dashed"
local p = ld.polyreg(0,1,6)
local P = ld.parallelep(M(-2,-2,-2),4*vecI,4*vecJ,4*vecK)
local V = g:Sortpolyfacet(P)
local list = {}
g:Filloptions("full","Crimson",1,true); -- true for even odd rule
g:Lineoptions("solid","Gold",8)
for _, F in ipairs(V) do
    local P1 = ld.plane(pt3d.isobar3d(F),F[1],F[2]) -- plane of facet F
    local A, u, v = ld.orthoframe(P1) -- orthonormal coordinate system on the facet with center of gravity as the
    ↪ origin
    local p1 = ld.map(function(z) return A+z.re*u+z.im*v end,p) -- hexagon reproduced on the facet
    table.insert(p1,2,"m")
    local color = "Crimson"
    if not g:Isvisible(F) then color = "Crimson!60!black" end
    g:Dpath3d( ld.concat(F,{"l"},p1,{"l","cl"}),"fill="..color ) -- drawing of the "perforated" facet with the hexagon
end
g:Show()
\end{luadraw}
```

Figure 38: Faces of a cube with holes in it and a regular hexagon



4) Circumscribed Sphere, Inscribed Sphere: circumsphere(), insphere()

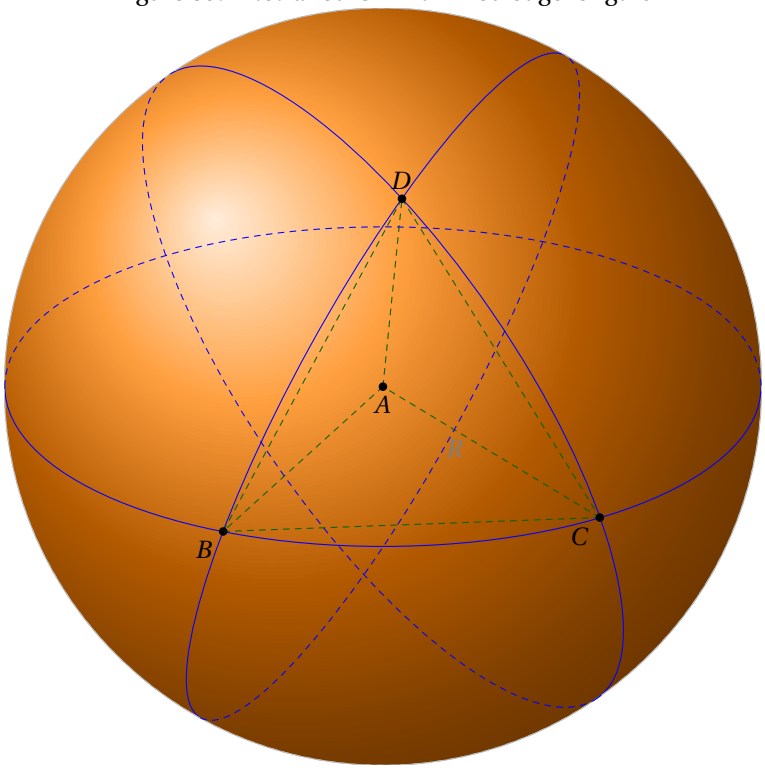
- The function **ld.circumsphere(A, B, C, D)**, where $\langle A\rangle$, $\langle B\rangle$, $\langle C\rangle$ and $\langle D\rangle$ are four non-coplanar 3D points, returns the sphere circumscribed about the tetrahedron formed by these four points, as a sequence: I, R, where I is the center of the sphere, and R its radius.
- The function **ld.insphere(A, B, C, D)**, where $\langle A\rangle$, $\langle B\rangle$, $\langle C\rangle$ and $\langle D\rangle$ are four non-coplanar 3D points, returns the sphere inscribed within the tetrahedron formed by these four points, as a sequence: I, R, where I is the center of the sphere, and R its radius.

5) Fixed-length tetrahedron: tetra_len()

The function **ld.tetra_len(ab, ac, ad, bc, bd, cd)** calculates the vertices A,B,C,D of a tetrahedron whose edge lengths are given, i.e., such that $AB = \langle ab \rangle$, $AC = \langle ac \rangle$, $AD = \langle ad \rangle$, $BC = \langle bc \rangle$, $BD = \langle bd \rangle$ and $CD = \langle cd \rangle$. The function returns the sequence of four points A,B,C,D. Vertex A is always the point $M(0,0,0)$ (`pt3d.Origin`) and vertex B is always the point `ab*pt3d.vecI` and vertex C in the xy plane. The tetrahedron as a polyhedron can then be constructed with the function **ld.tetra(A, B-A, C-A, D-A)**.

```
\begin{luadraw}{name=tetra_len}
local ld = luadraw
local g = ld.graph3d:new{window={-4,4,-4,4},margin={0,0,0,0},viewdir={25,65},size={10,10}}
ld.Hiddenlines = true; ld.Hiddenlinestyle = "dashed"
require 'luadraw_spherical'
local R = 4
local A,B,C,D = ld.tetra_len(R,R,R,R,R,R)
local T = ld.tetra(A,B-A,C-A,D-A)
g:Define_sphere({radius=R})
g:DSpolyline( ld.facetedges(T), {color="DarkGreen"})
g:DSgreatcircle( {B,C},{color="Blue"} )
g:DSgreatcircle( {B,D},{color="Blue"} )
g:DSgreatcircle( {C,D},{color="Blue"}  )
g:DSlabel("$R$",(2*A+C)/3,{pos="S"})
g:Dspherical()
g:Ddots3d({A,B,C,D})
g:Dlabel3d("$A$",A,{pos="S"},"$B$",B,{pos="SW"},"$C$",C,{},"$D$",D,{pos="N"} )
g:Show()
\end{luadraw}
```

Figure 39: A tetrahedron with fixed edge lengths



6) Triangles: sss_triangle3d(), sas_triangle3d(), asa_triangle3d()

These functions are the 3D version of the sss_triangle(), sas_triangle(), and asa_triangle() functions already described.

- The function **ld.sss_triangle3d(ab, bc, ca)**, where $\langle ab \rangle$, $\langle bc \rangle$, and $\langle ca \rangle$ are three lengths, computes and returns a list of three 3D points $\{A,B,C\}$ forming the vertices of a direct triangle in the xOy plane, whose side lengths are the arguments, i.e., $AB = \langle ab \rangle$, $BC = \langle bc \rangle$ and $CA = \langle ca \rangle$, when possible. Vertex A is always point $M(0,0,0)$ (`pt3d.Origin`) and vertex B is always point `ab*pt3d.vecI`. This triangle can be drawn with the method **g:Dpolyline3d()**.

- The function **ld.sas_triangle3d(ab, alpha, ca)** where ⟨*ab*⟩ and ⟨*ca*⟩ are two lengths, ⟨*alpha*⟩ an angle in degrees, computes and returns a list of three 3D points $\{A,B,C\}$ forming the vertices of a triangle in the plane xOy such that $AB = \langle ab \rangle$, $CA = \langle ca \rangle$, and such that the angle $(\vec{AB}, \vec{AC})$ has measure ⟨*alpha*⟩, when possible. Vertex A is always point $M(0,0,0)$ (`pt3d.Origin`) and vertex B is always point `ab*pt3d.vecI`. This triangle can be drawn with the method **g:Dpolyline3d()**.

- The function **ld.asa_triangle3d(alpha, ab, beta)** where ⟨*ab*⟩ is a length, ⟨*alpha*⟩ and ⟨*beta*⟩ are two angles in degrees, computes and returns a list of three 3D points $\{A,B,C\}$ forming the vertices of a triangle in the xOy plane such that $AB = \langle ab \rangle$, such that angle $(\vec{AB}, \vec{AC})$ has measure ⟨*alpha*⟩, and such that angle $(\vec{BA}, \vec{BC})$ has measure ⟨*beta*⟩, when possible. Vertex A is always point $M(0,0,0)$ (`pt3d.Origin`) and vertex B is always point `ab*pt3d.vecI`. This triangle can be drawn with the method **g:Dpolyline3d()**.

VII Matrix Calculus Transformations and Some Mathematical Functions

1) 3D Transformations

In the following functions:

- the argument ⟨L⟩ is either a 3D point, a polyhedron, a list of 3D points (facet), or a list of lists of 3D points (facet list),
- a line ⟨d⟩ is a list of two 3D points $\{A,u\}$: a point on the line (A) and a direction vector (u),
- a plane ⟨P⟩ is a list of two 3D points $\{A,n\}$: a point on the plane (A) and a normal vector to the plane (n).

The returned result is of the same type as ⟨L⟩.

Apply a transformation function: ftransform3d

The function **ld.ftransform3d(L, f)** returns the image of ⟨L⟩ by the function ⟨f⟩; this must be a function from $\mathbf{R}^3$ to $\mathbf{R}^3$ (to a 3D point it associates a 3D point).

Projections: proj3d, proj3dO, dproj3d

- The function **ld.proj3d(L, P)** returns the image of ⟨L⟩ by the orthogonal projection onto the plane ⟨P⟩.
- The function **ld.proj3dO(L, P, v)** returns the image of ⟨L⟩ by the projection onto the plane ⟨P⟩ parallel to the direction of the vector ⟨v⟩ (non-zero 3D point).
- The function **ld.dproj3d(L, d)** returns the image of ⟨L⟩ by the projection onto the line ⟨d⟩.

Projections onto axes or planes related to axes

- The function **ld.pxy(L [, z0])** returns the image of ⟨L⟩ by the orthogonal projection onto the $z = \langle z0 \rangle$ plane (by default $z0 = 0$).
- The function **ld.pyz(L [, x0])** returns the image of ⟨L⟩ by the orthogonal projection onto the $x = \langle x0 \rangle$ plane (by default $x0 = 0$).
- The function **ld.pxz(L [, y0])** returns the image of ⟨L⟩ by the orthogonal projection onto the $y = \langle y0 \rangle$ plane (by default $y0 = 0$).
- The function **ld.px(L)** returns the image of ⟨L⟩ by the orthogonal projection onto the x-axis.
- The function **ld.py(L)** returns the image of ⟨L⟩ by the orthogonal projection onto the y-axis.
- The function **ld.pz(L)** returns the image of ⟨L⟩ by the orthogonal projection onto the z-axis.

Symmetries: sym3d, sym3dO, dsym3d, psym3d

- The function **ld.sym3d(L, P)** returns the image of ⟨*L*⟩ by the orthogonal symmetry about the ⟨*P*⟩ plane.
- The function **ld.sym3dO(L, P, v)** returns the image of ⟨*L*⟩ by the symmetry about the ⟨*P*⟩ plane and parallel to the direction of the ⟨*v*⟩ vector (non-zero 3D point).
- The function **ld.dsym3d(L, d)** returns the image of ⟨*L*⟩ by the orthogonal symmetry with respect to the line ⟨*d*⟩.
- The function **ld.psym3d(L, point)** returns the image of ⟨*L*⟩ by the symmetry with respect to ⟨*point*⟩ (3D point).

Rotation: rotate3d, rotateaxe3d

- The function **ld.rotate3d(L, angle, d)** returns the image of ⟨*L*⟩ rotated along axis ⟨*d*⟩ (oriented by the direction vector, which is *d*[2]), and by *angle* degrees.
- The function **ld.rotateaxe3d(L, v1, v2 [, center])** returns the image of ⟨*L*⟩ rotated along axis passing through the 3D point ⟨*center*⟩, which transforms the vector ⟨*v1*⟩ into the vector ⟨*v2*⟩. These vectors are normalized by the function. The argument ⟨*center*⟩ is optional and defaults to the point `pt3d.Origin`.

Scaling: scale3d

The function **ld.scale3d(L , k [, center])** returns the image of ⟨*L*⟩ by the scaling with center at the 3D point ⟨*center*⟩, and ratio ⟨*k*⟩. The argument ⟨*center*⟩ is optional and defaults to the point `pt3d.Origin`.

Inversion: inv3d

The function **ld.inv3d(L, radius [, center])** returns the image of ⟨*L*⟩ by the inversion with respect to the sphere with center ⟨*center*⟩, and radius ⟨*radius*⟩. The argument ⟨*center*⟩ is optional and defaults to the point `pt3d.Origin`.

Stereography: projstereo and inv_projstereo

Function **ld.projstereo(L, S, N, h)**: the argument ⟨*L*⟩ denotes a 3D point or a list of 3D points or a list of lists of 3D points, all belonging to the sphere ⟨*S*⟩, where ⟨*S*⟩= {*C*, *r*} (*C* is the center of the sphere, and *r* the radius). The argument ⟨*N*⟩ denotes a point on the sphere that will be the pole of the projection. The argument ⟨*h*⟩ is a real number that defines the projection plane. This plane is perpendicular to the axis (*CN*), and passes through the point $I = C + h\frac{\vec{CN}}{CN}$ (with $h = 0$ it is the equatorial plane, with $h = -r$ it is the plane tangent to the sphere at the opposite pole). The function returns the image of ⟨*L*⟩ by the stereographic projection with respect to the sphere ⟨*S*⟩ with ⟨*N*⟩ as pole, and on the plane {*I*, *N* − *C*}.

Inverse function **ld.inv_projstereo(L, S, N)**: ⟨*S*⟩= {*C*, *r*} is the sphere with center *C* and radius *r*, ⟨*N*⟩ is a point on the sphere ⟨*S*⟩ (pole), and ⟨*L*⟩ is a 3D point or a list of 3D points or a list of lists of 3D points all belonging to the same plane orthogonal to the (*CN*) axis. The function returns the image of ⟨*L*⟩ by the inverse of the stereographic projection with respect to ⟨*S*⟩ and with pole ⟨*N*⟩.

Translation: shift3d

The function **ld.shift3d(L, v)** returns the image of ⟨*L*⟩ by the translation of vector ⟨*v*⟩ (3D point).

2) Matrix Calculus

If *f* is an affine mapping of the space $\mathbf{R}^3$, we will call the list (table) of *f* a matrix:

```
{f(pt3d.Origin),Lf(pt3d.vecI),Lf(pt3d.vecJ),Lf(pt3d.vecK)}
```

where *Lf* denotes the linear part of *f* (we have `Lf(pt3d.vecI)=f(pt3d.vecI)-f(pt3d.Origin)`, etc.). The identity matrix is denoted `ld.ID3d` in *luadraw*; it simply corresponds to the list:

```
{pt3d.Origin,pt3d.vecI,pt3d.vecJ,pt3d.vecK}
```

applymatrix3d and applyLmatrix3d

- The function **ld.applymatrix3d(A, M)** applies the matrix $\langle M\rangle$ to the 3D point $\langle A\rangle$ and returns the result (which is equivalent to calculating $f(A)$ if M is the matrix of f). If $\langle A\rangle$ is not a 3D point, the function returns $\langle A\rangle$.
- The function **ld.applyLmatrix3d(A, M)** applies the linear part of the matrix $\langle M\rangle$ to the 3D point $\langle A\rangle$ and returns the result (which is equivalent to calculating $Lf(A)$ if M is the matrix of f). If $\langle A\rangle$ is not a 3D point, the function returns $\langle A\rangle$.

composematrix3d

The function **ld.composematrix3d(M1, M2)** performs the matrix product $\langle M1\rangle\times\langle M2\rangle$ and returns the result.

getmatrix3d

The function **ld.getmatrix3d(T1, T2)** calculates and returns the matrix that transforms the tetrahedron $\langle T1\rangle$ into the tetrahedron $\langle T2\rangle$, with $\langle T1\rangle = \{a_1, b_1, c_1, d_1\}$ (a list of four non-coplanar 3D points) and $\langle T2\rangle = \{a_2, b_2, c_2, d_2\}$ (a list of four 3D points).

invmatrix3d

The function **ld.invmatrix3d(M)** calculates and returns the inverse of the matrix $\langle M\rangle$ when possible.

matrix3dof

The function **ld.matrix3dof(f)** calculates and returns the matrix of $\langle f\rangle$ which must be an affine mapping of the space $\mathbf{R}^3$ (to a 3D point it associates a 3D point).

mtransform3d and mLtransform3d

- The function **ld.mtransform3d(L, M)** applies the matrix $\langle M\rangle$ to the list $\langle L\rangle$ and returns the result. $\langle L\rangle$ must be a list of 3D points (a facet) or a list of lists of 3D points (a list of facets).
- The function **ld.mLtransform3d(L, M)** applies the linear part of the matrix $\langle M\rangle$ to the list $\langle L\rangle$ and returns the result. $\langle L\rangle$ must be a list of 3D points (a facet) or a list of lists of 3D points (a list of facets).

planetransform

The function **ld.planetransform(P, M)** calculates and returns the image of the plane $\langle P\rangle$ under the 3D matrix $\langle M\rangle$. The plane $\langle P\rangle$ is a list of the form $\{A, n\}$, where A is a point on the plane and n is a normal vector to the plane; the function returns a list of the same type.

3) Matrix associated with the 3D graph

When creating a graph in the *luadraw* environment, for example:

```
local ld = luadraw
local g = ld.graph3d:new{size={10,10}}
```

The created g object has a 3D transformation matrix that is initially the identity. All graphics methods automatically apply the graph's 3D transformation matrix. To manipulate this matrix, the following methods are available.

g:Composematrix3d()

The **g:Composematrix3d(M)** method multiplies the 3D matrix of the graph g by the matrix $\langle M\rangle$ (with $\langle M\rangle$ on the right), and the result is assigned to the graph's 3D matrix. The argument $\langle M\rangle$ must therefore be a 3D matrix.

g:Det3d()

The **g:Det3d()** method returns 1 when the 3D transformation matrix has a positive determinant, and −1 otherwise. This information is useful when we need to know whether the orientation of space has been changed or not.

g:IDmatrix3d()

The **g:IDmatrix3d()** method reassigns the identity to the 3D matrix of the graph g.

g:Mtransform3d()

The **g:Mtransform3d(L)** method applies the 3D graph matrix of g to L and returns the result. The argument L must be a list of 3D points (a facet) or a list of lists of 3D points (a list of facets).

g:MLtransform3d()

The **g:MLtransform3d(L)** method applies the linear part of the 3D matrix of the graph g to ⟨L⟩ and returns the result. The argument ⟨L⟩ must be a list of 3D points (a facet) or a list of lists of 3D points (a list of facets).

g:Rotate3d()

The method **g:Rotate3d(angle, axis)** modifies the 3D transformation matrix of the graph g by composing it with the rotation matrix of angle ⟨*angle*⟩ (in degrees) and axis ⟨*axis*⟩.

g:Scale3d()

The method **g:Scale3d(factor [, center])** modifies the 3D transformation matrix of the graph g by composing it with the homothety matrix of ratio ⟨*factor*⟩ and center ⟨*center*⟩. The argument ⟨*center*⟩ is a 3D point that defaults to `pt3d.Origin`.

g:Setmatrix3d()

The **g:Setmatrix3d(M)** method assigns the matrix ⟨M⟩ to the 3D transformation matrix of the graph g.

g:Shift3d()

The **g:Shift3d(v)** method modifies the 3D transformation matrix of the graph g by composing it with the translation matrix of vector ⟨v⟩, which must be a 3D point.

4) Additional Mathematical Functions

clippolyline3d()

The function **ld.clippolyline3d(L, poly [, exterior, close])** clips the 3D polygonal line ⟨L⟩ to the **convex** polyhedron ⟨*poly*⟩. If the optional argument ⟨*exterior*⟩ is `true`, then the part outside the polyhedron is returned (`false` by default). If the optional argument ⟨*close*⟩ is `true`, then the polygonal line is closed (`false` by default). ⟨L⟩ is a list of 3D points or a list of lists of 3D points.

Note: The result is not always satisfactory for the exterior part.

Special case : Clipping a 3D polygonal line ⟨L⟩ with the current 3D window can be done with this function as follows:

```
L=ld.clippolyline3d(L,g:Box3d())
```

Indeed, the **g:Box3d()** method returns the current 3D window as a parallelepiped.

clipline3d()

The function **ld.clipline3d(line, poly)** clips the line ⟨*line*⟩ with the **convex** polyhedron ⟨*poly*⟩; the function returns the part of the line inside the polyhedron. The argument ⟨*line*⟩ is a table of the form $\{A, u\}$ where A is a point on the line and u is a direction vector (two 3D points).

Special case : Clipping a line ⟨*d*⟩ with the current 3D window can be done with this function as follows:

```
d=clipline3d(d,g:Box3d())
```

Indeed, the **g:Box3d()** method returns the current 3D window as a parallelepiped (⟨*d*⟩ then becomes a segment).

cutpolyline3d()

The function **ld.cutpolyline3d(L, plane [, close])** intersects the 3D polygonal line ⟨*L*⟩ with the plane ⟨*plane*⟩. If the optional argument ⟨*close*⟩ is `true`, then the line is closed (`false` by default). ⟨*L*⟩ is a list of 3D points or a list of lists of 3D points, ⟨*plane*⟩ is a table of the form $\{A, n\}$ where A is a point in the plane and n is a normal vector (two 3D points).

The function returns three things:

- the part of ⟨*L*⟩ that is in the half-space containing the vector n,
- followed by the part of ⟨*L*⟩ that is in the other half-space,
- followed by the list of intersection points.

getbounds3d()

The function **ld.getbounds3d(L)** returns the bounds *xmin, xmax, ymin, ymax, zmin, zmax* (sequence) of the 3D polygonal line ⟨*L*⟩ (list of 3D points or a list of lists of 3D points).

interDP()

The function **ld.interDP(d, P)** calculates and returns (if it exists) the intersection between line ⟨*d*⟩ and plane ⟨*P*⟩.

interPP()

The function **ld.interPP(P1, P2)** calculates and returns (if it exists) the intersection between planes ⟨*P1*⟩ and ⟨*P2*⟩.

interDD()

The function **ld.interDD(D1, D2 [, epsilon])** calculates and returns (if it exists) the intersection between lines ⟨*D1*⟩ and ⟨*D2*⟩. The argument ⟨*epsilon*⟩ is 10^{-10} by default (used to test whether a given float is zero).

interCS()

The function **ld.interCS(C, S)** calculates and returns (if it exists) the intersection between the circle ⟨*C*⟩= $\{A, r, n\}$ (A is the center of the circle, r the radius, and n a normal vector to the plane of the circle), and the sphere ⟨*S*⟩= $\{B, R\}$ (B is the center of the sphere and R the radius). The function returns either `nil` (empty intersection), a single point, or two points (sequence).

interDS()

The function **ld.interDS(d, S)** calculates and returns (if it exists) the intersection between the line ⟨*d*⟩ and the sphere ⟨*S*⟩ which is a table of the form $\{C, r\}$ with C as the center (3D point) and r as the radius of the sphere. The function returns either `nil` (empty intersection), a single point, or two points.

interPS()

The function **ld.interPS(P, S)** calculates and returns (if it exists) the intersection between the plane ⟨*P*⟩ and the sphere ⟨*S*⟩= $\{C, r\}$ with C as the center (3D point) and r as the radius of the sphere. The function returns either `nil` (empty intersection) or a sequence of the form I, r, n, where I is a 3D point representing the center of a circle, r its radius, and n a normal vector to the plane of the circle. This circle is the desired intersection.

interSS()

The function **ld.interSS(S1, S2)** calculates and returns (if it exists) the intersection between the sphere $\langle S1\rangle = \{C1, r1\}$ and $\langle S2\rangle = \{C2, r2\}$. The function returns either `nil` (empty intersection) or a sequence of the form I, r, n, where I is a 3D point representing the center of a circle, r its radius, and n a normal vector to the plane of the circle. This circle is the desired intersection.

interSSS()

The function **ld.interSSS(S1, S2, S3)** calculates and returns (if it exists) the intersection between the spheres $\langle S1\rangle = \{C1, r1\}$, $\langle S2\rangle = \{C2, r2\}$ and $\langle S3\rangle = \{C3, r3\}$. The function returns either `nil` (empty intersection), a single point, or two points (sequence).

merge3d()

The function **ld.merge3d(L [, epsilon])** combines, if possible, the connected components of $\langle L\rangle$, which must be a list of lists of 3D points. The function returns the result. The argument $\langle epsilon\rangle$ defaults to 10^{-10}, it is used during comparisons.

split_points_by_visibility()

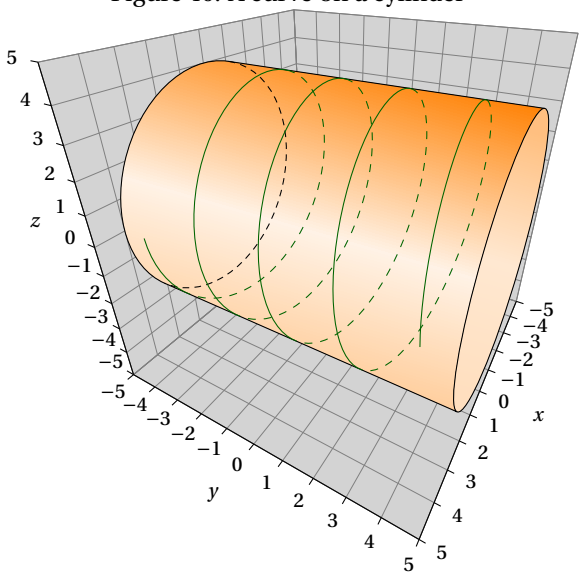
The function **ld.split_points_by_visibility(L, visible_function)**, where $\langle L\rangle$ is a list of 3D points, or a list of lists of 3D points, and where $\langle visible_function\rangle$ is a function such that `visible_function(A)` returns `true` if the 3D point A is visible, `false` otherwise, sorts the points of $\langle L\rangle$ according to whether they are visible or not. The function returns a sequence of two tables: *visible_points*, *hidden_points*.

```
\begin{luadraw}{name=curve_on_cylinder}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M, Mc = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, pt3d.Mc

local g = ld.graph3d:new{adjust2d=true,bbox=false,size={10,10}, viewdir="central"}
g:Labelsize("footnotesize")
ld.Hiddenlines = true; ld.Hiddenlinestyle = "dashed"
local curve_on_cylinder = function(curve,cylinder)
-- curve is a 3D polyline on a cylinder,
-- cylinder = {A,r,V,B}
    local A,r,V,B = table.unpack(cylinder)
    if B == nil then B = V; V = B-A end
    local U = B-A
    local visible_function
    if ld.projection_mode == "central" then
        visible_function = function(N)
            local I = ld.dproj3d(N,{A,U})
            local M1, M2 = ld.interCS({I,r,U},{ (I+ld.camera)/2, pt3d.abs(I-ld.camera)/2})
            local alpha = pt3d.angle3d(M1-I,ld.camera-I)
            return pt3d.angle3d(N-I,ld.camera-I) <= alpha
        end
    else
        visible_function = function(N)
            local I = ld.dproj3d(N,{A,U})
            return (pt3d.dot(N-I,g.Normal) >= 0)
        end
    end
    return ld.split_points_by_visibility(curve,visible_function)
end
-- test
local A, r, B = -5*vecJ, 4, 5*vecJ -- cylinder
local p = function(t) return Mc(r,t,t/3) end
local Curve = ld.rotate3d( ld.parametric3d(p,-4*math.pi,4*math.pi),90,{Origin,vecI})
local Vi, Hi = curve_on_cylinder(Curve,{A,r,B})
local curve_color = "DarkGreen"
g:Dboxaxes3d({grid=true,gridcolor="gray",fillcolor="LightGray"})
```

```
g:Dcylinder(A,r,B,{color="orange"})
g:Dpolyline3d(Vi,curve_color)
g:Dpolyline3d(Hi,curve_color.."."..ld.Hiddenlinestyle)
g:Show()
\end{luadraw}
```

Figure 40: A curve on a cylinder



VIII More Advanced Examples

1) The Box of Sugars

The problem[7] is to draw sugars in a box. You need to be able to position the desired number of pieces, and wherever you want them in the box[8] without having to rewrite the entire code. Another constraint: to keep the figure as light as possible, only the facets actually seen should be displayed. In the code below, we keep the default viewing angles, and:

- the sugars are cubes of side 1 (we then modify the 3D matrix of the graph to "elongate" them),
- each piece is identified by the coordinates (x, y, z) of the upper right corner of the front face, with x an integer between 1 and Lg, y an integer between 1 and lg, and z an integer between 1 and ht.
- to store the positions of the pieces, we use a three-dimensional matrix *positions*, one for x, one for y, and one for z, with the convention that *positions[x][y][z]* is 1 if there is a sugar at position (x, y, z), and 0 otherwise.
- For each piece, there are at most three visible faces: the top one, the right one, and the front one.[9], but we only draw the top face if there isn't another sugar cube above it, we only draw the right face if there isn't another sugar cube to the right, and we only draw the front face if there isn't another sugar cube in front. This builds the list of facets actually seen.
- In the scene display, you must **put the box first**, otherwise its facets will be cut off by the planes of the sugar cube facets. The sugar cube facets cannot be cut off by the box because they are all inside.

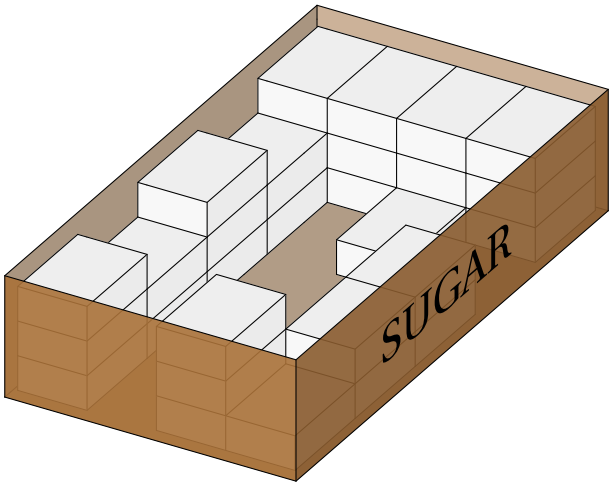
```
\begin{luadraw}{name=boite_sucres}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M, Mc = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, pt3d.Mc

local g = ld.graph3d:new{window={-9,8,-10,4},size={10,10}}
ld.Hiddenlines = false
local Lg, lg, ht = 5, 4, 3 -- length, width, height (box size)
```

[7] Problem posed in a forum, the objective being to use it as counting exercises for students.
[8] A piece must rest either on the bottom of the box or on top of another piece
[9] Provided you don't change the viewing angles!

```
local positions = {} -- 3-dimensional matrix initialized with 0s
for L = 1, Lg do
    local X = {}
    for l = 1, lg do
        local Y = {}
        for h = 1, ht do table.insert(Y,0) end
        table.insert(X,Y)
    end
    table.insert(positions,X)
end
local facetList = function() -- returns the list of facets to draw (pay attention to the orientation)
    local facet = {}
    for x = 1, Lg do -- loop over the positions matrix
        for y = 1, lg do
            for z = 1, ht do
                if positions[x][y][z] == 1 then -- there is a sugar in (x,y,z)
                    if (z == ht) or (positions[x][y][z+1] == 0) then -- no sugar on top so top side visible
                        table.insert(facet, {M(x,y,z),M(x-1,y,z),M(x-1,y-1,z),M(x,y-1,z)}) -- insert top face
                    end
                    if (y == lg) or (positions[x][y+1][z] == 0) then -- no sugar on the right so right side visible
                        table.insert(facet, {M(x,y,z),M(x,y,z-1),M(x-1,y,z-1),M(x-1,y,z)}) -- insert right face
                    end
                    if (x == Lg) or (positions[x+1][y][z] == 0) then -- no sugar in front so front side visible
                        table.insert(facet, {M(x,y,z),M(x,y-1,z),M(x,y-1,z-1),M(x,y,z-1)}) -- insert front face
                    end
                end
            end
        end
    end
    return facet
end
-- creation of the box (parallelepiped)
local O = Origin -0.1*M(1,1,1) -- so that the box does not stick to the sugars
local boite = ld.parallelep(O, (Lg+0.2)*vecI, (lg+0.2)*vecJ, (ht+0.5)*vecK)
table.remove(boite.facets,2) -- we remove the top of the box, this is facet number 2
-- sugars are positioned
for y = 1, 4 do for z = 1, 3 do  positions[1][y][z] = 1 end end
for x = 2, 5 do for z = 1, 2 do positions[x][1][z] = 1 end end
for z = 1, 3 do positions[5][3][z] = 1 end
for z = 1, 2 do positions[4][4][z] = 1 end
for z = 1, 2 do positions[3][4][z] = 1 end
positions[5][1][3] = 1; positions[3][1][3] = 1; positions[5][4][1] = 1; positions[2][3][1] = 1
g:Setmatrix3d({Origin,3*vecI,2*vecJ,vecK}) -- expansion on Ox and Oy to "lengthen" the cubes...
g:Dscene3d( -- drawing
    g:addPoly(boite,{color="brown",edge=true,opacity=0.9}),
    g:addFacet(facetList(), {backcull=true,contrast=0.25,edge=true})    )
g:Labelsize("huge"); g:Dlabel3d( "SUGAR", M(Lg/2+0.1,lg+0.1,ht/2+0.1), {dir={-vecI,vecK}})
g:Show()
\end{luadraw}
```

Figure 41: Box of Sugar Cubes



2) Stack of Cubes

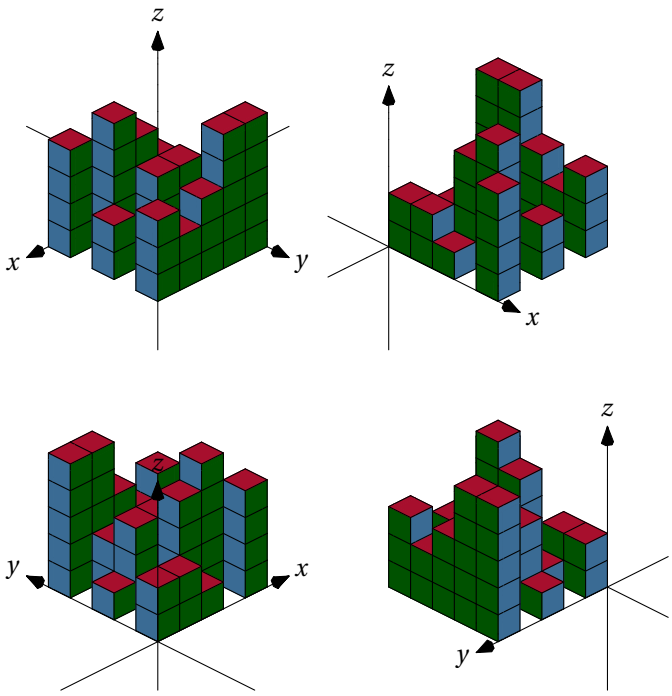
We can modify the previous example to draw a stack of randomly positioned cubes, with four views. We'll position the cubes by placing a random number per column, starting from the bottom. We'll create four views of the stack, adding axes to help us navigate between these different views. This slightly changes the search for potentially visible facets; there are five cases per cube, not just three (front, back, left, right, and top; we don't create bottom views). To make the stack more readable, we use three colors to paint the cube faces (two opposite faces have the same color).

```
\begin{luadraw}{name=cubes_empiles}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window3d={-6,6,-6,6,-6,6},size={10,10}}
ld.Hiddenlines = false
local Lg, lg, ht, a = 5, 5, 5, 2 -- length, width, height of the space to fill, size of a cube
local positions = {} -- 3-dimensional matrix initialized with 0s
for L = 1, Lg do
    local X = {}
    for l = 1, lg do
        local Y = {}
        for h = 1, ht do table.insert(Y,0) end
        table.insert(X,Y)
    end
    table.insert(positions,X)
end
for x = 1, Lg do  -- random positioning of cubes
    for y = 1, lg do
        local nb = math.random(0,ht) -- we put number of cubes in the column (x,y,*) starting from the bottom
        for z = 1, nb do positions[x][y][z] = 1 end
    end
end
local dessus,gauche,devant = {},{},{} -- to memorize the facets
for x = 1, Lg do -- loop over the positions matrix to determine the facets to draw
    for y = 1, lg do
        for z = 1, ht do
            if positions[x][y][z] == 1 then -- there is a cube in  (x,y,z)
                if (z == ht) or (positions[x][y][z+1] == 0) then -- no cube above, top face visible
                    table.insert(dessus,{M(x,y,z),M(x-1,y,z),M(x-1,y-1,z),M(x,y-1,z)}) -- insert top face
                end
                if (y == lg) or (positions[x][y+1][z] == 0) then -- right face visible
```

```
                    table.insert(gauche,{M(x,y,z),M(x,y,z-1),M(x-1,y,z-1),M(x-1,y,z)}) -- insert right face
                end
                if (y == 1) or (positions[x][y-1][z] == 0) then -- left face visible
                    table.insert(gauche,{M(x,y-1,z),M(x-1,y-1,z),M(x-1,y-1,z-1),M(x,y-1,z-1)}) -- insert left face
                end
                if (x == Lg) or (positions[x+1][y][z] == 0) then -- front face visible
                    table.insert(devant,{M(x,y,z),M(x,y-1,z),M(x,y-1,z-1),M(x,y,z-1)}) -- insert front face
                end
                if (x == 1) or (positions[x-1][y][z] == 0) then -- no cube behind so back face visible
                    table.insert(devant,{M(x-1,y,z),M(x-1,y,z-1),M(x-1,y-1,z-1),M(x-1,y-1,z)}) -- back face
                end
            end
        end
    end
end
g:Setmatrix3d({M(-a*Lg/2,-a*lg/2,-a*ht/2),a*vecI,a*vecJ,a*vecK}) -- to center the figure and have cubes of side a
local dessin = function()
    g:Dscene3d(
        g:addFacet(dessus, {backcull=true,color="Crimson"}), g:addFacet(gauche, {backcull=true,color="DarkGreen"}),
        g:addFacet(devant, {backcull=true,color="SteelBlue"}),
        g:addPolyline(ld.facetedges(ld.concat(dessus,gauche,devant))), -- drawing edges
        g:addAxes(Origin,{arrows=1}))
end
g:Saveattr(); g:Viewport(-5,0,0,5); g:Coordsystem(-11,11,-11,11); g:Setviewdir(45,60) -- top left
 dessin(); g:Restoreattr()
g:Saveattr(); g:Viewport(0,5,0,5);g:Coordsystem(-11,11,-11,11); g:Setviewdir(-45,60) -- top right
dessin(); g:Restoreattr()
g:Saveattr(); g:Viewport(-5,0,-5,0);g:Coordsystem(-11,11,-11,11); g:Setviewdir(-135,60) -- bottom left
dessin(); g:Restoreattr()
g:Saveattr(); g:Viewport(0,5,-5,0);g:Coordsystem(-11,11,-11,11); g:Setviewdir(135,60) -- bottom right
dessin(); g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 42: Stack of Cubes



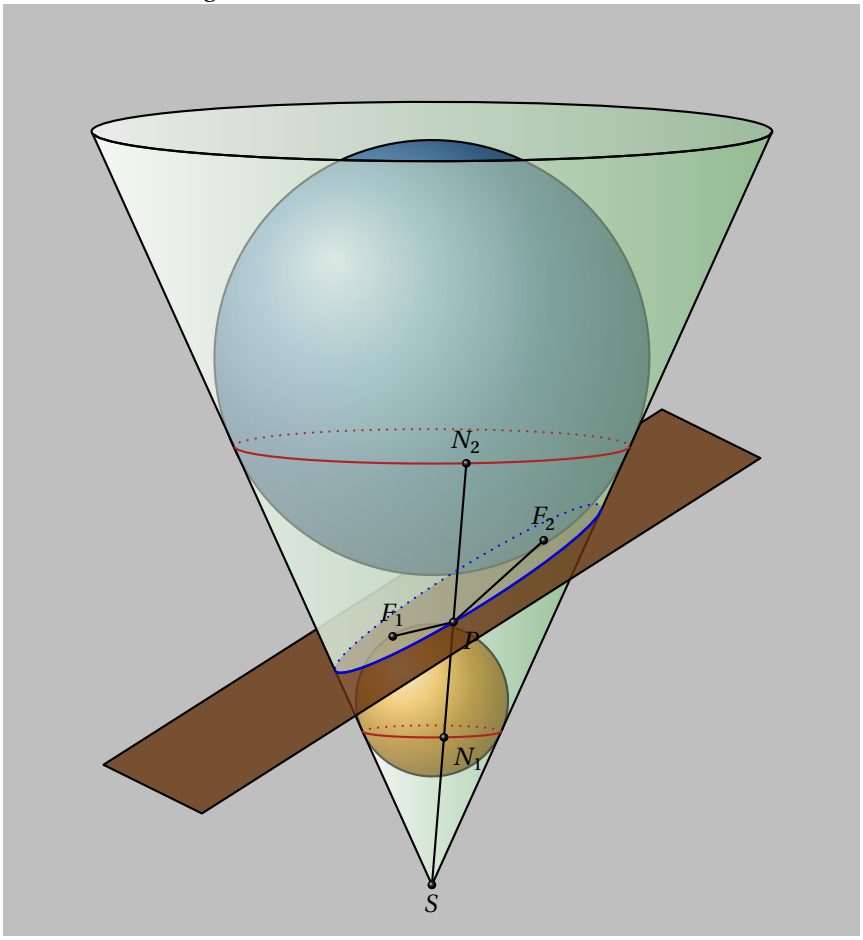
3) Illustration of Dandelin's Theorem

```
\begin{luadraw}{name=Dandelin}
local ld = luadraw
```

```
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window3d={-5,5,-5,5,-5,5}, window={-5,5,-5,6}, bg="lightgray",viewdir={-10,85}}
g:Linewidth(8)
local sqrt = math.sqrt
local sqr = function(x) return x*x end
local L, a = 4.5, 2
local R = (a+5)*L/sqrt(100+L^2) --large sphere center=M(0,0,a) radius=R
local S2 = ld.sphere(M(0,0,a),R,45,45)
local k = 0.35 --homothety ratio
local b, r = (a+5)*k-5, k*R -- small sphere center = M(0,0,b) radius = r
local S1 = ld.sphere(M(0,0,b),r,45,45)
local c = (b+k*a)/(1+k) --second center of homothety
local z = a+sqr(R)/(c-a) --image of c under inversion with respect to the large sphere
local M1 = M(0,sqrt(sqr(R)-sqr(z-a)),z)--point of the large sphere and the tangent plane
local N = M1-M(0,0,a) --normal vector to the tangent plane
local plane = {M(0,0,c),-N} --tangent plane
local z2 = a+sqr(R)/(-5-a) --image of the vertex under inversion with respect to the large sphere
local z1 = b+sqr(r)/(-5-b) --image of the vertex by the inversion with respect to the small sphere
local P2 = M(sqrt(R^2-(z2-a)^2),0,z2)
local P1= M(sqrt(r^2-(z1-b)^2),0,z1)
local S = M(0,0,-5)
local P = ld.interDP({P1,P2-P1},plane)
local C = ld.cone(M(0,0,-5),10*vecK,L,45,true)
local ellips = g:Intersection3d(C,plane)
local plane1 = {M(0,0,z1),vecK}
local plane2 = {M(0,0,z2),vecK}
local L1, L2 = g:Intersection3d(S1,plane1), g:Intersection3d(S2,plane2)
local F1, F2 = ld.proj3d(M(0,0,b), plane), ld.proj3d(M(0,0,a), plane) --focal points
local s1, s2 = g:Proj3d(M(0,0,a)), g:Proj3d(M(0,0,b))
local V, H = g:Classifyfacet(C) -- separate visible and invisible facets
local V1, V2 = ld.cutfacet(V,plane)
local H1, H2 = ld.cutfacet(H,plane)
-- Drawing
 -- faces not visible under the plane, fill only
g:Dpolyline3d( ld.border(H2),"left color=white, right color=DarkSeaGreen, draw=none")
g:Dsphere( M(0,0,b), r, {mode=ld.mBorder,color="Orange"}) -- small sphere
 -- visible faces below the plane
g:Dpolyline3d( ld.border(V2),"left color=white, right color=DarkSeaGreen, fill opacity=0.4")
g:Dpolyline3d({S,P}) -- segment [S,P] which is partially below the plane
g:Dfacet( g:Plane2facet(plane,0.75), {color="Chocolate", opacity=0.8}) -- the plane
 -- outline of faces not visible above the plane, fill only
g:Dpolyline3d( ld.border(H1),"left color=white, right color=DarkSeaGreen,draw=none,fill opacity=0.7")
g:Dsphere( M(0,0,a),R, {mode=ld.mBorder,color="SteelBlue"}) -- large sphere
 -- outline of faces visible above the plane
g:Dpolyline3d( ld.border(V1),"left color=white, right color=DarkSeaGreen, fill opacity=0.6")
g:Dcircle3d(M(0,0,5),L,vecK) -- opening of the cone
g:Dpolyline3d({{P,F1},{F2,P,P2}})
g:Dedges(L1,{hidden=true,color="FireBrick"})
g:Dedges(L2,{hidden=true,color="FireBrick"})
g:Dedges(ellips,{hidden=true, color="blue"})
g:Dballdots3d({F1,F2,S,P1,P,P2},nil,0.75)
g:Dlabel3d( "$F_1$",F1,{pos="N"}, "$F_2$",F2,{}, "$N_2$",P2,{},"$S$",S,{pos="S"},
 "$N_1$",P1,{pos="SE"}, "$P$",P,{pos="SE"} )
g:Show()
\end{luadraw}
```

Figure 43: Illustration of Dandelin's Theorem



We want to draw a cone with a section through a plane and two spheres inside this cone (and tangent to the plane), but without drawing any spheres or faceted cones. The starting point, however, is the creation of these faceted solids, the spheres *S1* and *S2* (lines 11 and 8 of the listing) as well as the cone *C* in line 23. The drawing principle is as follows:

1. We separate the facets of the cone into two categories: the visible facets (facing the observer) and the others (variables *V* and *H* in line 30), which actually correspond to the front of the cone and the back of the cone.
2. We divide the two lists of facets with the plane (lines 31 and 32). Thus, *V1* corresponds to the front facets located above the plane and *V2* corresponds to the front facets located below the plane (same thing with *H1* and *H2* for the back).
3. We then draw the outline of *H2* with a gradient fill (only) (line 34).
4. We draw the small sphere (in orange, line 35).
5. We draw the outline of *V2* with a gradient fill and transparency to see the small sphere (line 36).
6. We draw the segment $[S,P]$ (line 37) then the plane as a transparent facet (line 38).
7. We draw the outline of *H1* with a gradient fill (line 39). This is the back part above the plane.
8. We draw the large sphere (line 40).
9. Finally, we draw the outline of *V1* with a gradient fill (line 41) and transparency to see the sphere (this is the front part of the cone above the plane), then the opening of the cone (line 42).
10. We draw the intersections between the cone and the spheres (lines 44 and 45) as well as between the cone and the plane (line 46).

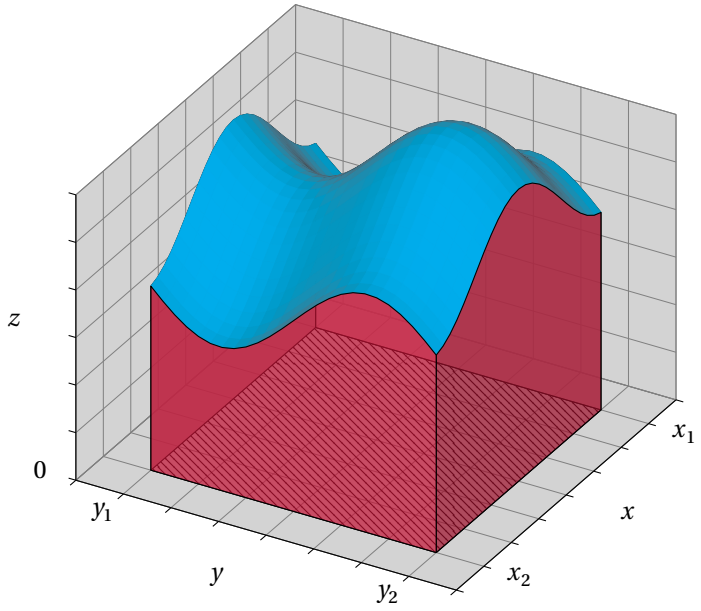
4) Volume defined by a double integral

```
\begin{luadraw}{name=volume_integrale}
local ld = luadraw
local M = ld.pt3d.M
```

```
local pi, sin, cos = math.pi, math.sin, math.cos

local g = ld.graph3d:new{window3d={-4,4,-4,4,0,6},adjust2d=true,margin={0,0,0,0},size={10,10}}
local x1, x2, y1, y2 = -3,3,-3,3 -- bornes
local f = function(x,y) return cos(x)+sin(y)+5 end -- function to integrate
local p = function(u,v) return M(u,v,f(u,v)) end -- surface parameterization z=f(x,y)
local Fx1 = ld.concat({ld.pxy(p(x1,y2)), ld.pxy(p(x1,y1))}, ld.parametric3d(function(t) return p(x1,t)
 ↪  end,y1,y2,25,false,0)[1])
local Fx2 = ld.concat({ld.pxy(p(x2,y1)), ld.pxy(p(x2,y2))}, ld.parametric3d(function(t) return p(x2,t)
 ↪  end,y2,y1,25,false,0)[1])
local Fy1 = ld.concat({ld.pxy(p(x1,y1)), ld.pxy(p(x2,y1))}, ld.parametric3d(function(t) return p(t,y1)
 ↪  end,x2,x1,25,false,0)[1])
local Fy2 = ld.concat({ld.pxy(p(x2,y2)), ld.pxy(p(x1,y2))}, ld.parametric3d(function(t) return p(t,y2)
 ↪  end,x1,x2,25,false,0)[1])
g:Dboxaxes3d({grid=true, gridcolor="gray",fillcolor="LightGray",labels=false})
g:Filloptions("fdiag","black"); g:Dpolyline3d( {M(x1,y1,0),M(x1,y2,0),M(x2,y2,0),M(x2,y1,0)}) -- below
g:Dfacet( {Fx1,Fy1},{mode=ld.mShaded,opacity=0.7,color="Crimson"} )
g:Dfacet(ld.surface(p,x1,x2,y1,y2), {mode=ld.mShadedOnly,color="cyan"})
g:Dfacet( {Fx2,Fy2},{mode=ld.mShaded,opacity=0.7,color="Crimson"} )
g:Dlabel3d("$x_1$", M(x1,4.75,0),{}, "$x_2$", M(x2,4.75,0),{},
"$y_1$", M(4.75,y1,0),{}, "$y_2$", M(4.75,y2,0),{}, "$0$",M(4,-4.75,0),{})
g:Show()
\end{luadraw}
```

Figure 44: Volume corresponding to $\int_{x_1}^{x_2}\int_{y_1}^{y_2} f(x,y)dxdy$



Here, the solid represented has lateral faces (*Fx1*, *Fx2*, *Fy1* and *Fy2*) with one side being a parametric curve. We therefore take the points of this parametric curve (its first connected component) and add the projections of the two ends onto the xy-plane. Care must be taken with the direction of travel so that the faces are correctly oriented (normal outward). This normal is calculated from the first three points of the face; it is best to start the face with the two projections onto the plane to be sure of the orientation. We draw the bottom first, then the lateral faces, and finish with the surface.

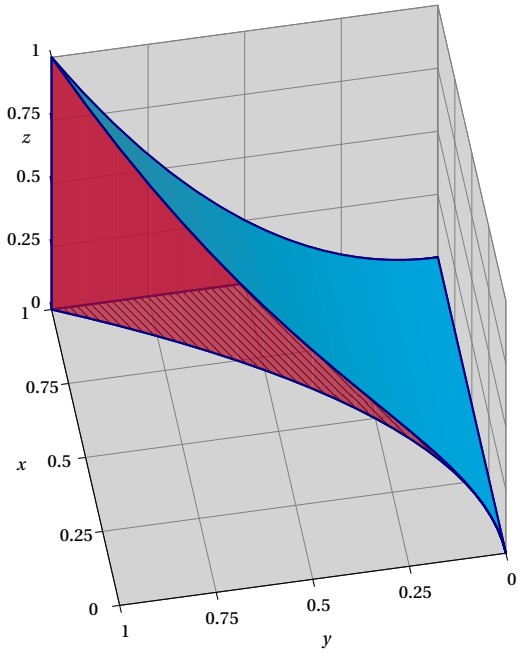
5) Volume defined on something other than a block

```
\begin{luadraw}{name=volume2}
local ld = luadraw
local M = ld.pt3d.M
local pi, sin, cos = math.pi, math.sin, math.cos

local g = ld.graph3d:new{window3d={0,1,0,1,0,1}, margin={0,0,0,0},adjust2d=true,viewdir={170,40}, size={10,10}}
```

```
g:Labelsize("scriptsize")
local f = function(t) return M(t,t^2,0) end
local h = function(t) return M(1,t,t^2) end
local C = ld.parametric3d(f,0,1,25,false,0)[1] -- curve y=x^2 in the plane z=0 (first connected component)
local D = ld.parametric3d(h,1,0,25,false,0)[1] -- curve z=y^2 in the plane x=1, in the opposite direction
local dessous = ld.concat({M(1,0,0)},C) -- forms the underside
local arriere = ld.concat({M(1,1,0)},D) -- forms the back face
local  avant, dessus, A, B = {}, {}, nil, C[1]
for k = 2, #C do --We construct the front and top faces facet by facet, starting from the points of C
    A = B; B = C[k]
    table.insert(avant, {B,A,M(A.x,A.y,A.y^2),M(B.x,B.y,B.y^2)})
    table.insert(dessus, {M(B.x,B.y,B.y^2),M(A.x,A.y,A.y^2),M(1,A.y,A.y^2),M(1,B.y,B.y^2)})
end
g:Dboxaxes3d({grid=true, gridcolor="gray",fillcolor="LightGray", drawbox=false,
    xyzstep=0.25, zlabelstyle="W",zlabelsep=0})
g:Lineoptions(nil,"Navy",8)
g:Dpolyline3d(arriere,true,"fill=Crimson, fill opacity=0.6") -- rear face (flat)
g:Filloptions("fdiag","black"); g:Dpolyline3d(dessous,true) -- below
g:Dmixfacet(avant,{color="Crimson",opacity=0.7,mode=ld.mShadedOnly}, dessus,{color="cyan",opacity=1})
g:Filloptions("none"); g:Dpolyline3d(ld.concat(ld.border(avant),ld.border(dessus)))
g:Show()
\end{luadraw}
```

Figure 45: Volume : $0 \leqslant x \leqslant 1$; $0 \leqslant y \leqslant x^2$; $0 \leqslant z \leqslant y^2$



In this example, the surface has the equation $z = y^2$ (parabolic cylinder), but we are no longer on a block. The front face is not flat; we construct it like a cylinder (line 14) with vertical facets resting on curve C at the bottom, and on curve $t \mapsto M(t, t^2, t^4)$ at the top.

Similarly, the top face (the surface) is constructed like a horizontal cylinder resting on curves D and $t \mapsto M(t, t^2, t^4)$.

We could not construct the surface by hand (called *dessus* in the code), and instead draw the following surface (after the front face):

```
g:Dfacet( ld.surface(function(u,v) return M(u,v*u^2,v^2*u^4) end, 0,1,0,1), {mode=ld.mShadedOnly, color="cyan"})
```

but it has many more facets (25×25) than the cylinder-shaped construction (21 facets), which is less interesting.

Chapter 3

Appendices

I Extensions

1) The *luadraw_polyhedrons* module

This module is still in draft form and is expected to be expanded in the future. As its name suggests, it contains the definition of polyhedra. All numerical data comes from the Visual Polyhedra website.

Usage: This module does not add new graphics methods, but it returns a table of functions for constructing polyhedra. Therefore, it is used as follows (example):

```
local poly = require 'luadraw_polyhedrons'
local M = pt3d.M
local T = poly.tetrahedron( M(0,0,0), M(1,1,1) )
```

All functions follow the same model: **<name>(C, S [, all])** where *C* is the center of the polyhedron (3D point) and *S* is a vertex of the polyhedron (3D point). When *C* or *S* have the value *nil*, the untransformed polyhedron (centered at the origin) is returned. The optional argument *all* is a boolean. When it has the value *true*, the function returns four things: *P, V, E, F* where:

- *P* is the solid as a polyhedron,
- *V* the list (table) of vertices,
- *E* the list (table) of edges (with 3D points),
- *F* the list of facets (with 3D points). Some polyhedra have multiple facet types; in this case, the returned result is of the form: *P, V, E, F1, F2, ...*, where $F1$, $F2$, ... are lists of facets. This can allow them to be drawn with different colors, for example.

The argument *all* is set to *false*, which is the default value; the function only returns the polyhedron.

Here are the solids currently contained in this module:

- The Platonic solids, these solids have only one face type:
  - The function **tetrahedron(C, S [, all])** allows the construction of a regular tetrahedron with center $\langle C\rangle$ (3d point) and one vertex at $\langle S\rangle$ (3d point).
  - The function **octahedron(C, S [, all])** allows the construction of an octahedron with center $\langle C\rangle$ (3d point) and one vertex at $\langle S\rangle$ (3d point).
  - The function **cube(C, S [, all])** allows the construction of a cube with center $\langle C\rangle$ (3d point) and one vertex at $\langle S\rangle$ (3d point).
  - The function **icosahedron(C, S [, all])** allows the construction of an icosahedron with center $\langle C\rangle$ (3d point) and one vertex at $\langle S\rangle$ (3d point).
  - The function **dodecahedron(C, S [, all])** allows the construction of a dodecahedron with center $\langle C\rangle$ (3d point) and one vertex at $\langle S\rangle$ (3d point).

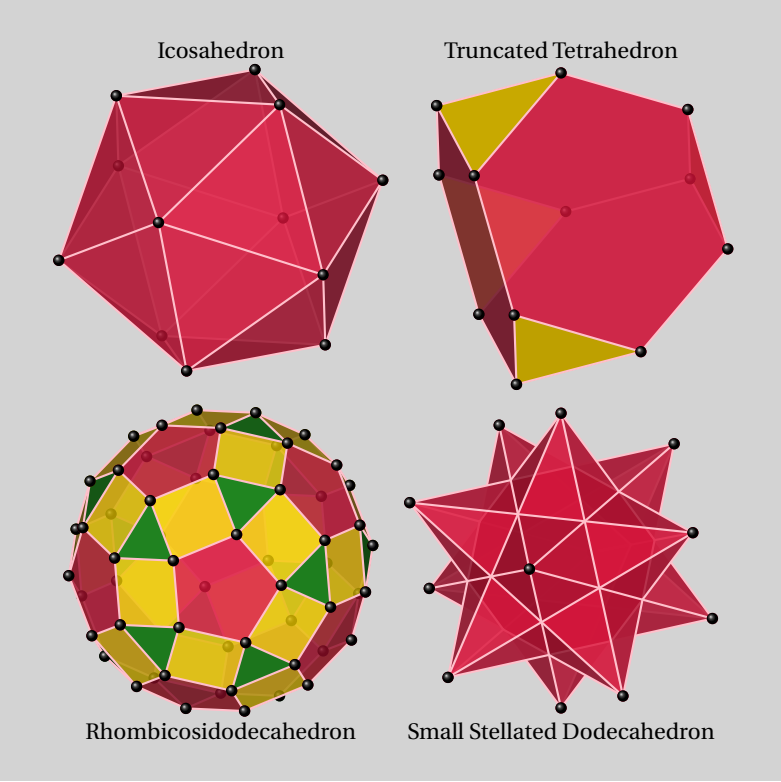
- The Archimedean Solids:
  - The function **cuboctahedron(C, S [, all])** allows the construction of a cuboctahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **icosidodecahedron(C, S [, all])** allows the construction of an icosidodecahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **lsnubcube(C, S [, all])** allows the construction of a snub cube (form 1) with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **lsnubdodecahedron(C, S [, all])** allows the construction of a snub dodecahedron (form 1) with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **rhombicosidodecahedron(C, S [, all])** allows the construction of a rhombicosidodecahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has three types of faces.
  - The function **rhombicuboctahedron(C, S [, all])** allows the construction of a rhombicuboctahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **rsnubcube(C, S [, all])** allows the construction of a snub cube (shape 2) with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **rsnubdodecahedron(C, S [, all])** allows the construction of a snub dodecahedron (shape 2) with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **truncatedcube(C, S [, all])** allows the construction of a truncated cube with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **truncatedcuboctahedron(C, S [, all])** allows the construction of a truncated cuboctahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has three types of faces.
  - The function **truncateddodecahedron(C, S [, all])** allows the construction of a truncated dodecahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **truncatedicosahedron(C, S [, all])** allows the construction of a truncated icosahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **truncatedicosidodecahedron(C, S [, all])** allows the construction of a truncated icosidodecahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has three types of faces.
  - The function **truncatedoctahedron(C, S [, all])** allows the construction of a truncated octahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **truncatedtetrahedron(C, S [, all])** allows the construction of a truncated tetrahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
- Other solids:
  - The function **octahemioctahedron(C, S [, all])** allows the construction of an octahemioctahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has two types of faces.
  - The function **small_stellated_dodecahedron(C, S [, all])** allows the construction of a small stellated dodecahedron with center $\langle C \rangle$ (3d point) and one vertex at $\langle S \rangle$ (3d point). This solid has only one type of face.

```
\begin{luadraw}{name=polyhedrons}
local ld = luadraw
local M, Origin = ld.pt3d.M, ld.pt3d.Origin
local i = ld.cpx.I
local poly = require 'luadraw_polyhedrons' -- chargement du module

local g = ld.graph3d:new{bg="LightGray", size={10,10}}
g:Labelsize("small"); ld.Hiddenlines = false
-- en haut à gauche
g:Saveattr(); g:Viewport(-5,0,0,5); g:Coordsystem(-5,5,-5,5,true)
local T,S,A,F = poly.icosahedron(Origin,M(0,2,4.5),true)
g:Dscene3d(
```

```
    g:addFacet(F, {color="Crimson",opacity=0.8}),
    g:addPolyline(A, {color="Pink", width=8}),
    g:addDots(S) )
g:Dlabel("Icosahedron",5*i,{})
g:Restoreattr()
-- en haut à droite
g:Saveattr()
g:Viewport(0,5,0,5); g:Coordsystem(-5,5,-5,5,true)
local T,S,A,F1,F2 = poly.truncatedtetrahedron(Origin,M(0,0,5),true) -- Full output, display in a 3D scene
g:Dscene3d(
    g:addFacet(F1, {color="Crimson",opacity=0.8}),
    g:addFacet(F2, {color="Gold"}),
    g:addPolyline(A, {color="Pink", width=8}),
    g:addDots(S) )
g:Dlabel("Truncated Tetrahedron",5*i,{})
g:Restoreattr()
-- en bas à gauche
g:Saveattr(); g:Viewport(-5,0,-5,0); g:Coordsystem(-5,5,-5,5,true)
local T,S,A,F1,F2,F3 = poly.rhombicosidodecahedron(Origin,M(0,0,4.5),true)
g:Dscene3d(
    g:addFacet(F1, {color="Crimson",opacity=0.8}),
    g:addFacet(F2, {color="Gold",opacity=0.8}), g:addFacet(F3, {color="ForestGreen"}),
    g:addPolyline(A, {color="Pink", width=8}), g:addDots(S) )
g:Dlabel("Rhombicosidodecahedron",-5*i,{})
g:Restoreattr()
-- en bas à droite
g:Saveattr(); g:Viewport(0,5,-5,0); g:Coordsystem(-5,5,-5,5,true)
local T,S,A,F1 = poly.small_stellated_dodecahedron(Origin,M(0,0,5),true)
g:Dscene3d(
    g:addFacet(F1, {color="Crimson",opacity=0.8}),
    g:addPolyline(A, {color="Pink", width=8}),
    g:addDots(S) )
g:Dlabel("Small Stellated Dodecahedron",-5*i,{})
g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 1: Polyhedra from the *luadraw_polyhedrons* module



2) The *luadraw_spherical* Module

This module allows you to draw a number of objects on a sphere (such as circles, spherical triangles, etc) without having to manually manage the visible or invisible parts. Drawing is done in three steps:

1. We define the characteristics of the sphere (center, radius, color, etc.)
2. We define the objects to be added to the scene using dedicated methods.
3. We display everything with the **g:Dspherical()** method.

Of course, all 2D and 3D drawing methods remain usable.

Usage: This module adds new graphics methods to the *ld.graph3d* class; it does not return anything. The variables introduced by this module go in the *luadraw* namespace.

Global Module Functions

By default, these functions apply to the current sphere, but some of them can be applied to a sphere other than the current one; these have a final argument named ⟨*sph*⟩, which is optional and defaults to `nil`, but when specified, it must be a table of the form {center, radius} (representing a sphere).

- **ld.sM(x, y, z [, sph])**: returns a point on the sphere; this is the point I on the sphere such that the half-line $[O,I)$ (O being the center of the sphere) passes through the point A with Cartesian coordinates (x,y,z). It is the projection of the point $M(x,y,z)$ onto the sphere from the center.
- **ld.sM(theta, phi [, sph])**: where ⟨*theta*⟩ and ⟨*phi*⟩ are angles in degrees, returns a point on the sphere, whose spherical coordinates are *(R,theta,phi)* where R is the radius of the sphere.
- **ld.smidpoint(A, B, x [, sph])** : ⟨A⟩ and ⟨B⟩ are two points on the sphere and ⟨x⟩ is a number between 0 and 1; the function returns the point at curvilinear coordinate ⟨x⟩ along the great-circle arc $\overset{\frown}{AB}$, which is therefore a point on the sphere. For example, with $x = 0.5$, the returned point is the midpoint of the arc $\overset{\frown}{AB}$.
- **ld.sbarycenter(list [, sph])** : where ⟨*list*⟩ = {A1,c1,A2,c2,…,An,cn}, the arguments ⟨*A1*⟩, …,⟨*An*⟩ are points on the sphere and the arguments ⟨*c1*⟩, …,⟨*cn*⟩ are real numbers; the function calculates the barycenter of these points using the given coefficients and returns its projection onto the sphere from the center.
- **ld.toSphere(A [, sph])**: returns the projection of point ⟨A⟩ onto the sphere from the center.
- **ld.interSphericalC(P1, P2)**: returns, as a sequence, the points of intersection (if they exist) between two circles belonging to the sphere (not necessarily great circles). The two arguments ⟨*P1*⟩ and ⟨*P2*⟩ are two planes, and it is their intersection with the sphere that forms the two circles whose intersection we are looking for.
- **ld.interGreatC(C1,C2)**: returns, as a sequence, the two points of intersection of the two great circles ⟨*C1*⟩ and ⟨*C2*⟩ belonging to the sphere. A great circle of the sphere is a list of two points on the sphere **not collinear with the center**.
- **ld.projstereo_Scircle(P, N, h)**: returns the stereographic projection of a circle drawn on the sphere as a path. The argument ⟨P⟩ is a plane, and its intersection with the sphere forms the circle to be projected. The argument ⟨N⟩ designates a point on the sphere, which will be the pole of the projection. The argument ⟨h⟩ is a real number that defines the projection plane. This plane is perpendicular to the axis (CN), where C is the center of the sphere, and passes through the point $I = C + h\frac{\vec{CN}}{CN}$. With $h = 0$, this is the equatorial plane; with $h = -R$, where R is the radius of the sphere, this is the plane tangent to the sphere at the opposite pole.
- **ld.projstereo_Sfacet(L, N, h [, close])**: returns the stereographic projection of a spherical facet (drawn onto the sphere) as a path. The argument ⟨L⟩ is a list of points on the sphere that form the vertices of the facet; two consecutive vertices are connected by a great-circle arc (the angular distance between two consecutive vertices must not exceed 180 degrees). The argument ⟨N⟩ designates a point on the sphere that will be the pole of the projection. The argument ⟨h⟩ is a real number that defines the projection plane. This plane is perpendicular to the axis (CN), where C is the center of the sphere, and passes through the point $I = C + h\frac{\vec{CN}}{CN}$. With $h = 0$, this is the equatorial plane; with $h = -R$, where R is the radius of the sphere, this is the plane tangent to the sphere at the opposite pole. The optional argument ⟨*close*⟩ indicates whether the list ⟨L⟩ should be closed (`true` by default).

The module also offers the following two methods:

- The method **g:Split_facet(facet [, sph])**, where ⟨*facet*⟩ is a list of points **that must lie on the sphere**. The method returns a sequence of the form `front, back`; these are two 3D paths, where the first represents the outline of the visible part of the spherical facet (the edges are great-circle arcs) and the second path represents the outline of the part of the facet located on the hidden side of the sphere.
- The method **g:Split_region(L [, sph])**, where ⟨*L*⟩ is a list of 3D points representing a **closed curve drawn on the sphere**. The method returns a sequence of the form `front, back, out`; the first element is a 3D path representing the outline of the visible part of the spherical region, the second element is a 3D path representing the outline of the part of the region located on the hidden side of the sphere, and the last element is a table containing the points corresponding to the endpoints of the hidden sections.

Sphere Definition

The sphere is defined using the method **g:Define_sphere(options)**, where ⟨*options*⟩ is a table allowing adjustment of each parameter. These are as follows (with their default values):

- `center=pt3d.Origin`,
- `radius=3`,
- `color="orange"`,
- `opacity=1`,
- `mode=ld.mBorder`, sphere display mode (possible values: `ld.mWireframe` or `ld.mGrid` or `ld.mBorder`),
- `edgecolor="lightgray"`,
- `edgestyle="solid"`,
- `hiddenstyle=ld.Hiddenlinestyle`,
- `hiddencolor="gray"`,
- `edgewidth=4`,
- `show=true`, to show or hide the sphere,
- `back=true`, to show or hide what is behind the sphere or on the hidden side,
- `inside=true`, to show or hide what is inside the sphere,
- `front=true`, to show or hide what is in front of the sphere or on the visible side,
- `insidelabelcolor="darkgray"`: Defines the color of labels whose anchor point is inside the sphere,
- `arrowBstyle="->"`: Type of arrow at the end of the line,
- `arrowAstyle="<-"`: Type of arrow at the beginning of the line,
- `arrowABstyle="<->"`: Very rarely used because most of the time the lines drawn on the sphere must be cut,
- `hiddendelayed=false`: with the value `false` the hidden parts are drawn at the end of the instruction **g:Dspherical()**, with the value `true` they are drawn at the very end of the current graph which can be useful if you have added elements after the sphere which hide part of it (However, this behavior can be modified locally with the option `hidden=true/false`).

The **g:Clear_spherical()** method allows you to remove objects that have been added to the scene, and resets the values to default.

Adding a label to graphic elements

Most of the following graphic elements allow a label to be added via their options. The options regarding this optional label are:

- `label=""` : the label to be added.
- `anchor1d=nil` : a number between 0 and 1 indicating the label's position along the object's 2D projection (0 for the start of the line, 1 for the end of the line).
- `anchor=nil` : a 3D point representing the label's anchor point in space.
- `anchor2d=cpx.Z(0.5,0.5)` : a complex number representing the label's position within the $[0;1] \times [0;1]$ square corresponding to the 2D bounding box of the object's 2D projection; thus, by default, the label is centered within this box. The order of priority is: `anchor`, `anchor1d`, `anchor2d` (if the first option is `nil`, the second is chosen; if that is also `nil`, then the third is chosen).
- `pos="center"`, indicates the label's position relative to the anchor point; possible values are `"center"` (centered), `"N"` (north), `"NE"` (northeast), `"E"` (east), `"SE"` (southeast), `"S"` (south), `"SW"` (southwest), `"W"` (west), and `"NW"` (northwest). The default value is `center`, in which case the label is centered on the anchor point.
- `dist=0`, distance in cm between the label and its anchor point when `pos` is not equal to `"center"`.
- `dir={}`, this option is a table in the form `{dirX, dirY [,dep]}` specifying the writing direction. The three values `dirX`, `dirY`, and `dep` are 3D vectors; the first two indicate the orientation of the text, while the third represents a displacement (translation) of the label relative to the anchor point. The `dep` vector is null by default.
- `node_options=""` is a string (empty by default) intended to hold options that will be passed directly to TikZ within the *node[]* instruction.

Add a circle: g:DScircle

The **g:DScircle(P, options)** method allows you to add a circle to the sphere. The argument P is a table of the form $\{A, n\}$ that represents a plane (passing through A and normal to n, two 3D points). The circle is then defined as the intersection of this plane with the sphere. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `out=nil`, if we assign a list variable to this ⟨*out*⟩ parameter, then the function adds to this list the two points corresponding to the ends of the hidden arc, if any, which allows us to retrieve them without having to calculate them.
- options for adding a label: see page 165.

Add a great circle: g:DSgreatcircle

The method **g:DSgreatcircle(AB, options)** adds a great circle to the sphere. The argument ⟨*AB*⟩ is a table of the form $\{A, B\}$ where A and B are two distinct points on the sphere. The great circle is then the circle centered at the center of the sphere, and passing through A and B. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,

- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `out=nil`, if we assign a list variable to this ⟨*out*⟩ parameter, then the function adds to this list the two points corresponding to the ends of the hidden arc, if any, which allows us to retrieve them without having to calculate them.
- options for adding a label: see page 165.

Add a great circle arc: g:DSarc

The method **g:DSarc(AB, sens, options)** allows you to add a great circle arc to the sphere. The argument ⟨*AB*⟩ is a table of the form $\{A,B\}$ where A and B are two distinct points on the sphere. The great circle arc is then drawn from A to B. The argument ⟨*sens*⟩ is equal to 1 or -1 to indicate the direction of the arc. When A and B are not diametrically opposed, the plane OAB (where O is the center of the sphere) is oriented with $\vec{OA} \wedge \vec{OB}$. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `arrows=0`, three possible values: 0 (no arrow), 1 (one arrow at B), 2 (arrow at A and B).
- `normal=nil`, allows you to specify a normal vector to the OAB plane when these three points are aligned.
- options for adding a label: see page 165.

Add an angle: g:DSangle

The method **g:DSangle(B, A, C, r, sens, options)**, where ⟨*A*⟩, ⟨*B*⟩, and ⟨*C*⟩ are three points on the sphere, allows you to draw a great circle arc on the sphere to represent the angle $(\vec{AB}, \vec{AC})$ with a radius of ⟨*r*⟩. The argument ⟨*sens*⟩ is 1 or -1 to indicate the direction of the arc; the plane ABC is oriented with $\vec{AB} \wedge \vec{AC}$. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `arrows=0`, three possible values: 0 (no arrow), 1 (one arrow at B), 2 (arrow at A and B).
- `normal=nil`, allows you to specify a normal vector to the OAB plane when these three points are aligned.
- Options for adding a label:
  - `label=""` : text to be displayed.
  - `node_options=""` : string defining options for the label.
  - `pos="auto"` : specifies the label's position relative to the anchor point; other possible values are the standard ones for label positioning: `"center"`, `"N"`, `"NW"`, etc. By default, the anchor point is located at the intersection of the circular arc and the angle bisector.

- `dist=r` : distance between the anchor point and the circle's center (A); by default, this is the circle's radius $\langle r \rangle$.
- `angle=0` : rotation angle (in degrees) applied to the default anchor point around the circle's center (A) within the circle's plane.
- `rotate="none"` : indicates whether the label should be rotated around its anchor point **in the screen plane**. Other possible values are: `"auto"` (label follows the direction of the bisector) or `"ortho"` (label is written perpendicular to the bisector).
- `rotate3d="none"` : indicates whether the label should be rotated around its anchor point **in the** (ABC) **plane**. Other possible values are: `"auto"` (label follows the direction of the bisector) or `"ortho"` (label is written perpendicular to the bisector, still within the (ABC) plane). With the value `"none"`, the label is written in the screen plane.
- `fill=""` and `fillopacity=0.5`: when the `fill` option specifies a color name, the angular (spherical) sector is filled with that color and the opacity specified by the `fillopacity` option.

Add a spherical facet: g:DSfacet

The method **g:DSfacet(F, options)**, where $\langle F \rangle$ is a list of points on the sphere, allows you to draw the facet represented by $\langle F \rangle$, the edges being great circle arcs. The $\langle options \rangle$ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `fill=""`, string representing the fill color (none by default),
- `fillopacity=0.3`, opacity of the fill color.
- `front=nil`, `back=nil`: if a table-type variable is assigned to these options, the function adds to these tables the path corresponding to the outline of the visible part for the first option (front), and the path corresponding to the outline of the hidden part for the second option (back).
- options for adding a label: see page 165.

Add a spherical curve: g:DScurve

The method **g:DScurve(L, options)**, where $\langle L \rangle$ is a list of points on the sphere, allows you to draw the curve represented by $\langle L \rangle$. The $\langle options \rangle$ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `out=nil`. If we assign a table-type variable to this `out` option, then the function adds the points corresponding to the ends of the hidden parts to this list.
- options for adding a label: see page 165.

Adding a spherical region: g:DSregion

The method **g:DSregion(L, options)**, where ⟨*L*⟩ is a list of 3D points representing a simple closed spherical curve. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `fill=""`: string representing the fill color (none by default),
- `fillopacity=0.3`: opacity of the fill color.
- `dir={1,1}`: allows changing the direction of the circular arcs that "replace" the hidden segments (for the first value) or the visible segments (for the second value). A value of 1 indicates counter-clockwise (direct) direction, and −1 indicates clockwise (indirect) direction.
- `out=nil`: if a table-type variable is assigned to this `out` option, the function adds the points corresponding to the endpoints of the hidden segments to that list.
- `front=nil`, `back=nil`: if a table-type variable is assigned to these options, the function adds to these tables the path corresponding to the outline of the visible part for the first option (front), and the path corresponding to the outline of the hidden part for the second option (back).
- options for adding a label: see page 165.

We will now deal with objects that are not necessarily on the sphere, but that may pass through it, or be inside it, or outside it.

Add a segment: g:DSseg

The **g:DSseg(AB, options)** method allows you to add a segment. The argument ⟨*AB*⟩ is a table of the form $\{A, B\}$ where A and B are two points in space. The function handles interactions with the sphere. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `arrows=0`, three possible values: 0 (no arrow), 1 (one arrow in B), 2 (arrow in A and B).
- options for adding a label: see page 165.

Add a line: g:DSline

The **g:DSline(d, options)** method allows you to add a line. The argument ⟨*d*⟩ is a table of the form $\{A, u\}$ where A is a point on the line and u is a direction vector (two 3D points). The function handles interactions with the sphere. The drawn segment is obtained by intersecting the line with the 3D window; it may be empty if the window is too narrow. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,

- color=<current line color>,
- width=<current line thickness in tenths of a point>,
- opacity=<current line opacity>,
- hidden=ld.Hiddenlines,
- arrows=0, three possible values: 0 (no arrow), 1 (one arrow in B), 2 (arrow in A and B).
- scale=1, allows you to change the size of the plotted segment.
- options for adding a label: see page 165.

Add a polygonal line: g:DSpolyline

The **g:DSpolyline(L, options)** method allows you to add a polygonal line. The argument $\langle L \rangle$ is a list of points in space, or a list of lists of points in space. The function handles interactions with the sphere. The $\langle options \rangle$ argument is a table whose fields define the options, which are (with their default value):

- style=<current line style>,
- color=<current line color>,
- width=<current line thickness in tenths of a point>,
- opacity=<current line opacity>,
- hidden=ld.Hiddenlines,
- arrows=0, three possible values: 0 (no arrow), 1 (one arrow in B), 2 (arrow in A and B).
- close=false, indicates whether the line should be closed.
- options for adding a label: see page 165.

Adding axes: g:DSaxes

The **g:DSaxes(O, options)** method allows you to add the axes: $(\langle O \rangle, vecI)$, $(\langle O \rangle, vecJ)$, and $(\langle O \rangle, vecK)$; the $\langle O \rangle$ argument is a 3D point representing the intersection of the three axes. The function handles interactions with the sphere. The $\langle options \rangle$ argument is a table whose fields define the options, which are (with their default value)

- style=<current line style>,
- color=<current line color>,
- width=<current line thickness in tenths of a point>,
- opacity=<current line opacity>,
- hidden=ld.Hiddenlines,
- arrows=0, three possible values: 0 (no arrow), 1 (arrow at the end), 2 (arrows at both start and end).
- xyzlimits=nil: By default, the interval considered for each of the three axes is the one defined by the 3D window when the graph is created; however, this can be changed (without altering the window)—for example, with xyzlimits={0,5}, all three intervals will be set to $[0;5]$. These three intervals can also be modified individually using the xlimits, ylimits, and zlimits options. The axes do not have tick marks.
- legend=true: Automatically adds a label at the end of each axis; these labels are controlled by the labels={"x","y","z"} option.
- When legend=true, the labels are positioned at the far end of the segment plus an offset equal to 1/40 of the segment's length. This distance can be adjusted either globally using the xyzdist=0 option (defaulting to zero length) or individually via the xdist, ydist, and zdist options; by default, these three options inherit the value of xyzdist.

Add a plane: g:DSplane

The **g:DSplane(P, options)** method allows you to add the contour of a plane. The argument ⟨*P*⟩ is a table of the form {*A*, *n*}, where *A* is a point on the plane and *n* is a normal vector. The function draws a parallelogram representing the plane ⟨*P*⟩, processing the interactions with the sphere. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `scale=1`, allows you to change the size of the parallelogram,
- `angle=0`, angle in degrees, allows you to rotate the parallelogram around the perpendicular line passing through the center of the sphere.
- `trace=true`, allows you to draw, or not, the intersection of the plane with the sphere when it is not empty.
- options for adding a label: see page 165.

Add a label: g:DSlabel

The **g:DSlabel(text1, anchor1, options1, text2, anchor2, options2, ...)** method allows you to add one or more labels using the same principle as the **g:Dlabel3d()** method, except that here the function handles cases where the anchor point is inside the sphere, behind the sphere, or in front of the sphere. When it is inside, the label color is given by the sphere option `insidelabelcolor`, which defaults to `"darkgray"`.

Add points: g:DSdots and g:DSstars

The **g:DSdots(dots, options)** method allows you to add points to the scene. The ⟨*dots*⟩ argument is a list of 3D points. The function draws points by managing interactions with the sphere. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `hidden=ld.Hiddenlines`,
- `mark_options=""`, a string that will be passed directly to the *\draw* instruction.

If a point is inside the sphere, or on the hidden face, the point's color is given by sphere option `insidelabelcolor`, which defaults to `"darkgray"`.

The **g:DSstars(dots, options)** method allows you to add points **onto** the sphere. The ⟨*dots*⟩ argument is a list of 3D points that will be projected onto the sphere. The function draws these points as an asterisk. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `style=<current line style>`,
- `color=<current line color>`,
- `width=<current line thickness in tenths of a point>`,
- `opacity=<current line opacity>`,
- `hidden=ld.Hiddenlines`,
- `scale=1`, allows you to change the size,
- `circled=false`, allows you to add a circle around the star,

- `fill=""`, string representing a color. When not empty, the asterisk is replaced by a circled hexagonal facet and filled with the color specified by this option.

The points on the hidden face of the sphere have the color given by the sphere option `insidelabelcolor`, which defaults to `"darkgray"`.

Inverse Stereography: g:DSinvstereo_curve and g:DSinvstereo_polyline

The method **g:DSinvstereo_curve(L, options)**, where ⟨*L*⟩ is a 3D polygonal line representing a curve drawn on a plane with equation *z* =cte, draws the image of ⟨*L*⟩ on the sphere by inverse stereography, the pole being the point *C+r*vecK*, where *C* is the center of the sphere and *r* is the radius.

The method **g:DSinvstereo_polyline(L, options)**, where ⟨*L*⟩ is a 3D polygonal line drawn on a plane with equation *z* =cte, draws the image of ⟨*L*⟩ on the sphere by inverse stereography, the pole being the point *C+r*vecK*, where *C* is the center of the sphere and *r* is the radius.

In both cases, the ⟨*options*⟩ are the same as for the **g:DScurve()** method.

Adding paths

The module displays the scene elements in four stages:

- Stage 1: display of elements located behind the sphere.
- Stage 2: display of elements located inside the sphere.
- Stage 3: display of elements located in front of the sphere.
- Stage 4: display of ”hidden” elements.

The following methods allow for the addition of other elements (paths), though these elements are not ”verified” by the module.

- The method **g:DSaddback(L [, draw_options, hidden, hidden_options])** displays the path, or polyline, ⟨*L*⟩ after stage 1 and before stage 2; this could be, for example, a path drawn on the hidden side of the sphere. ⟨*draw_options*⟩ is a string passed to the *\draw* instruction. ⟨*hidden*⟩ is a boolean (`false` by default) indicating whether the hidden part should be drawn. ⟨*hidden_options*⟩ is a string passed to the *\draw* instruction.
- The method **g:DSaddinside(L [, draw_options, hidden, hidden_options])** displays the path, or polyline, ⟨*L*⟩ concurrently with stage 2 (inside the sphere). The options are the same as for the previous method.
- The method **g:DSaddfront(L [, draw_options])** displays the path, or polyline, ⟨*L*⟩ after stage 2 and before stage 3; this could be, for example, a path drawn on the visible side of the sphere. ⟨*draw_options*⟩ is a string that will be passed to the *\draw* instruction.

Additional methods: g:Dhalf_sphere, g:Dquater_sphere, g:Deighth_sphere

These three methods are independent of the sphere created by the **g:Define_sphere()** method, though they do utilize certain functions from this module.

- **g:Dhalf_sphere(O, r, dir, options)**: draws a hemisphere, where ⟨*O*⟩ is the center, ⟨*r*⟩ is the radius, and ⟨*dir*⟩ is a non-zero 3D vector. The hemisphere is obtained by cutting the sphere with the plane defined by {*O*, dir} and retaining the portion within the half-space that contains the vector ⟨*dir*⟩. It consists of one spherical face and one circular planar face.
- **g:Dquater_sphere(O, r, dir1, dir2, options)**: draws a quarter-sphere, where ⟨*O*⟩ is the center, ⟨*r*⟩ is the radius, and ⟨*dir1*⟩ and ⟨*dir2*⟩ are two non-zero, orthogonal 3D vectors. The quarter-sphere is obtained by cutting the sphere with the planes defined by {*O*, dir1} and {*O*, dir2} and retaining the portion within the quarter-space that contains the vectors ⟨*dir1*⟩ and ⟨*dir2*⟩. It consists of one spherical face and two semi-circular planar faces.

- **g:Deighth_sphere(O, r, dir1, dir2, dir3, options)**: draws an eighth of a sphere, where ⟨*O*⟩ is the center, ⟨*r*⟩ is the radius, and ⟨*dir1*⟩, ⟨*dir2*⟩, and ⟨*dir3*⟩ are three non-zero 3D vectors that are mutually orthogonal. The eighth of a sphere is obtained by cutting the sphere with the planes defined by {*O*, dir1}, {*O*, dir2}, and {*O*, dir3}, and retaining the portion located within the octant that contains the vectors ⟨*dir1*⟩, ⟨*dir2*⟩, and ⟨*dir3*⟩. It consists of one spherical face and three planar, quarter-circle faces.

For these three methods, the ⟨*options*⟩ argument is a table whose fields define the available options (along with their default values):

- `color="white"`: fill color for the faces.
- `mode=ld.mShaded`: representation mode; by default, the faces are shaded (filled) and drawn with visible edges. Other possible modes include:
  - `ld.mShadedHidden`: shaded faces, with both visible and hidden edges drawn.
  - `ld.mShadedOnly`: shaded faces, with no edges drawn.
  - `ld.mWireframe`: edges only; the faces are not shaded.
- `opacity=default opacity`: opacity for the face fill.
- `edgecolor=default color`: color of the edges.
- `edgewidth=default thickness`: thickness of the edges (in tenths of a point).
- `edgestyle=default style`: line style for visible edges (character string).
- `hiddenstyle=ld.Hiddenlinestyle`: line style for hidden edges, when they are drawn.
- `outline=false`: if set to `true`, the outline of the visible spherical portion is drawn using the attributes of the visible edges.

Examples

```
\begin{luadraw}{name=cube_in_sphere}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

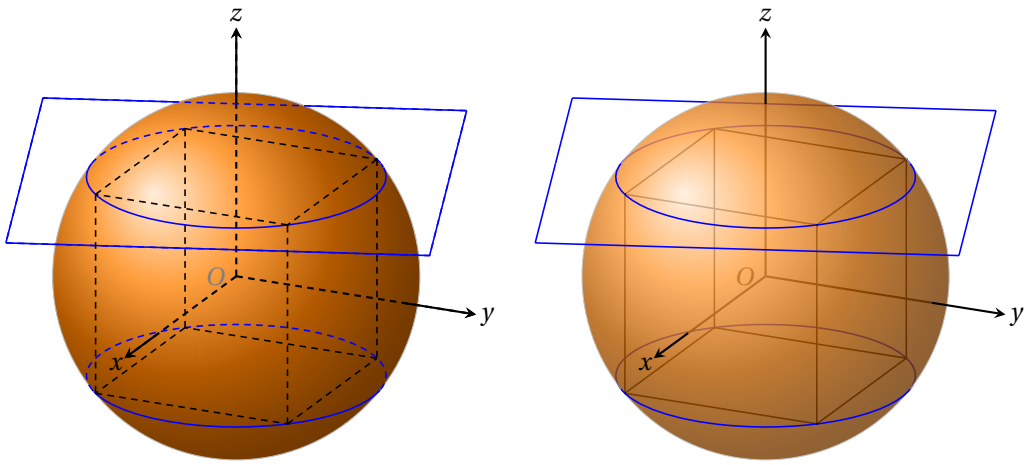
local g = ld.graph3d:new{window={-9,9,-4,5},viewdir={25,70},size={16,8}}
require 'luadraw_spherical'
g:Linewidth(6); ld.Hiddenlinestyle = "dashed"
local a = 4
local O = Origin
local cube = ld.parallelep(O,a*vecI,a*vecJ,a*vecK)
local G = pt3d.isobar3d(cube.vertices)
cube = ld.shift3d(cube,-G) -- to center the cube at the origin
local R = pt3d.abs(cube.vertices[1])

local dessin = function()
    g:DSaxes(Origin, {width=8, arrows=1, xyzlimits={0,5}, xdist=0.1})
    g:DSplane({a/2*vecK,vecK},{color="blue",scale=0.9,angle=20});
    g:DScircle({-a/2*vecK,vecK},{color="blue"})
    g:DSpolyline( ld.facetedges(cube) ); g:DSlabel("$O$",O,{pos="W"})
    g:Dspherical()
end

g:Saveattr(); g:Viewport(-9,0,-4,5); g:Coordsystem(-5,5,-5,5)
ld.Hiddenlines = true; g:Define_sphere({radius=R, arrowBstyle = "-stealth"})
dessin()
g:Dlabel("Hiddenlines=true",0.5-4.5*cpx.I,{})
g:Restoreattr()
```

```
g:Saveattr(); g:Viewport(0,9,-4,5); g:Coordsystem(-5,5,-5,5)
ld.Hiddenlines = false; g:Define_sphere({radius=R,opacity=0.8, arrowBstyle = "-stealth"} )
dessin()
g:Dlabel("Hiddenlines=false, opacity=0.8",0.5-4.5*cpx.I,{})
g:Restoreattr()
g:Show()
\end{luadraw}
```

Figure 2: Cube in a Sphere



Hiddenlines=true

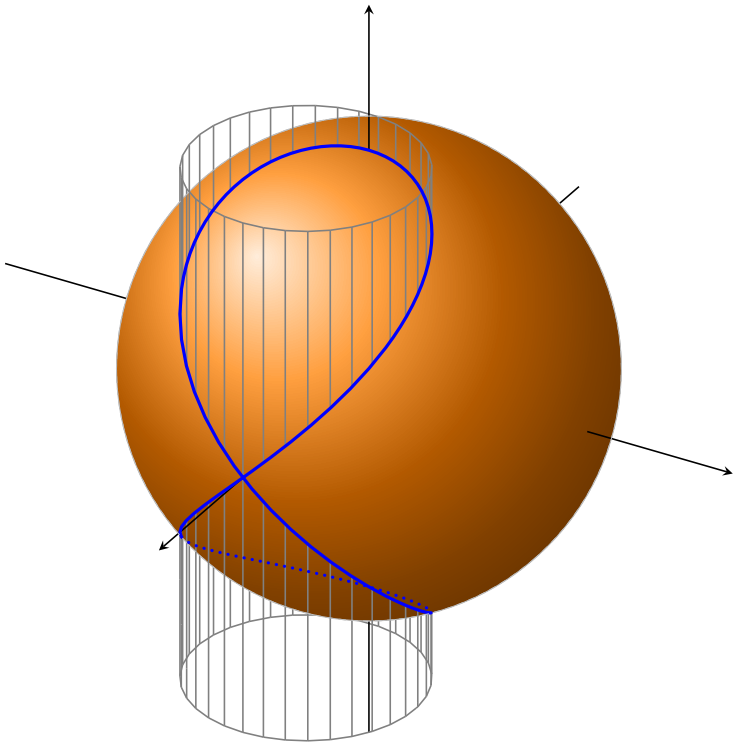
Hiddenlines=false, opacity=0.8

Spherical curve

```
\begin{luadraw}{name=courbe_spherique}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Origin, vecI, vecJ, vecK, M, Ms = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, pt3d.Ms

local g = ld.graph3d:new{window={-4.5,4.5,-4.5,4.5},viewdir={30,60},margin={0,0,0,0},size={10,10}}
require 'luadraw_spherical'
g:Linewidth(6); ld.Hiddenlinestyle = "dotted"
ld.Hiddenlines = false;
local C = ld.cylinder(M(1.5,0,-3.5),1.5,M(1.5,0,3.5),35,true)
local L = ld.parametric3d( function(t) return Ms(3,t-math.pi/2,t) end, -math.pi,math.pi) -- the curve
g:Define_sphere({arrowBstyle = "-stealth"})
g:DSpolyline(ld.facetedges(C),{color="gray"}) -- drawing cylinder
g:DSpolyline({{-5*vecI,5*vecI},{-5*vecJ,5*vecJ},{-5*vecK,5*vecK}},{arrows=1}) --axes
ld.Hiddenlines=true; g:DScurve(L,{width=12,color="blue"}) -- curve with hidden part
g:Dspherical()
g:Show()
\end{luadraw}
```

Figure 3: Viviani window



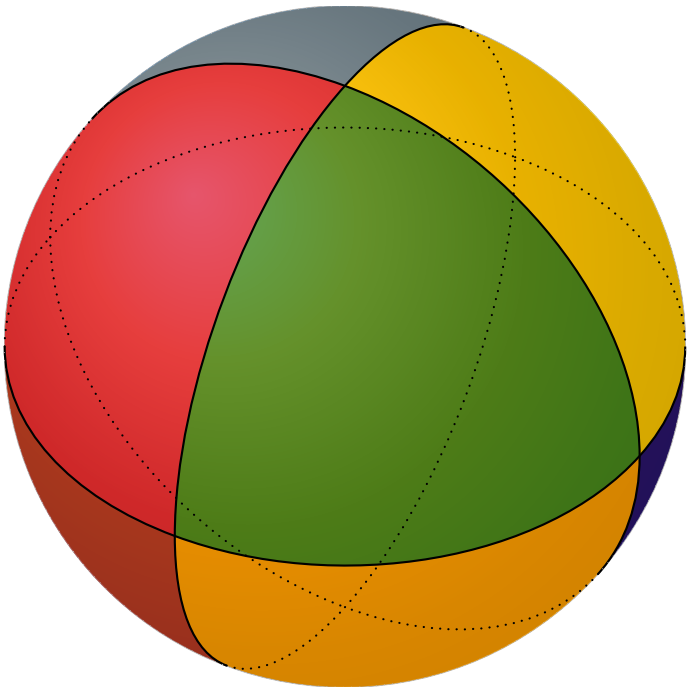
To avoid compromising the readability of the drawing, the hidden parts have not been displayed except for the curve.

A spherical tiling

```
\begin{luadraw}{name=pavage_spherique}
local ld = luadraw
local Origin = ld.pt3d.Origin

local g = ld.graph3d:new{window={-3,3,-3,3},viewdir={30,60},size={10,10}}
require 'luadraw_spherical'
local poly = require "luadraw_polyhedrons"
g:Linewidth(6); ld.Hiddenlines = true; ld.Hiddenlinestyle = "dotted"
local P = ld.poly2facet( poly.octahedron(Origin, ld.sM(30,10)) )
local colors = {"Crimson","ForestGreen","Gold","SteelBlue","SlateGray","Brown","Orange","Navy"}
g:Define_sphere()
for k,F in ipairs(P) do
    g:DSfacet(F,{fill=colors[k],style="noline",fillopacity=0.7})  --facets without edges
end
for _, A in ipairs(ld.facetedges(P)) do
    g:DSarc(A,1,{width=8}) -- each edge is an arc of a great circle
end
g:Dspherical()
g:Show()
\end{luadraw}
```

Figure 4: A spherical tiling



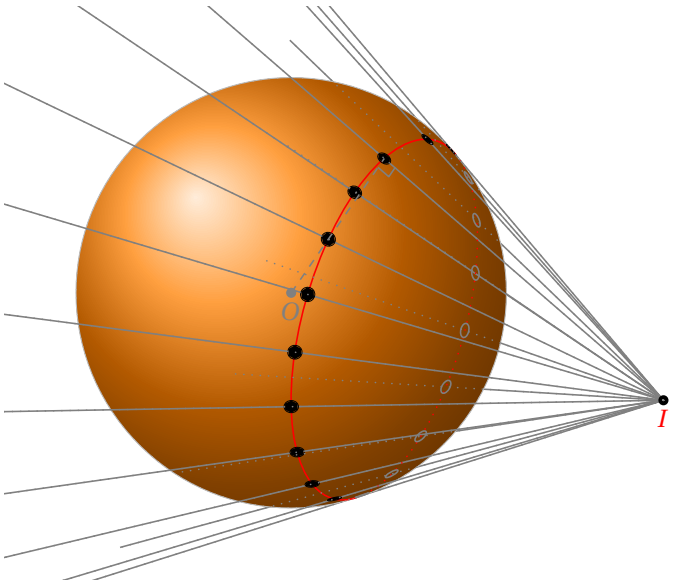
For this spherical tiling, we chose a regular octahedron with a center identical to that of the sphere and with one vertex on the sphere (and therefore all vertices are on the sphere).

Tangents to the sphere from a point

```
\begin{luadraw}{name=tangent_to_sphere}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, M = pt3d.Origin, pt3d.M

local g = ld.graph3d:new{window={-4,5.5,-4,4},viewdir={30,60},size={10,10}}
require 'luadraw_spherical'
ld.Hiddenlines=true; g:Linewidth(6)
local O, I = Origin, M(0,6,0)
local S,S1 = {O, 3}, {(I+O)/2,pt3d.abs(I-O)/2}
-- the circle of tangency is the intersection between spheres S and S1
local C,r,n = ld.interSS(S,S1)
local L = ld.circle3d(C,r,n)[1] -- list of 3D points on the circle
local dots, lines = {}, {}
-- draw
g:Define_sphere({opacity=1})
g:DScircle({C,n},{color="red"})
for k = 1, math.floor(#L/4) do
    local A = L[4*(k-1)+1]
    table.insert(dots,A)
    table.insert(lines,{I, 2*A-I})
end
g:DSpolyline(lines ,{color="gray"})
g:DSstars(dots) -- drawing points on the sphere
g:DSdots({O,I});  -- points in the scene
g:DSlabel("$I$",I,{pos="S",node_options="red"},"$O$",O,{})
g:Dspherical()
g:Dseg3d({O,dots[1]},"gray,dashed"); g:Dangle3d(O,dots[1],I,0.2,"gray")
g:Show()
\end{luadraw}
```

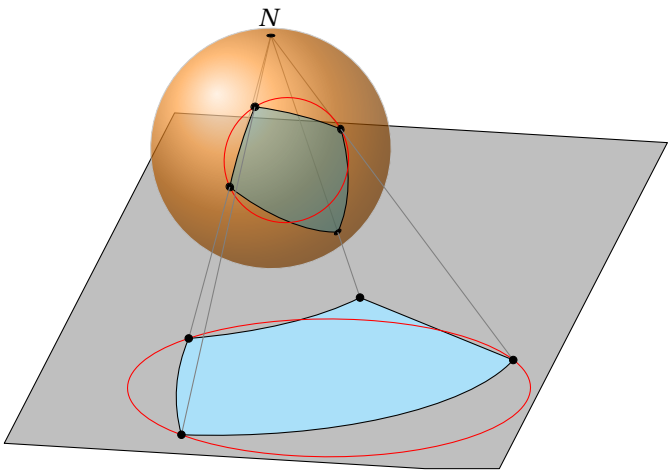
Figure 5: Tangents to the sphere from a point



Stereographic Projection

```
\begin{luadraw}{name=projstereo_Sfacet}
local ld = luadraw
local pt3d = ld.pt3d
local M = pt3d.M
require 'luadraw_spherical'
local sM = ld.sM
local g = ld.graph3d:new{window={-7,10,-8,4}, size={10,10}, viewdir={10,70}}
local O, R = M(0,0,0), 3
g:Define_sphere( {color="orange", opacity=0.7} )
local A, B, C, D, N = sM(-10,90), sM(0, 50), sM(50,65), sM(50,120), sM(0,0)
local F = {A,B,C,D}
local p = ld.projstereo(F, {O,R}, N, -R)
local T1 = ld.projstereo_Sfacet(F, N, -R)
local T2 = ld.projstereo_Scircle(ld.plane(A,B,C), N, -R)
g:Dplane({M(2,2,-R), M(0,0,1)}, M(0,1,0), 12.5,24.5, "fill=lightgray")
g:DSpolyline( {{N,A},{N,B},{N,C},{N,D}}, {color="gray", hidden=false})
g:DSfacet(F, {fill="cyan"})
g:DScircle(ld.plane(A,B,C), {color="red"})
g:DSstars({A,B,C,D,N}, {fill="black"})
g:Dspherical()
g:Dpath3d(T1, "fill=cyan!30")
g:Dpath3d(T2, "red")
for k,A in ipairs(F) do
    g:Dpolyline3d( {A,p[k]}, "gray")
end
g:Ddots3d(p)
g:Dlabel3d("$N$",N,{pos="N"})
g:Show()
\end{luadraw}
```

Figure 6: Stereographic Projection of a Circle and a Spherical Facet

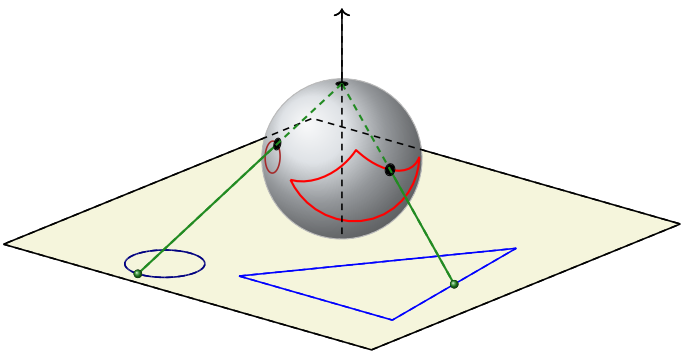


Inverse Stereography

```
\begin{luadraw}{name=stereographic_curve}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, M, vecJ, vecK = pt3d.Origin, pt3d.M, pt3d.vecJ, pt3d.vecK

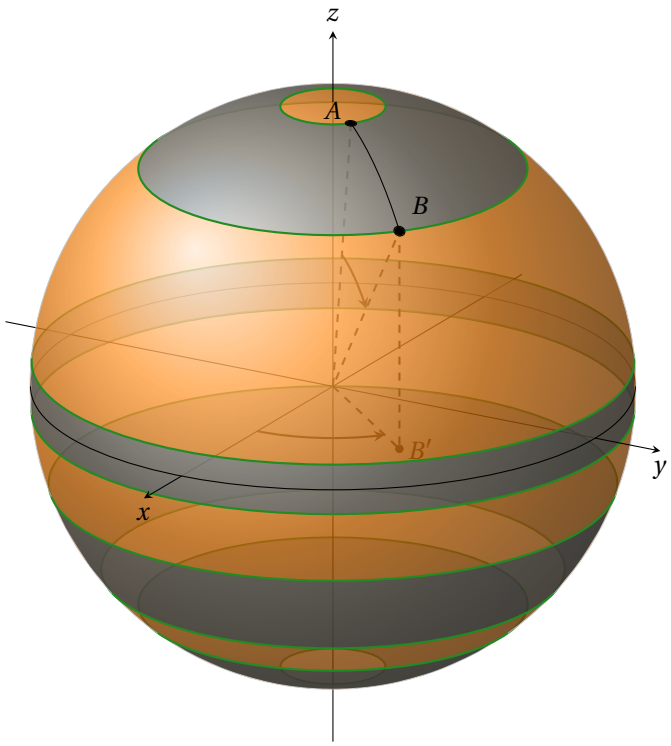
local g = ld.graph3d:new{window3d={-5,5,-2,2,-2,2},window={-4.25,4.25,-2.5,2},size={10,10}, viewdir={40,70}}
ld.Hiddenlines = true; ld.Hiddenlinestyle="dashed"; g:Linewidth(6)
require 'luadraw_spherical'
local C, R = Origin, 1
local a = -R
local P = ld.planeEq(0,0,1,-a)
local L = {M(2,0,a), M(2,2.5,a), M(-1,2,a)}
local L2 = ld.circle3d(M(2.25,-1,a),0.5,vecK)[1]
local A, B = (L[2]+L[3])/2, L2[20]
local a,b = table.unpack( ld.inv_projstereo({A,B},{C,R},C+R*vecK) )
g:Dplane(P,vecJ,6,6,15,"draw=none,fill=Beige")
g:Define_sphere( {center=C,radius=R, color="SlateGray!30", show=true} )
g:DSpolyline(L,{color="blue",close=true}); g:DSinvstereo_polyline(L,{color="red",width=8,close=true})
g:DSpolyline(L2,{color="Navy"}); g:DSinvstereo_curve(L2,{color="Brown",width=6})
g:DSplane(P,{scale=1.5})
g:DSpolyline({{C+R*vecK,A},{C+R*vecK,B}}, {color="ForestGreen",width=8})
g:DSpolyline({{-vecK,2*vecK}}, {arrows=1})
g:DSstars({C+R*vecK,a,b}, {scale=0.75})
g:Dspherical()
g:Dballdots3d({A,B},"ForestGreen",0.75)
g:Show()
\end{luadraw}
```

Figure 7: *DSinvstereo_curve* and *DSinvstereo_polyline* methods



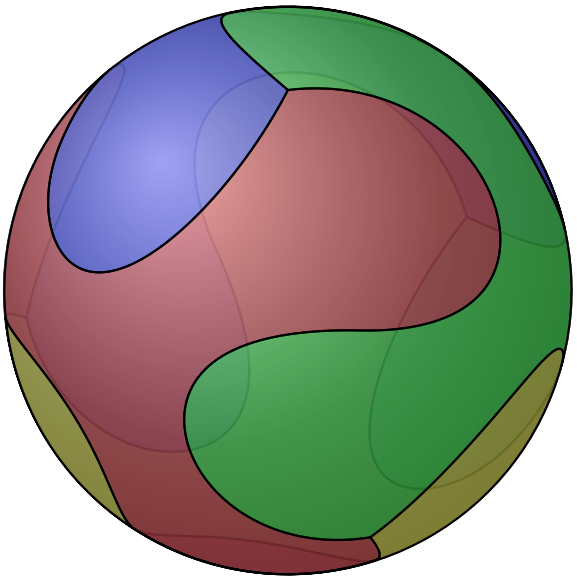
```
\begin{luadraw}{name=spherical_strip}
local ld = luadraw
local M, Ms = ld.pt3d.M, ld.pt3d.Ms
local g = ld.graph3d:new{ viewdir={30,70} }
require 'luadraw_spherical'
local R = 4
g:Define_sphere({radius=R, arrowBstyle="-stealth", opacity=0.8})
local sphere_strip = function(p1,p2,n)
    n = n or math.ceil((p2-p1)/5)
    p1 = p1*ld.deg; p2 = p2*ld.deg
    return ld.surface( function(u,v) return Ms(R,u,v) end, 2*math.pi, 0, p1, p2,{50,n})
end
local color1, color2 = "SteelBlue", "orange"
local strips ={ {10,40}, {85,95}, {110, 130}, {140,170} } -- angles
local O, I, J, K, A, B = M(0,0,0), M(1,0,0), M(0,1,0), M(0,0,1), ld.sM(50,10), ld.sM(50,40)
local B1 = ld.pxy(B)
local S = {}
for _, p in ipairs(strips) do
    ld.insert(S, sphere_strip(p[1],p[2]))
end
local Sv, Sh = g:Classifyfacet(S)
local back = ld.polyline2path3d( ld.border(Sh) )
local front = ld.polyline2path3d( ld.border(Sv) )
--drawing
g:DSaddback(back, "draw=none,even odd rule,left color="..color1.."!50,right color="..color1.."!80, fill opacity=0.6")
g:DSaddfront(front, "draw=none,even odd rule,ball color="..color1..", fill opacity=0.6")
for _, p in ipairs(strips) do
    g:DScircle({ld.sM(0,p[1]), M(0,0,1)}, {color="ForestGreen", width=8})
    g:DScircle({ld.sM(0,p[2]), M(0,0,1)}, {color="ForestGreen", width=8})
end
g:DSaddinside({A,O,B,B1,O}, "line width=0.8,dashed") -- polyline
g:DSaddinside({I,O,B1,2,1,"ca"}, "line width=0.8,-stealth") -- path
g:DSaddinside({A,O,B,2,1,"ca"}, "line width=0.8,-stealth")
g:DSstars({A,B}, {fill="black", scale=0.75}); g:DSdots(B1)
g:DSlabel("$B'$",B1,{pos="E"}, "$A$",1.05*A,{pos="W"}, "$B$",1.05*B,{pos="NE"})
g:DSarc({A,B},1)
g:DSpolyline( {{M(-5,0,0), M(5,0,0)}, {M(0,-5,0), M(0,5,0)}, {M(0,0,-5), M(0,0,5)}}, {arrows=1}) -- axes
g:DScircle( {O,K})
g:Dspherical()
g:Dlabel3d("$x$",5*I,{pos="S"}, "$y$",5*J,{}, "$z$",5*K,{pos="N"})
g:Show()
\end{luadraw}
```

Figure 8: Adding paths




```
\begin{luadraw}{name=DSregion}
local ld = luadraw
local pt3d = ld.pt3d
local Ms, O = pt3d.Ms, pt3d.Origin
local g = ld.graph3d:new{window={-4,4,-4,4},margin={0,0,0,0},viewdir={30,65},size={10,10}}
ld.Hiddenlines = false
ld.Hiddenlinestyle = "dashed"
g:Linewidth(8)
require 'luadraw_spherical'
local poly = require 'luadraw_polyhedrons'
local R = 3
local T = poly.tetrahedron(O, Ms(R,30*ld.deg,20*ld.deg))
local A, B, C, D = table.unpack(T.vertices)
local function edge(A,B,cut) -- cut = true/false, to remove the first point
    cut = cut or false
    local m = (A+B)/2
    local u = pt3d.normalize(B-A)
    local v = ld.rotate3d(u,90,{O,m})
    local L = ld.path3d({A,"m",A+2*v+u/2, m+2*v+u/2,m,"b",m-2*v-u/2,B-2*v-u/2,B,"b"},100)[1]
    if cut then table.remove(L,1) end
    return ld.map(ld.toSphere, L)
end
local ABD = ld.concat(edge(A,B), edge(B,D,true), edge(D,A,true) )
local ABC = ld.concat(edge(A,B), edge(B,C,true), edge(C,A,true) )
local ADC = ld.concat(edge(A,D), edge(D,C,true), edge(C,A,true) )
local BCD = ld.concat(edge(B,C), edge(C,D,true), edge(D,B,true) )
g:Define_sphere({radius=R, color="SlateGray", opacity=0.9})
g:DSregion(ABD,{fill="red"})
g:DSregion(ABC,{fill="green"})
g:DSregion(ADC,{fill="blue"})
g:DSregion(BCD,{fill="yellow"})
g:Dspherical()
g:Show()
\end{luadraw}
```

Figure 9: Paint a spherical region



```
\begin{luadraw}{name=Split_facet}
local ld = luadraw
local pt3d = ld.pt3d
local M, Origin, vecI, vecJ, vecK = pt3d.M, pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK
local g = ld.graph3d:new{ size={10,10}, margin=0 }
require 'luadraw_spherical'
local r1, dr = 4.5, 0.75
local r2, r3, r4 = r1-dr, r1-2*dr, r1-3*dr
-- teal
g:Dhalf_sphere(Origin, r1, -vecJ, {color="teal", mode=ld.mShadedOnly})
-- yellow
g:Dpath3d({r2*vecI,Origin,-r2*vecI,r2,-1,vecJ,"ca"}, "draw=none,fill=yellow!80!black")
g:Dquater_sphere(Origin, r2, vecJ, -vecK, {color="yellow", mode=ld.mShadedOnly})
-- gray
g:Dpath3d({r3*vecI,Origin,-r3*vecI,r3,-1,vecJ,"ca"}, "draw=none,fill=gray")
g:Dpath3d({r3*vecI,Origin,-r3*vecI,r3,1,vecK,"ca"},"draw=none,fill=gray")
g:Deighth_sphere(Origin, r3, -vecI, vecJ, vecK, {color="gray", mode=ld.mShadedOnly})
-- orange
g:Deighth_sphere(Origin, r4, vecI, vecJ, vecK, {color="orange", mode=ld.mShadedOnly})
g:Show()
\end{luadraw}
```

Figure 10: Example with **g:Dhalf_sphere** and Co



3) The *luadraw_palettes* Module

The **luadraw_palettes** Module[1] defines 261 color palettes, each with a name. A palette is a list (table) of colors, which are themselves lists of three numerical values between 0 and 1 (red, green, and blue components). The list of these palettes, as well as their rendering, can be viewed in this document.

This module does not add any new graphics methods, but it returns a table of color definitions, plus the function **getPal(palname, options)**. Therefore, it is used as follows (example):

```
local pal = require 'luadraw_palettes'
local color = pal.Blackbody
local BlackbodyTransformed = pal.getPal( -- returns a new palette
    color, -- a palette name
    {
    extract = {2, 5, 8, 9}, -- color numbers to extract
    shift = 1, -- offset among the extracted colors, which gives here: 5,8,9,2
    reverse = true -- reversing the order, which gives here: 2,9,8,5
    }
)
```

4) The *luadraw_compile_tex* module

Warning:

- This module adds new graphics methods to the *ld.graph* and *ld.graph3d* classes, as well as several functions in the *luadraw* namespace, but it does not return anything.
- This module has been tested under Linux, but not (yet) under Windows or macOS.
- This module requires that the following programs be installed on your system: *pdflatex*, *pdf2ps*, and *pstoedit*.

The **ld.compile_tex_default(options)** function allows you to modify the default settings. These parameters are (with their default values):

- `pdflatexcmd="pdflatex"`,
- `pstoeditcmd="pstoedit"`,

[1] This module is a contribution of Christophe BAL.

- `pdf2pscmd="pdf2ps"`,
- `preamble="\\documentclass[12pt]{article}\n"`,
- `usepackage="\\usepackage{amsmath,amssymb}\n\\usepackage{fourier}\n"`.

Depending on your operating system, you may need to modify these variables to add the program path, for example :

```
ld.compile_tex_default( {pstoeditcmd="/usr/bin/pstoedit"} )
```

This module allows you to:

1. compile a text fragment in TeX,
2. convert the resulting file into an *eps* file containing *flattened postscript*,
3. read the content of the *eps* file and return its content as a list of paths, with the line thickness at the beginning of each path, and the fill instruction at the end.
4. The list thus obtained can be:
   (a) drawn on the screen,
   (b) converted into 3D paths in a given plane and drawn,
   (c) converted into 3D polygonal lines in a given plane (the thickness and fill command are then lost) and drawn.

Part One: Compilation and Reading

Warning : This step requires compiling the document with the *-shell-escape* or *-enable-write18* option. Without this option, the fragment will not be compiled, which is not a problem if the <*filename*>*.eps* file already exists and you did not intend to modify it.

The first step is handled by the **ld.compile_tex(text [, filename, conv_stroke])** function. The ⟨*text*⟩ argument is a string; this is the fragment to be compiled. The optional ⟨*filename*⟩ argument is also a string; this is the name of the file that will be created. This name must not contain **a path or an extension**. By default, this name is *"tex2FlatPs"*. It is created in the current directory (but will then be deleted). The optional argument ⟨*conv_stroke*⟩ is a boolean that indicates whether polygonal lines (*stroke* instructions) should be converted into filled strips with a *fill*, or not (`false` by default).

The process unfolds in several steps:

1. Creation of the TeX file. This uses the parameters `preamble` and `usepackage`. Compilation is done with *pdflatex*.
2. The resulting file is converted to PostScript using the *pdf2ps* utility.
3. The resulting PS file is then converted using the *pstoedit* utility into an EPS file in flattened PostScript (all content is in the form of paths).
4. The resulting file <*filename*>*.eps* is copied to the working directory of *luadraw* (the name of this directory is in the global variable `ld.cachedir`), and all compilation remnants are erased.
5. The contents of the file thus created are automatically read by the function **ld.read_compiled_tex(filename)**, which returns a list of paths. Each path is a list starting with the line thickness, followed by dots and instructions like a regular path, and ending with the fill command (*"fill"*, *"eofill"*, or *"stroke"*).

Part Two: Using the Result

In 2D The result can be drawn using the method **g:Dcompiled_tex(L, anchor, options)** where ⟨*L*⟩ is the result returned by the function **compile_tex()**. The ⟨*anchor*⟩ argument is a complex number; it represents the center of the bounding box of the drawing contained in ⟨*L*⟩. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `pos="center"`, indicates the position of the content of ⟨*L*⟩ relative to the anchor point, the possible values are the same as for the `pos` option of labels, that is: `"center"`, `"N"`, `"NE"`, `"E"`, `"SE"`, `"S"`, `"SW"`, `"W"`, `"NW"`.

- `scale=1`, allows you to adjust the size of the drawing, this option can be a number or a table of two numbers: *{scaleX, scaleY}*,
- `color=<current default color>`,
- `dir=nil`, writing direction (`nil` means the usual direction). Generally, it is a table consisting of two vectors `dir={v1, v2}` indicating the writing direction,
- `hollow=false`, enables or disables the filling of shapes. With the value `true`, only the outlines are drawn,
- `drawbox=false`: allows you to draw or not draw the bounding box,
- `draw_options=""`: string containing the options that will be passed directly to the *\draw* command.

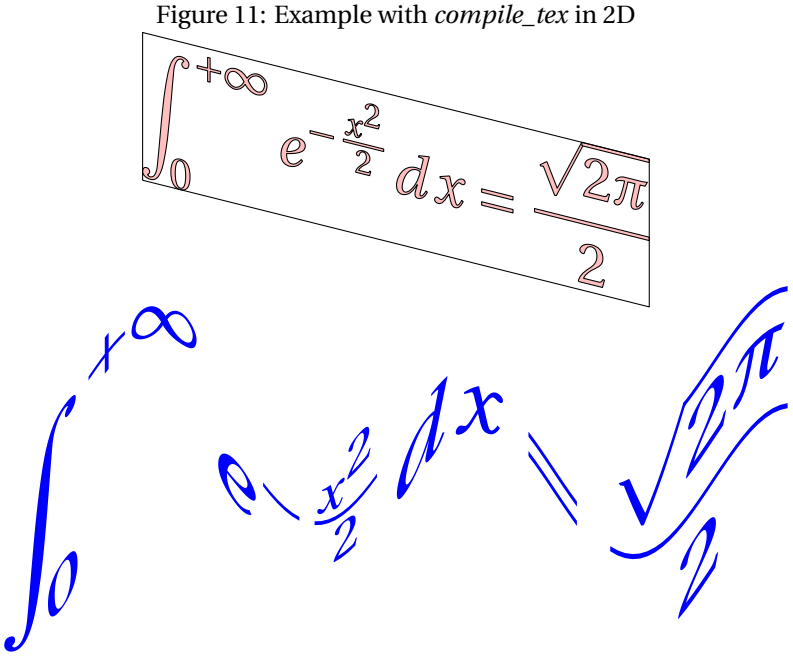
The result can also be transformed into a polygonal line using the function **ld.compiled_tex2polyline(L [, scale])** where ⟨*L*⟩ is the result returned by the **ld.compile_tex()** function. The optional argument ⟨*scale*⟩ allows you to adjust the size; it can be a number or a table of two numbers: *{scaleX, scaleY}*.

```
\begin{luadraw}{name=compile_tex2d}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{bbox=false}
require 'luadraw_compile_tex'
local i = cpx.I
local text = "\\[\\int_0^{+\\infty} e^{-\\frac{x^2}2}dx = \\frac{\\sqrt{2\\pi}}2\\]" -- text to compile
-- compile with -shell-escape the first time to create the file
local L = ld.compile_tex(text,"gauss_integral",true) -- with true the lines (strokes) are changed into strips
g:Shift(2*i) -- a first drawing
g:Dcompiled_tex(L,0,{scale=2,hollow=true, drawbox=true, draw_options="fill=pink", dir={1-i/4,i}}) -- we draw L

g:Shift(-4*i) -- a second drawing
L = ld.compiled_tex2polyline(L,{3,3}) --L is converted to a polygonal line
local f = function(z) return Z(z.re,z.im+math.sin(z.re*1.5)) end  -- this function produces sinusoidal waves
L = ld.ftransform(L,f) -- we apply f to L
g:Dpath( ld.polyline2path(L), 'draw=none,fill=blue') -- we draw L as a path
g:Show()
\end{luadraw}
```

Figure 11: Example with *compile_tex* in 2D



In 3D The result can be converted to 3D using the method **g:Compiled_tex2path3d(L, options)** where ⟨*L*⟩ is the result returned by the function **ld.compile_tex()**. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `scale=1`, allows you to adjust the size of the drawing, this option can be a number or a table of two numbers: *{scaleX, scaleY}*,
- `anchor=pt3d.Origin`, 3D point which represents the center of the bounding box of the drawing, `pos="center"`, indicates the position of the content of ⟨*L*⟩ relative to the anchor point, the possible values are the same as for the `pos` option of labels, that is: `"center"`, `"N"`, `"NE"`, `"E"`, `"SE"`, `"S"`, `"SW"`, `"W"`, `"NW"`,
- `color=<current default color>`,
- `dir={pt3d.vecJ,pt3d.vecK}`, basis of the plane in which the result will be located (this plane will also contain the `anchor` point), these two vectors indicate the direction of writing,
- `polyline=false`, with the value `true` the returned result will be a list of lists of 3D points and can therefore be drawn with the method **g:Dpolyline3d()**, however, the information: line thickness and fill command, are lost. With the value `false` the result is a list of paths, each path is a list starting with the line thickness, followed by 3D points and instructions like an ordinary 3D path, and ending with the fill command (*"fill"*, or *"eofill"* or *"stroke"*).

With the option `polyline=false` (default value), the output can be drawn using the method **g:Dcompiled_tex3d(L, options)** where ⟨*L*⟩ is the result of the method **g:Compiled_tex2path3d()**. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `color=<default current color>`,
- `hollow=false`, enables or disables the filling of shapes. With the value `true`, only the outlines are drawn.
- `drawbox=false`: allows you to draw or not draw the bounding box,
- `draw_options=""`: string containing the options that will be passed directly to the *\draw* command.

```
\begin{luadraw}{name=compile_tex3d}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Origin, vecI, vecJ, vecK, M, Mc = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, pt3d.Mc

local g = ld.graph3d:new{ window={-3,3,-4,4}, margin={0,0,0,0}, size={10,10}, viewdir={-50,60}}
require 'luadraw_compile_tex'

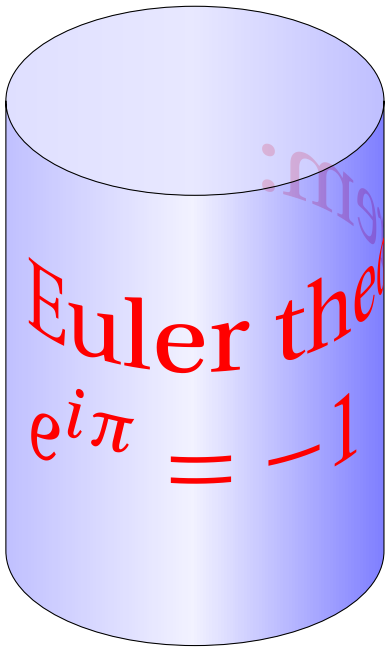
function curve_on_cylinder(curve,cylinder,screenNormal)
-- curve is a 3D polyline on a cylinder
-- cylinder = {A,r,B} estle cylindre
-- this function separate the visible part from the hidden part of the curve
    local A,r,B = table.unpack(cylinder)
    local U = B-A
    local visibility_function = function(N)
        local I = ld.dproj3d(N,{A,U})
        return (pt3d.dot(N-I,screenNormal) >= 0)
    end
    return ld.split_points_by_visibility(curve,visibility_function)
end

local A, r, B = -3*vecK, 2, 2.5*vecK -- the cylinder
local text = "Euler theorem: \\par \\(e^{i\\pi}=-1\\)"
local L = ld.compile_tex(text, "essai") -- compile with shell-escape the first time to create "essai.eps" file
local C = g:Compiled_tex2path3d(L,{scale=3, anchor=M(r,0,0), dir={vecJ,vecK}, polyline=true})
--C is the text converted into 3D polylines, in the plane passing through anchor and basic direction dir, with a scale
 ↪ of 3.

local f = function(A) return Mc(r,A.y/r,A.z) end -- returns the image of a point A on the cylinder by winding
C = ld.ftransform3d(C,f) -- plane curve -> cylindrical curve transformation
```

```
local Cv, Ch = curve_on_cylinder(C, {A,r,B}, g.Normal) -- visible part and hidden part of C, this may take some time
Ch = ld.polyline2path3d(Ch) -- hidden part, conversion to path
g:Dpath3d(Ch, "draw=none,fill=red!30")
g:Dcylinder(A,r,B,{color="blue",opacity=0.5})
Cv = ld.polyline2path3d(Cv) -- visible part, conversion to path
g:Dpath3d(Cv, "draw=none,fill=red")
g:Show()
\end{luadraw}
```

Figure 12: Write on a cylinder



5) The *luadraw_cvx_polyhedra_nets* module

This module adds a graphics method to the class *ld.graph3d*, as well as several functions in the namespace *luadraw*, but it does not return anything.

Basic Function

The module *luadraw_cvx_polyhedra_nets* allows you to « unfold » a **convex** polyhedron to obtain a net. The function that performs the unfolding is:

ld.unfold_polyhedron(P, options)

The argument ⟨*P*⟩ must be a convex polyhedron. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `opening=1`, a value between 0 and 1 representing the "opening rate". With the value 1, the polyhedron is fully unfolded; the facets returned by the function will therefore all be in the same plane. With the value 0, the function returns the polyhedron's faces without modification.
- `root=1`, the number of the polyhedron face that will serve as the root, because the function represents the polyhedron as a tree by determining, for each face, its neighbors (adjacent facets), as well as any shared edges and angles. This option allows you to choose the face that will serve as the starting point.
- `model=nil`, a list of facet number lists to impose a pattern model, for example `model={{1,6},{1,3},{1,4},{1,5,2}}`, the sublist *{1,5,2}* means that facet 1 is the ancestor of facet 5, and that facet 5 is the ancestor of facet 2, that is to say that facets 5 and 1 are adjacent, and facet 5 will rotate around its common edge with facet 1 (same for 5 and 2). For the model to be consistent, all facets of the polyhedron EXCEPT one (which will be the facet `root`), must have one and

only one ancestor; If facets 1 and 5 are not adjacent in the polyhedron, the function stops and displays an error in the terminal. When the `model` option is set to `nil` (the default value), the algorithm calculates a consistent model itself.

- `to2d=false`, is a boolean value that returns a 2D version of the pattern in the screen plane coordinate system. With the value `true`, the `opening` option automatically takes the value 1, and the facets returned by the function will have vertices expressed as complex numbers.
- `tabs=false`, is a boolean value that allows adding or excluding tabs from the pattern in the 2D version. With the value `true`, the `to2d` option automatically takes the value `true` as well. The facets returned by the function will have vertices expressed as complex numbers in the screen coordinate system, and the function also returns a 2D polygonal line representing tabs for certain edges (these are determined automatically).
- `tabs_wd=0.2`, a numeric value representing the thickness of the tabs when the `tabs` option is set to `true`.
- `tabs_lg=0.5`, a numerical value between 0 and 1, determines the length of the shorter side of the tabs. This length is equal to the length of the edge (which is the longer side) multiplied by `tabs_lg` (when the `tabs` option is set to `true`).
- `rotate=0`, when the `to2d` option is set to `true`, rotates the drawing by an angle equal to `rotate` (in degrees) around its center. In the 3D version, the drawing is rotated by an angle equal to `rotate` (in degrees) around the axis passing through the centroid of the facet *root* and oriented by a normal vector to this facet pointing outwards from the polyhedron.

The function returns a table containing the following fields:

- The field *facets*: which contains the list of facets, with vertices in 2D (complex numbers) if the `to2d` option is `true`, or vertices in 3D (3D points) otherwise.
- The field *tree*: which is a list of the form:

  $$\{\{ancestor, n1, n2, angle, vertices\}, \ldots\}$$

  Each element of this list represents a facet, with the following information for each facet:
  - *ancestor*: the number of the ancestor facet, its position in the list *tree* (the facet that served as the root has the number 0 as its ancestor, which does not correspond to any facet).
  - *n1, n2*: the number of the vertices of the ancestor facet representing the common edge.
  - *angle*: the angle in degrees with the ancestor facet.
  - *vertices*: the list of vertices (3D points) of the facet.
- The *bounds* field: which contains, as a list, the bounding box of the facets (either 2D or 3D)
- When the `tabs` option is set to `true`, there are two additional fields in the result:
  - The *tabs* field: which contains a 2D polygonal line (a list of lists of complex numbers) representing the tabs, only when the `tabs` option is set to `true`.
  - The *twins* field: which contains a list of the form $\{\{\{a1,b1\},\{a2,b2\}\}, \ldots\}$ representing the list of twin edge pairs (twin edges coincide when the polyhedron is closed). $a1$, $b1$, $a2$, $b2$ are complex numbers representing the endpoints of the edges in the 2D version of the polyhedron net. This list is calculated only when the `tabs` option is set to `true`.

The Drawing Method

This is the method **g:Dpolyhedron_net(P, options)** where $\langle P \rangle$ denotes a convex polyhedron. The options are those of the previous function, plus the following:

- In the case of a 2D pattern (when the `to2d` option, or the `tabs` option, has the value `true`):
  - `facet_name=false`, with the value `true` the facet number (preceded by the letter 'F') will be displayed in the center of each facet.

- `edge_name=false`, with the value `true`, the edge number (preceded by the letter 'e') will be displayed in the center of each edge, allowing you to identify twin edges and therefore neighboring facets.
- `tabs_options=""`, string representing TikZ drawing options for the tabs if the `tabs` option has the value `true`.
- `facet_options=""`, string representing TikZ drawing options for the **g:Dpolyline()** method that will draw the facets.

- In the case of a 3D pattern, there is only the following additional information:
  - `facet_options={}`, a list of drawing options for the **g:Dfacet()** method that will draw the facets.

The drawing is accompanied by a display in the terminal of its 2D bounding box.

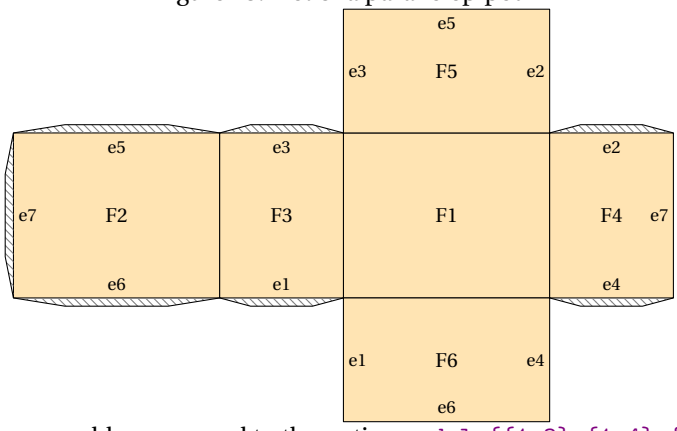
Examples

In this example, we display the default 2D net of a parallelepiped *P* with hatched tabs, the facet numbers (this number is the position in the *P.facets* list), and the edge numbers to see which ones need to be glued together:

```
\begin{luadraw}{name=parallelep_net}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Origin, vecI, vecJ, vecK = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK

local g = ld.graph3d:new{viewdir={30,60},window={-8.5,8,-5,5},bbox=false, size={10,10}}
require 'luadraw_cvx_polyhedra_nets'
P = ld.parallelep(Origin, 4*vecI,5*vecJ,3*vecK)
g:Dpolyhedron_net(P, {tabs=true, tabs_options="pattern=north west lines, pattern color=gray",
  facet_options="fill=Orange!30", facet_name=true, edge_name=true})
g:Show()
\end{luadraw}
```

Figure 13: Net of a parallelepiped



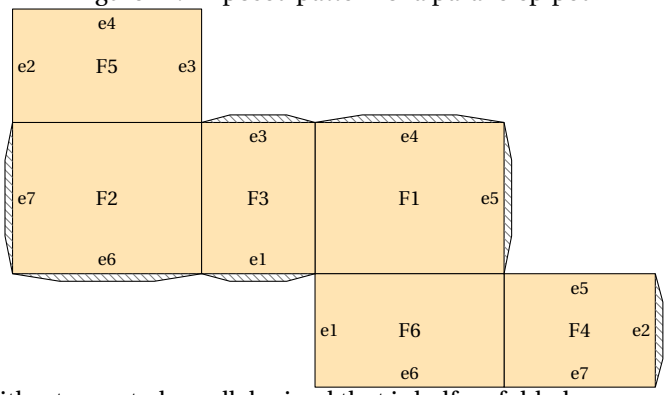
The default pattern here would correspond to the option `model={{1,3},{1,4},{1,5},{1,6},{3,2}}`[2], but we might want to impose a different model, for example, with the same parallelepiped:

```
\begin{luadraw}{name=parallelep_net2}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Origin, vecI, vecJ, vecK = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK

local g = ld.graph3d:new{viewdir={30,60},window={-9,9,-5,5},bbox=false,size={10,10}}
require 'luadraw_cvx_polyhedra_nets'
P = ld.parallelep(Origin, 4*vecI,5*vecJ,3*vecK)
g:Dpolyhedron_net(P, {model={{4,6,1,3,2,5}},tabs=true,
  tabs_options="pattern=north west lines, pattern color=gray",
  facet_options="fill=Orange!30", facet_name=true,
 edge_name=true, rotate=-90})
g:Show()
\end{luadraw}
```

[2]The algorithm takes the first face, then looks for its neighbors, then the neighbors of the first neighbor, etc.

Figure 14: Imposed pattern of a parallelepiped

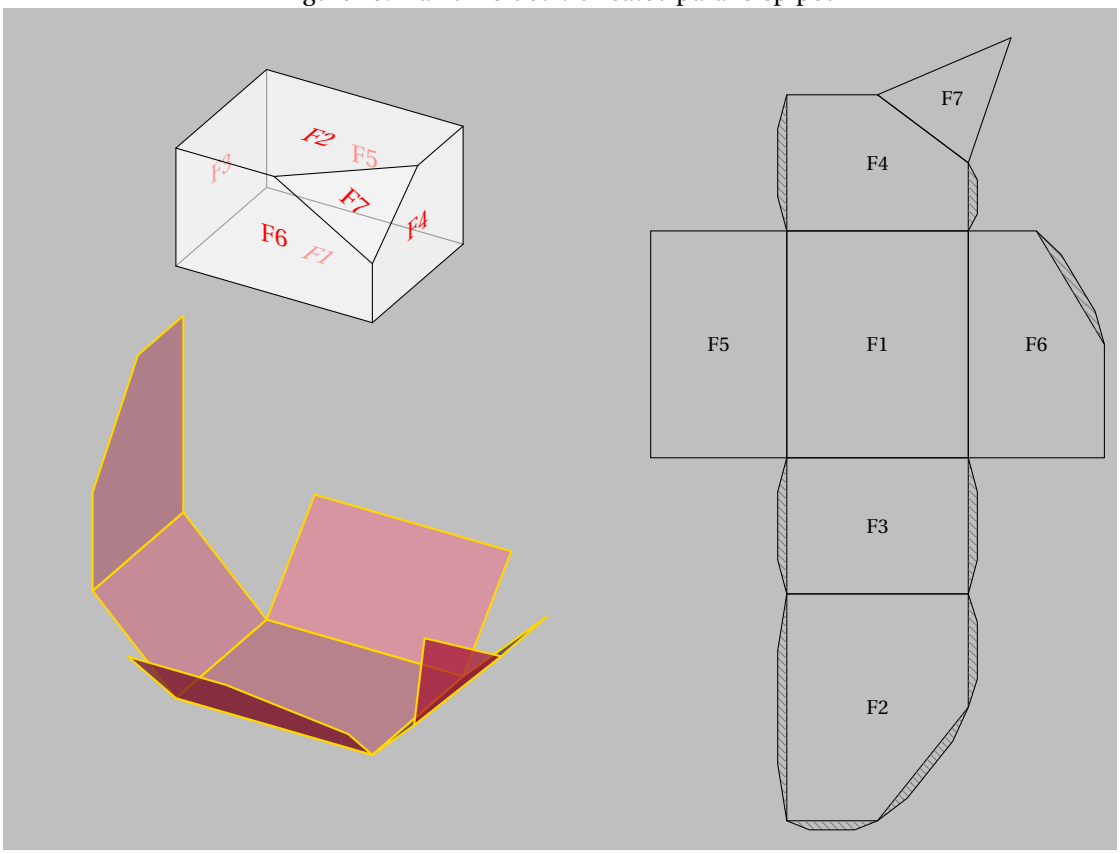


Here is an example with a truncated parallelepiped that is half-unfolded:

```
\begin{luadraw}{name=parallelep_net3}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-9,15,-9,9,0.6,0.6},bg="lightgray", viewdir={30,60}, margin={0,0,0,0}}
require 'luadraw_cvx_polyhedra_nets'
P = ld.parallelep(Origin, 4*vecI,5*vecJ,3*vecK)
local A, B, C = M(4,2.5,3), M(2,5,3), M(4,5,1.5)
P = ld.cutpoly(P, ld.plane(A,B,C), true) -- P is truncated with the plane
g:Shift3d(M(0,-4,5))
g:Dpolynames(P,"facet") -- this function shows facet numbers of P
-- half unfolded P
g:Shift3d(M(0,0,-11))
g:Dpolyhedron_net(P,{opening=0.5, facet_options={color="Crimson", opacity=0.7, edgecolor="Gold", edgewidth=8}})
-- 2D net
g:Shift(10)
g:Dpolyhedron_net(P,{tabs=true, tabs_options="pattern=north west lines, pattern color=gray",
 facet_name=true, rotate=90})
g:Show()
\end{luadraw}
```

Figure 15: Half unfolded truncated parallelepiped



NB: The functions **ld.unfold_polyhedron()** and **g:Dpolyhedron_net()** apply to any convex polyhedron, but they will not give the expected result with a non-convex polyhedron.

The *unfold_tree()* function

It can be useful to retrieve the tree generated by the **ld.unfold_polyhedron()** function to avoid recalculating it multiple times, for example, during an animation. The **ld.unfold_tree(tree, opening [, num])** function also allows you to unfold the polyhedron. The argument ⟨*tree*⟩ is the tree provided by the **ld.unfold_polyhedron()** function, the optional argument ⟨*opening*⟩ is a number between 0 and 1 that represents the opening rate (1 by default), the optional argument ⟨*num*⟩ is the number of the facet you want to open (and all descendants of that facet will rotate in the same way), when this argument is omitted, all facets rotate.

Example of animation:

```
\begin{luacode*}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK

ld.nbimages = 70
-- images creation
local g = ld.graph3d:new{ viewdir={"central",30,60}, bg="gray", size={10,10}, margin={0,0,0,0} }
-- declarations
local poly = require 'luadraw_polyhedrons'
require 'luadraw_cvx_polyhedra_nets'
local p = ld.linspace(0,1,36)
local T = ld.linspace(0,360, ld.nbimages+1)
local P = poly.dodecahedron(Origin, -2*vecI)
local net = ld.unfold_polyhedron(P)
local tree = net.tree
-- create the image number k, this function must be global
function ld.makeframe(k) -- do not modify this line
    local r = k
    if k > 36 then r = 72-k end
    local P1 = ld.rotate3d( ld.unfold_tree(tree,p[r]), T[k],{Origin,vecK})
    g:Dfacet(P1, {color="Crimson", edgecolor="Gold", edgewidth=8})
    -- send image number k
    g:Sendtotex()  -- send the TikZpicture to TeX
    g:Cleargraph()
end
\end{luacode*}
```

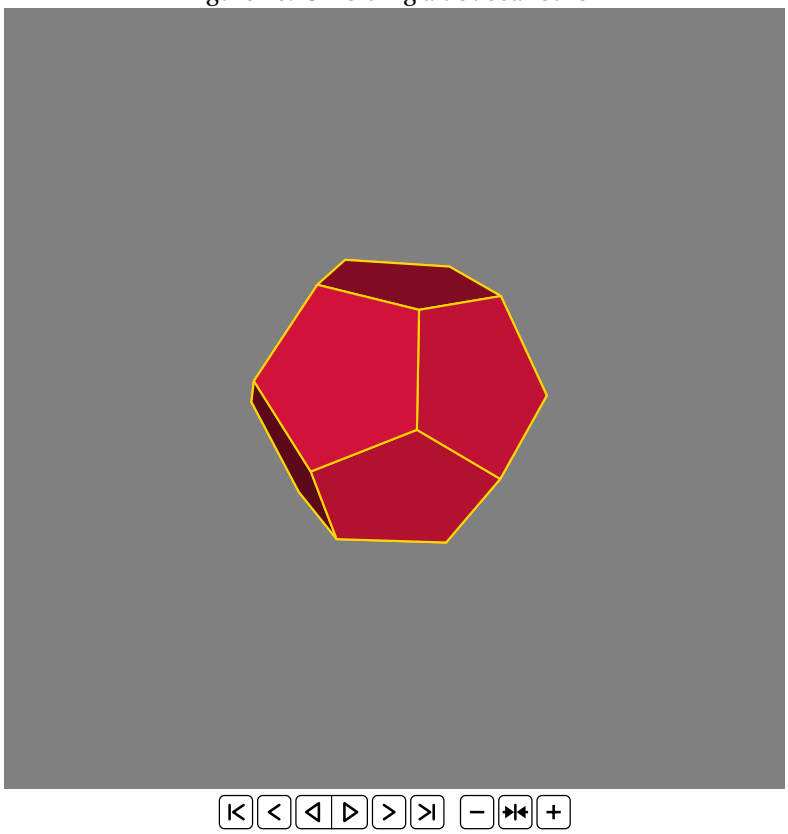
The TEX code (with the *animate* package):

```
\newcommand*\nb{\directlua{tex.print(luadraw.nbimages)}}
\newcommand*\makeframe[1]{\directlua{luadraw.makeframe(#1)}}%

\begin{animateinline}[poster=first,controls,loop]{8}
\multiframe{\nb}{ik=1+1}{%
\makeframe{\ik}%
}%
\end{animateinline}
```

The result :

Figure 16: Unfolding a dodecahedron



6) The *luadraw_fields* module

This module does not return anything, it adds new graphing methods to the *ld.graph* class and new functions to the *luadraw* namespace.

2D Fields

These are the methods given at the end of Chapter 1 for drawing vector fields and gradient fields. They include:

- The function **ld.field(f, x1, x2, y1, y2 [, grid, length])** returns the vector field (list of segments). The argument $\langle f\rangle$ is a function $\langle f\rangle\colon (x,y)\mapsto f(x,y))\in \mathbf{R}^2$ ($f(x,y)$ is a list of two real numbers). The field is calculated over the cuboid $[x_1;x_2]\times[y_1;y_2]$. The argument $\langle grid\rangle$ defaults to $\{25,25\}$, which is the number of subdivisions of the interval $[x_1;x_2]$ and the number of subdivisions of the interval $[y_1;y_2]$. The argument $\langle length\rangle$ allows you to impose the length of the vectors (all vectors have the same length), by default a length is calculated based on the steps on each of the two axes.
- The method **g:Dvectorfield(f, options)** draws the vector field associated with the function $\langle f\rangle$. The $\langle options\rangle$ argument is a table whose fields define the options, which are (with their default value):
  - `view=<default window>`, a list of the form `view={x1,x2,y1,y2}` used to define the tile $[x_1;x_2]\times[y_1;y_2]$. By default, this is the window chosen when creating the graph.
  - `grid={25,25}`, a list of two integers used to define the subdivisions along the two axes.
  - `length=nil`, allows you to specify the length of the vectors; by default, a length is calculated based on the steps on each of the two axes.
  - `draw_options=""`, a string containing the drawing options that will be passed to TikZ.
- The method **g:Dgradientfield(f, options)** draws the gradient field associated with the function **numérique** $\langle f\rangle\colon (x,y)\mapsto f(x,y)\in\mathbf{R}$. The argument $\langle options\rangle$ is identical to that of the method **Dvectorfield()**..

An example of these two methods has already been given on page **??**.

3D Fields

These are vector fields tangent to a 3D surface.

The surface is defined by a parameterization $p: (u,v) \mapsto p(u,v) \in \mathbb{R}^3$ with $(u,v) \in [u_1;u_2] \times [v_1;v_2]$.

The field is defined by a function $f: (u,v) \mapsto (f_1(u,v);f_2(u,v)) \in \mathbb{R}^2$, with $(u,v) \in [u_1;u_2] \times [v_1;v_2]$.

The vectors are calculated at the center of each mesh and are defined by the formula:

$$f_1(u,v)\frac{\partial p}{\partial u}(u,v) + f_2(u,v)\frac{\partial p}{\partial v}(u,v), \text{ then these are normalized.}$$

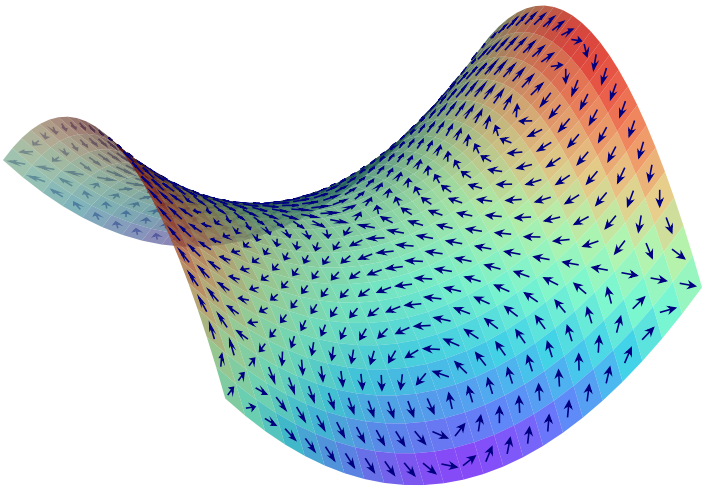
- The function **ld.surfacefield(p, f, u1, u2, v1, v2 [, grid, length, arrows])** returns the vector field (a list of lists of 3D points). The argument ⟨*p*⟩ is the surface parameter, the argument ⟨*f*⟩ defines the vector field, and calculations are performed in the domain $[u_1;u_2] \times [v_1;v_2]$. The argument ⟨*grid*⟩ defaults to {25,25}, which is the number of subdivisions of the interval $[u_1;u_2]$ and the interval $[v_1;v_2]$. The argument ⟨*length*⟩ allows you to specify the length of the vectors (all vectors have the same length); by default, a length is calculated based on the steps on each of the two axes. The argument ⟨*arrows*⟩ is a boolean (`false` by default), with the value `true` an arrow (basic) is added to the end of each vector, these are drawn in the plane tangent to the surface.

- The method **g:Dsurfacefield(p, f, options)** draws the vector field associated with the function ⟨*f*⟩, and (optionally) the surface field associated with ⟨*p*⟩. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

  - `domain={u1,u2,v1,v2}` defines the domain $[u_1;u_2] \times [v_1;v_2]$. By default, these are the intervals of the x and y coordinates of the 3D window chosen when creating the graph.
  - `grid={25,25}` is a list of two integers that defines the subdivisions of the domain.
  - `length=nil` sets the length of the vectors; by default, a length is calculated based on the chosen grid.
  - `color=<current line color>`, vector color.
  - `width=<current line thickness>`, vector line thickness.
  - `arrows="auto"`, arrow style for vectors. By default, it's an arrow drawn in the plane tangent to the surface. For example, with `arrows="-stealth"`, TikZ will draw arrows in the screen plane. With `arrows="-"`, there will be no arrows, only line segments.
  - `clip=false`, allows you to clip the drawing to the current 3D window.
  - `field_options=""`, string containing the vector drawing options that will be passed to TikZ. This option should not be necessary.
  - `surface_options=nil`, with the value `nil` the surface is neither calculated nor drawn; only the vectors will be displayed. In other cases, this option must be an options table for the **g:Dfacet()** method, which will draw the surface. Note that if this table contains the option `opacity=0`, then the surface facets will not be rendered.

```
\begin{luadraw}{name=surface-quiver}
local ld = luadraw
local pt3d, cpx = ld.pt3d, ld.cpx
local M, Z = pt3d.M, cpx.Z
require 'luadraw_fields'
local surface = function(x, y) return M(x, y, 0.15*x^2 - 0.15*y^2) end
local field = function(x, y) return {math.sin(y), math.sin(x)} end
local g = ld.graph3d:new{ window={-5.5, 6, -6, 4}, viewdir={65, 60}, margin=0, size={10, 10} }
g:Dsurfacefield(surface, field, {
    domain = {-4, 4, -4, 4},
    color = "Navy",
    width = 6,
    surface_options = {mode=ld.mShadedOnly, usepalette={ld.palRainbow,"z"}, opacity=0.8}
    })
g:Dlabel("$z=0.15x^2-0.15y^2$,\\quad $f(x,y)=(\\sin y,\\sin x)$", Z(0,-4.5), {pos="S"})
g:Show()
\end{luadraw}
```

Figure 17: The *g:Dsurfacefield()* method

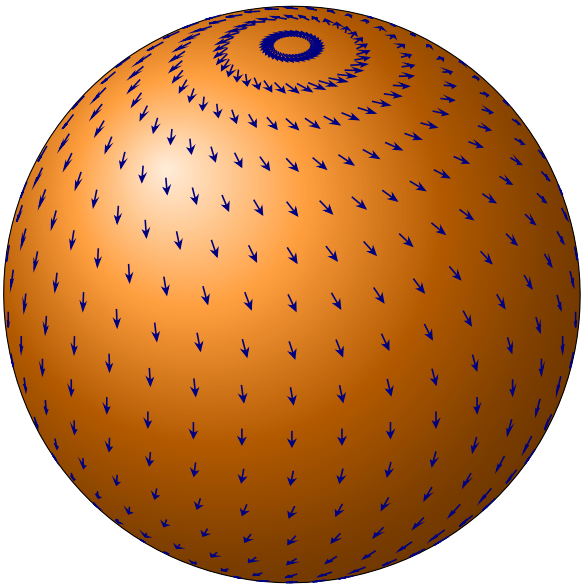


$$z = 0.15x^2 - 0.15y^2, \quad f(x,y) = (\sin y, \sin x)$$

In the second example, we use surface calculation to eliminate the non-visible facets (and therefore the associated vectors as well) with the option `backcull=true`, but we do not draw the surface, thanks to the option `opacity=0`, to put a prettier version of the sphere in its place.

```
\begin{luadraw}{name=surface-quiver2}
local ld = luadraw
local pt3d, cpx = ld.pt3d, ld.cpx
local M, Ms, Z, sin, cos, pi = pt3d.M, pt3d.Ms, cpx.Z, math.sin, math.cos, math.pi
require 'luadraw_fields'
local surface = function(u,v) return Ms(4,u,v) end
local field = function(u,v) return {cos(v)/(1-cos(u)), sin(v)/(1-cos(u))} end
local g = ld.graph3d:new{ window={-5, 5.5, -5.5, 5}, viewdir={65, 60}, margin=0, size={10, 10} }
g:Dsphere( M(0,0,0), 4, {color="orange", mode=ld.mBorder})
g:Dsurfacefield(surface, field, {
    domain = {pi, -pi, 0, pi},
    grid = {37,20},
    color = "Navy",
    width = 6,
    surface_options = {backcull=true, opacity=0}
    })
g:Dlabel("$f(u,v)= (\\frac{\\cos(v)}{1-\\cos(u)},\\frac{\\sin(v)}{1-\\cos(u)})$", Z(0,-4.5), {pos="S"})
g:Show()
\end{luadraw}
```

Figure 18: On a sphere



$$f(u,v) = \left(\frac{\cos(v)}{1-\cos(u)}, \frac{\sin(v)}{1-\cos(u)}\right)$$

7) The *luadraw_shadedforms* module

This module does not return anything; it adds new graphics methods to the *ld.graph* class. It allows you to draw polygonal lines or fill a shape using a color gradient.

Dshadedpolyline

The function **g:Dshadedpolyline(L, palette, options)** enables the rendering of the two-dimensional polygonal line $\langle L\rangle$ with a continuous color gradient determined by the computational method and the selected $\langle palette\rangle$. The argument $\langle L\rangle$ may be either a list of complex numbers or a list of lists of complex numbers. The $\langle palette\rangle$ is a list of colors, each color being represented as a list of three real numbers between 0 and 1, corresponding respectively to the red, green, and blue components. The $\langle options\rangle$ argument is a table whose fields define the options, which are (with their default value):

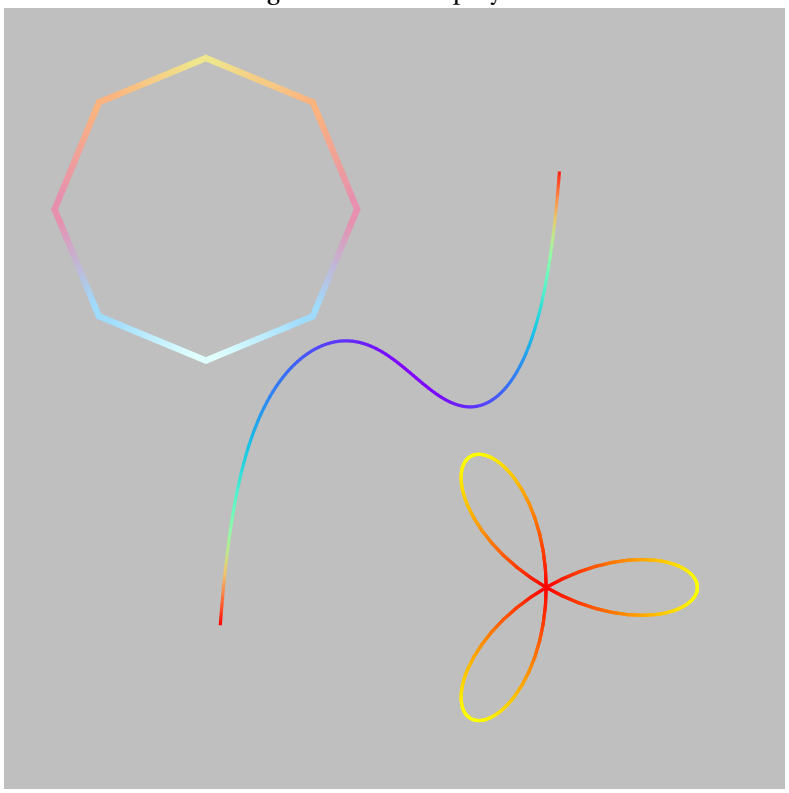
- `values="x"`: for each point in $\langle L\rangle$, a numerical value is computed, which determines the associated color according to the chosen palette. The `values` option specifies the evaluation mode and may take one of the following forms:
  - `"x"` (default): the value corresponds to the point's abscissa;
  - `"y"`: the value corresponds to the point's ordinate;
  - a function `f`: $(x,y) \mapsto f(x,y) \in \mathbf{R}$: the value for each point (x,y) in $\langle L\rangle$ is given by $f(x,y)$.
- `width=<current line width>`: specifies the line thickness in tenths of a point.
- `close=false`: a Boolean value indicating whether the polygonal line should be closed.
- `clip=nil` — this option can be either `nil` (default) or a table *{x1,x2,y1,y2}*. In the former case, the line is clipped by the current two-dimensional window **after** applying the graph's 2D transformation matrix; in the latter, the line is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** the transformation.

This procedure transforms $\langle L\rangle$ into a sequence of trapezoids, which are subsequently filled using a smooth color gradient.

```
\begin{luadraw}{name=shading_polyline}
local ld = luadraw
local cpx = ld.cpx
local i, Z = cpx.I, cpx.Z
```

```
local g = ld.graph:new{size={10,10},bg="lightgray", margin={0,0,0,0}}
require 'luadraw_shadedforms'
-- first example diff. equation y'= x^2+y^2-1 (=f(x,y))
local x1,x2,y1,y2 = -3,3,-3,3
local A = Z(0,1/2) -- initial condition
local f = function(x,y)
    return x^2+y^2-1
end
local S = ld.odesolve(f, A.re, A.im, x1, x2, 150) -- S is a matrix {X,Y}
local L = {} -- to convert {X,Y} into the complex numbers list L
for k = 1, #S[1] do table.insert(L, Z(S[1][k],S[2][k])) end
L = ld.clippolyline(L,-2.5,2.4,y1,y2)[1] -- L is the solution curve
g:Dshadedpolyline(L, ld.palRainbow, {values=f, width=12}) -- solution drawn with rainbow color map using function f
-- second example
L = ld.polar(function(t) return 2*math.cos(3*t) end, -math.pi, math.pi)
local f = function(x,y) return cpx.abs(Z(x,y)) end -- here the value will be the modulus
g:Shift(2-2.5*i)
g:Dshadedpolyline(L, ld.palAutumn, {values=f, width=12})
-- third example
g:Shift(-4.5+5*i)
g:Dshadedpolyline( ld.polyreg(0,2,8), ld.palGasFlame, {values="y", width=24, close=true})
g:Show()
\end{luadraw}
```

Figure 19: Shaded polyline



Dcolorbar

The method **g:Dcolorbar(A, pal, options)** allows you to draw a rectangle with a color gradient from a palette, optionally with a gradation. The argument ⟨*A*⟩ is a complex number; it is the reference point for constructing the rectangle. The argument ⟨*pal*⟩ is a color palette (a list of lists of the form $\{r,g,b\}$ where r, g, and b are between 0 and 1). The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `minmax={0,1}`, a list containing the minimum value (which will be assigned to the first color of the palette) and the maximum value (which will be assigned to the last color of the palette). Thus, any numeric value between *min* and *max* corresponds to a color in the palette.
- `dir=cpx.I`, direction of the longer side of the rectangle (this vector is automatically normalized), so by default the rectangle is vertical.

- `length=8`, length of the rectangle.
- `width=0.5`, width of the rectangle. The vertices of the rectangle are:

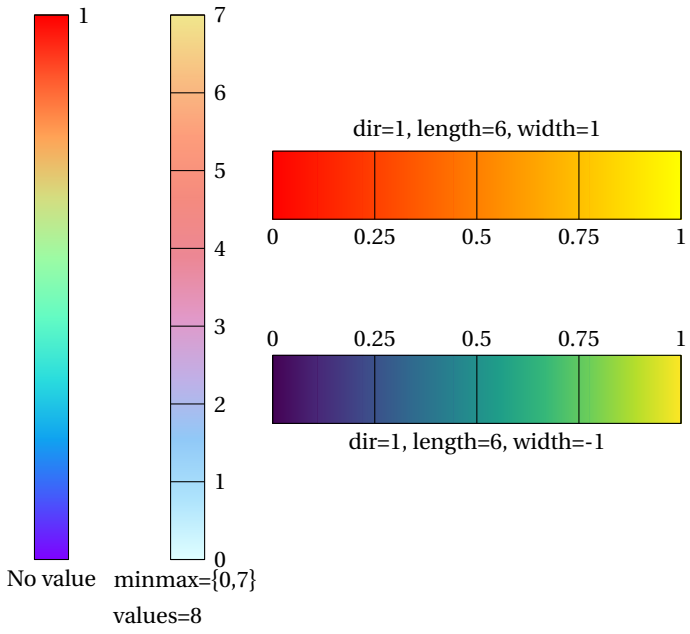
```
A, A+length*dir, A+length*dir+width*cpx.I*dir, A+width*cpx.I*dir
```

- `values=0`, this option allows you to define either the number of numeric values displayed equally within the interval `minmax` (none by default), or the list of numeric values displayed (in this case `values` must be a list of numeric values within the interval `minmax`).
- `addvalues=nil`, this option allows you to define a list of numeric values to display in addition to the *min* and *max* values. This option takes precedence over the previous one.
- `digits=2`, the number of decimal places for the numeric displays.
- `labelpos="E"` option positions the labels relative to anchor points (`"N"`, `"NE"`, `"E"`, `"SE"`, `"S"`, `"SW"`, "W", `"NW"`). The labels are positioned along the *(A, dir)* axis.

```
\begin{luadraw}{name=Dcolorbar}
local ld = luadraw
local Z = ld.cpx.Z

local g = ld.graph:new{size={10,10}, bbox=false}
g:Labelsize("small")
require 'luadraw_shadedforms'
local pal = require 'luadraw_palettes'
g:Dcolorbar(Z(-4,-4), pal.Rainbow)
g:Dcolorbar(Z(-2,-4), pal.GasFlame, {minmax={0,7},values=8})
g:Dcolorbar(Z(-1,1), pal.Autumn, {dir=1,length=6,width=1,addvalues={0.25,0.5,0.75},labelpos="S"})
g:Dcolorbar(Z(-1,-1), pal.Viridis, {dir=1,length=6,width=-1,addvalues={0.25,0.5,0.75},labelpos="N"})
g:Dlabel("No value", Z(-4.25,-4),{pos="S"},
    "\\parbox{1.95cm}{minmax=\\{0,7\\}\\\\values=8}", Z(-2.25,-4), {},
    "dir=1, length=6, width=1", Z(2,2), {pos="N"},
    "dir=1, length=6, width=-1", Z(2,-2),{pos="S"})
g:Show()
\end{luadraw}
```

Figure 20: Color bars



Dshadedrectangle

NB: Using this method with the `bilinear=true` option requires the *shadings* library.

The **g:Dshadedrectangle(x1, x2, y1, y2, pal, options)** method fills the rectangle $[x_1;x_2]\times[y_1;y_2]$ with a gradient of colors extracted from the $\langle pal\rangle$ palette. Each point (x,y) in the rectangle has a color from the palette, calculated from a value

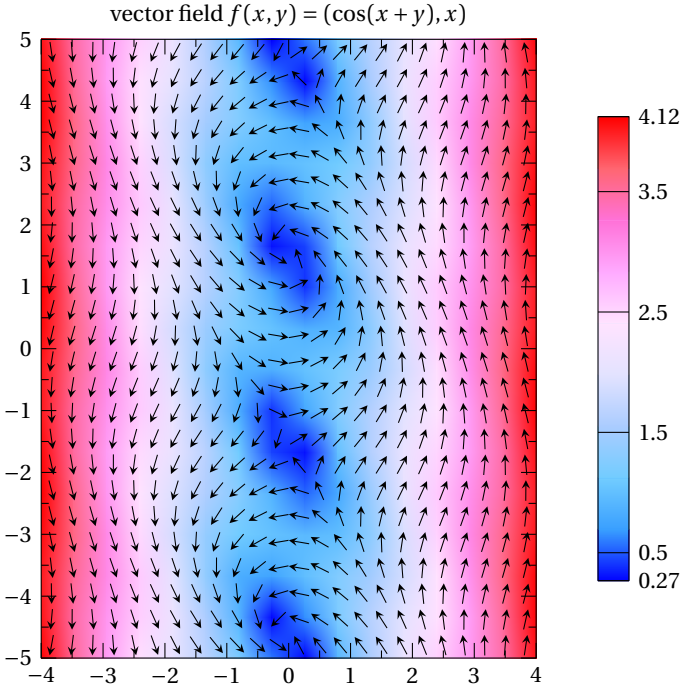
$f(x,y)$ where f is a numeric function defined on the rectangle. The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `values=function(x,y) return cpx.abs(Z(x,y)) end`, this option defines the function f used to calculate the color of each point. The maximum value of f on the rectangle will correspond to the last color in the palette, and the minimum value of f will correspond to the first color in the palette. By default, this function is the modulus.
- `grid={15,15}`, this option defines the number of subdivisions for the interval $[x_1;x_2]$ and for the interval $[y_1;y_2]$. The finer the subdivision, the longer the display will take.
- `bilinear=true`: with the value `true` (default), bilinear shading is used on each cell. With the value `false`, each cell is filled with a solid color.
- `bar="none"` allows you to add or not add a legend using the *Dcolorbar* method. This option can be: `"none"`, `"right"`, `"bottom"`, `"left"`, or `"top"`.
- `bardist=1`, the distance between the rectangle and the legend, if there is one.
- `baroptions={}`, a list of options for the **Dcolorbar()** method if there is a legend.
- `out=nil`, if a list variable is assigned to this parameter `out`, then the method adds the two values *min* and *max* of the f function to this list (which allows you to retrieve these two values if necessary).

```
\begin{luadraw}{name=Dshadedrectangle}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{window={-4,6,-5.5,5.5}, size={10,10}, bbox=false}
require 'luadraw_shadedforms'
require 'luadraw_fields'
local pal = require 'luadraw_palettes'
g:Labelsize("small")
local f = function(x,y) return {math.cos(x+y), x} end -- vector field
local Nf = function(x,y) local A = f(x,y); return cpx.abs(Z(A[1],A[2])) end -- modulus of f(x,y)
g:Dshadedrectangle(-4,4,-5,5, pal.Picnic, {values=Nf, bar="right", baroptions={addvalues={0.5,1.5,2.5,3.5}}})
g:Dvectorfield(f,{view={-4,4,-5,5},draw_options="-stealth"})
g:Dgradbox({Z(-4,-5),Z(4,5),1,1}, {title="vector field $f(x,y)=(\\cos(x+y),x)$"})
g:Show()
\end{luadraw}
```

Figure 21: Shaded rectangle



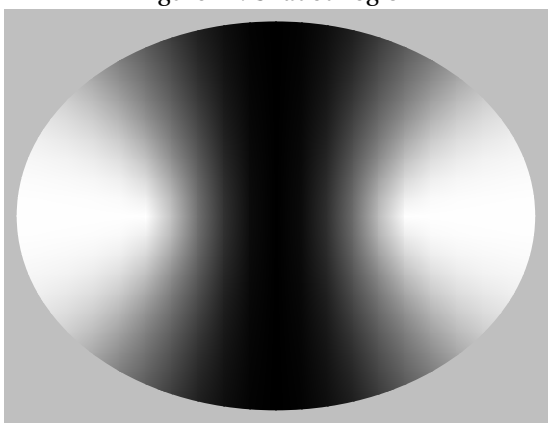
Dshadedregion

The method **g:Dshadedregion(apath, pal, options)** fills the region defined by the path ⟨*apath*⟩ with a gradient of colors extracted from the palette ⟨*pal*⟩. This method uses the previous one (**Dshadedrectangle()**) on the rectangle defined by the bounding box of the path. Each point (x, y) in this rectangle has a color from the palette, calculated from a value $f(x, y)$ where f is a numeric function defined on this rectangle. The drawing is clipped to the path. The argument ⟨*options*⟩ is the same as that of the previous method **Dshadedrectangle()**.

```
\begin{luadraw}{name=Dshadedregion}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{window={-4.5,6,-4.5,5}, size={10,10},bg="lightgray", bbox=false}
g:Labelsize("small")
require 'luadraw_shadedforms'
local pal = require 'luadraw_palettes'
local L1, L2 = -2, 2
local f = function(x,y)
    local z = Z(x,y)
    return (cpx.abs(z-L1)-cpx.abs(z-L2))^2
end
g:Dshadedregion({4,0,4,3,"e"}, pal.getPal(pal.Grays,{reverse=true}), {values=f, grid={20,20}})
g:Show()
\end{luadraw}
```

Figure 22: Shaded region



8) The *luadraw_povray* Module

This module does not return anything; it adds new graphics methods to the *ld.graph3d* class, as well as a function in the namespace *luadraw*.

Prerequisites and introduction

First and foremost, the POV-Ray software must be installed on your computer in its command-line version (non-graphical version).

- On Linux: POV-Ray is always a command-line application and is installed using the package manager.
- On macOS: the command-line version of POV-Ray can be installed via Homebrew.
- On Windows: the installer can be downloaded from the POV-Ray website.

This module enables the creation of (relatively simple) source files for the POV-Ray software (command-line version) and allows them to be compiled on the fly. Once created, the resulting image (in *png* format) can be automatically included in the current graphic, allowing further drawing both underneath and on top of it. The image is inserted with perfect alignment, meaning that the 3D reference frames of the graphic and of the image will match exactly, provided that the system operates

in **orthographic projection** mode. Source files and generated images are created in the working directory of *luadraw*. The source files can of course be compiled separately by the user; the parameters to be passed to POV-Ray are written in comments at the beginning of the source file. The source file is divided into three parts: a preamble, the object declarations, and the rendering of those objects.

Default Settings

Using this module requires certain default settings that may vary depending on the operating system. These default values can be modified using the function:

ld.povray_default(options)

The ⟨*options*⟩ argument is a table whose fields define the options, which are (with their default value):

- `bg=""` This option defines the background. It can be an empty string (default value), in which case the background will be transparent (essential if drawing under the image), or a non-empty string, in which case it must be a color name known to POV-Ray, or an array of the form $\{r,g,b\}$ representing a color with the values r, g, and b between 0 and 1.
- `shadow=true`, this boolean indicates whether objects should cast a shadow or not (this boolean can be modified locally for each object created).
- `imagescale=1`, this option allows you to change the size of the generated *png* image; values greater than 1 obviously increase its file size.
- `param="-V +A +FN"`, these are the basic parameters passed to POV-Ray. The image width and height will be automatically added to these, as well as the "+UA" option if the background should be transparent.
- `pov_cmd=`, the name of the command to run POV-Ray. On Unix, the default value is `"povray"`, and on Windows, the default value is `"pvengine64.exe"`.
- `pov_cmd_ext=""`, a string that is appended to the end of each command sent to POV-Ray.
- `win_param_ext="/RENDER /EXIT"`, additional parameters that only apply to Windows.
- `include={}`, this option allows POV-Ray to include **.inc* files, for example:

  ```
  include={"textures.inc", "colors.inc"}
  ```

  These two files are normally distributed with POV-Ray; the first defines textures, the second colors. By default, no inclusions are made.
- `arrowscale={1,1}`, table of two numbers, the first is a scale factor for the radius of the base of the arrows (these are cones), the second is a scale factor for the height of the arrows.

If you modify one of these options, the value you assign becomes the new default value for the rest of the document. However, it's not necessary to repeat this function in every graph in your document using POV-Ray; it only needs to be done once in the preamble. For example, on macOS, if you installed POV-Ray via *Homebrew*, then this is suitable:

```
\documentclass{standalone}%
\usepackage[svgnames]{xcolor}
\usepackage[3d]{luadraw}
\directlua{%
require "luadraw_povray"
luadraw.povray_default( {pov_cmd = "/opt/homebrew/bin/povray"} )
}%
...
```

Of course, if the default values are suitable for your system (this should be the case under Linux), you do not need to use the function **ld.povray_default()**.

Before Creating Objects

The POV-Ray must be initialized using the method:

g:Pov_new(options).

The argument ⟨*options*⟩ is exactly the same as for the previous function **ld.povray_default()**, except that the effect will be local to the current graph only.

Creation of Objects

Objects are created using methods following the format **g:Pov_<command>(<data>, options)**. Each object is first declared in the POV-Ray source file with a name (similar to a variable), and then this object is "rendered" in the third section of the source file with a texture defined from the options.

The argument ⟨*options*⟩ is a table whose fields specify the options of the object. The following are the **options common to all objects**, along with their default values:

- `name="object<num>"`, a string representing the name of the created object. By default, it is the word "object" followed by a number (its order of appearance). Assigning a name is useful when the object needs to be reused later.
- `shadow=true`, a Boolean indicating whether the object casts a shadow.
- `render=true`, a Boolean indicating whether the object should be displayed. It may be desirable not to render an object if it is only used in the construction of another one.
- Options defining the base texture, composed of a *pigment* (color + opacity) and a *finish* (ambient + diffuse + phong):
  - `color=ld.White`, this option may be either a table in the form $\{r, g, b\}$ (with r, g, and b between 0 and 1), or a string that must represent a color known to POV-Ray. It can also refer to a texture defined only by a *pigment* (for example, `color="Blue_Sky3"`, where this name is defined in the file *textures.inc*).
  - `usepalette=nil`, this option allows the use of a color palette. The syntax is `usepalette={palette, func, minmax}` where ⟨*palette*⟩ is the list (table) of colors, each of which is a list in the format $\{r, g, b\}$ with r, g, and b between 0 and 1. The argument ⟨*func*⟩ is a string that defines the gradient type; it can be either `"x"`, `"y"`, or `"z"` (in this case, the argument ⟨*minmax*⟩ must be a list containing the minimum and maximum values), or a string of the form `"function { <expression depending on x, y, z> }"`. The expression that depends on x, y, and z is a function (for POV-Ray) that must return a value between 0 and 1 (in this case, the ⟨*minmax*⟩ argument is not needed).
  - `opacity=1`, a real number between 0 and 1 defining the object's opacity.
  - `ambient=0.35`, `diffuse=0.8`, and `phong=0.5`: three parameters defining the *finish*.
  - `mytexture=nil`, this parameter allows specifying a texture directly as a string or referencing an existing POV-Ray texture. In that case, the previous parameters are ignored. For example: `mytexture="texture{Silver_Metal}"` (this texture is declared in the file *textures.inc*).
- `matrix=nil`, a 3D transformation matrix applied locally to the object. The global 3D transformation matrix of the graphic is also taken into account.
- `clipbox=nil`, this option defines a list (table) of objects used to clip the object being constructed. These objects may be: an existing object, referenced by its name as a string; a box, specified as a table of the form *{M(xmin,ymin,zmin), M(xmax,ymax,zmax)}* (representing the box diagonal); or a sphere, given as a table *{center,radius}*, where *center* is a 3D point and *radius* a positive number.
- `clipplane=nil`, this option defines a list (table) of planes to clip the object being constructed. Each plane must be of the form $\{A, n\}$, where A is a point on the plane (a 3D point) and n the normal vector to the plane. Only the part of the object lying in the half-space containing n is preserved.

List of Predefined Objects

The following is the list of objects that can be drawn using the corresponding methods:

- **Implicit surfaces** defined by the equation $f(x,y,z)=0$. The corresponding method is:

  g:Pov_implicit(povfunction, luafunction, options).

  The argument ⟨*povfunction*⟩ is a string containing the expression of $f(x,y,z)$. POV-Ray supports standard mathematical functions; however, note that the power function is written as *pow*, i.e. $x \rightarrow \text{pow}(x,n)$, and that POV-Ray does not handle numerical errors such as division by zero.

  The options are those already described, plus the specific option `containedby=<current 3D window>`, which specifies the box (or sphere) within which the computations are carried out. By default, this box is the current 3D window of the graphic. A box is given as a table of the form *{M(xinf,yinf,zinf), M(xsup,ysup,zsup)}* (representing a diagonal of the box), and a sphere is a table of the form $\{C,r\}$, where C is the center (a 3D point) and r the radius. Example:

```
local f = function(x,y,z) return x^2+y^2+z^2 - 1 end
local r = 1.1
g:Pov_implicit("x*x+y*y+z*z-1", f, {color=ld.SteelBlue, containedby={M(-r,-r,-r), M(r,r,r)}})
```

- **Parameterized Surfaces**. Two possible syntaxes:

  1. The method: **g:Pov_surface(f, u1, u2, v1, v2, options)**

     draws the (smoothed) surface parameterized by the function $\langle f\rangle\colon (u,v) \rightarrow f(u,v) \in \mathbf{R}^3$. The interval for the parameter u is given by ⟨*u1*⟩ and ⟨*u2*⟩. The interval for the parameter v is given by ⟨*v1*⟩ and ⟨*v2*⟩. The options are those already provided, plus two specific options:

     - `grid={25,25}`, this option defines the number of points to calculate for the parameter u followed by the number of points to calculate for the parameter v (25 by default).
     - `clip=false`, with the value `true` the surface is clipped to the current 3D window.

     Example:

```
local f = function(x,y) return M(x,y,x^2+y^2) end
g:Pov_surface(f,-2,2,-2,2, {color=ld.SteelBlue, clip=true})
```

  2. The method: **g:Pov_surface(xfunc, yfunc, zfunc, u1, u2, v1, v2, options)**

     draws the surface parameterized by $(u,v) \rightarrow (x(u,v),y(u,v),z(u,v))$. The arguments ⟨*xfunc*⟩, ⟨*yfunc*⟩, and ⟨*zfunc*⟩ are three strings containing the expressions $x(u,v)$, $y(u,v)$, and $z(u,v)$, respectively. The arguments ⟨*u1*⟩ and ⟨*u2*⟩, respectively ⟨*v1*⟩ and ⟨*v2*⟩, define the interval bounds for the parameter u and for the parameter v, respectively. The options are those already given, plus two specific options:

     - `containedby=<current 3D window>`, this indicates in which box (or sphere) the calculations will be performed; by default, this box is the 3D window of the current graph. A box is a table of the form *{M(xinf,yinf,zinf), M(xsup,ysup,zsup)}* (this represents a diagonal of the box), a sphere is a table of the form $\{C,r\}$, where C is the center (3D point) and r is the radius.
     - `max_grad=nil`, numerical value (optional) allowing calculations to be optimized, here is what the POV-Ray help says about this number:

```
The max_gradient is the maximum magnitude of all six partial derivatives
over the specified ranges of u and v.
Take dx/du, dx/dv, dy/du, dy/dv, dz/du, and dz/dv and calculate them over
the entire range.
The max_gradient should be at least the maximum (absolute value) of all of those values.
Choosing a too small of a value will create holes or artifacts in the object.
```

     Example:

```
g:Pov_surface("x", "y", "x*x+y*y",-2,2,-2,2, {color=ld.SteelBlue, max_grad=4})
```

     Note: This is the same surface in both examples, but the second method is longer, and if the `max_grad=4` option is not specified, then the entire surface is not drawn.

- **Polyhedron or list of facets**. This is the method:

  g:Pov_facet(F, options).

  The argument ⟨*F*⟩ is either a polyhedron or a list of facets. The options include the common ones, plus the following specific options (with their default values):

  – `edge=false`, boolean indicating whether edges should be drawn.

  – `edgestyle=<current line style>`, style of the edges.

  – `edgecolor=ld.Black`, color of the edges.

  – `edgewidth=<current line width>`, width of the edges.

  – `hidden=ld.Hiddenlines`, boolean indicating whether hidden edges should be drawn.

  – `hiddenstyle=ld.Hiddenlinestyle`, style of hidden edges.

  – `hiddenscale=ld.Hiddenlinescale`: a number representing a percentage; the thickness of hidden lines is equal to that of visible lines multiplied by this number. `ld.Hiddenlinescale` is a global variable that defaults to `2/3`.

  – `csg=false`, with the value `true`, the 3D object defined by ⟨*F*⟩ can be used in CSG operations (union, intersection, difference, merge), **provided** that this object is closed and correctly oriented.

  Example:

```
local T1 = tetra(M(-1,-1,-1), 3*vecI, 3*vecJ, M(1,1,3))
g:Pov_facet(T1, {color=ld.SteelBlue, edge=true, hidden=true})
```

  Note: drawing hidden edges is not always optimal; sometimes it is better to draw them with TikZ over the rendered image.

- **Polygonal line**. This is the method:

  g:Pov_polyline(L, options).

  The argument ⟨*L*⟩ is either a list of 3D points or a list of lists of 3D points. The options include the common ones, plus the following specific options (with their default values):

  – `style=<current line style>`, line style.

  – `width=<current line width>`, line width.

  – `close=false`, boolean indicating whether the line should be closed.

  – `clip=false`, a boolean indicating whether the line should be clipped by the current 3D window.

  – `arrows=0`, three possible values: 0 (no arrow), 1 (arrow at the end), or 2 (arrows at both start and end).

  – `arrowscale={1,1}` allows you to adjust the size of the arrows (width for the first number, height for the second). When `arrowscale` is a number, the second value is considered equal to the first.

  – `hiddenstyle=ld.Hiddenlinestyle`, style of hidden edges.

  – `hiddenscale=ld.Hiddenlinescale`: a number representing a percentage; the thickness of hidden lines is equal to that of visible lines multiplied by this number. `ld.Hiddenlinescale` is a global variable that defaults to `2/3`.

  Example: drawing the axes

```
g:Pov_polyline({{-5*vecI,5*vecI},{-5*vecJ,5*vecJ},{-5*vecK,5*vecK}}, {arrows=1,width=8})
```

- **3D points**. This is the method:

  g:Pov_dots(L, options).

  The argument ⟨*L*⟩ is a list of 3D points. The options include the common ones, plus the following specific options (with their default values):

  – `style="ball"`, two possible styles: `"ball"` (sphere) or `"box"` (a box with faces parallel to the 3D window).

  – `dotscale=1`, scale factor for dot size.

- **A plane**. This is the method:

  g:Pov_plane(P, options).

  The argument ⟨*P*⟩ is a table of the form {*A*, *n*}, where *A* is a point on the plane ⟨*P*⟩ (a 3D point) and *n* is a vector normal to the plane. The plane is automatically clipped by the 3D window. The options are the options of **g:Pov_facet()**, plus the specific option `scale=1`.

- **A circle**. This is the method:

  g:Pov_circle(A, R, N, options).

  It draws the circle with center ⟨*A*⟩, radius ⟨*R*⟩, and normal vector ⟨*N*⟩ specifying the plane of the circle. These are the common options.

- **Axes**.This is the method:

  g:Pov_axes(O, options).

  It draws the axes using point ⟨*O*⟩ (3D point) as the intersection point. The options are the same as for **g:Pov_polyline()**. There are no graduations or legend.

- **Basic solids**. These are the methods:

  - **g:Pov_sphere(center, radius, options)**, which draws a sphere with center ⟨*center*⟩ (3D point) and radius ⟨*radius*⟩. These are the common options.
  - **g:Pov_torus(center, R, r, N, options)**, which draws a torus centered at ⟨*center*⟩ (3D point), with major radius ⟨*R*⟩, minor radius ⟨*r*⟩, lying in the plane normal to vector ⟨*N*⟩. These are the common options.
  - **g:Pov_cylinder(A, R, B, options)**, which draws a cylinder along the axis (*AB*) from ⟨*A*⟩ to ⟨*B*⟩ (3D points), with radius ⟨*R*⟩. These are the common options plus the specific option `hollow=false`, indicating whether the cylinder is hollow or not.
  - **g:Pov_cone(A, R, B [, r, options])**, which draws a cone along the axis (*AB*) from ⟨*A*⟩ to ⟨*B*⟩ (3D points), with radius ⟨*R*⟩ at end ⟨*A*⟩ and radius ⟨*r*⟩ at end ⟨*B*⟩. The radius ⟨*r*⟩ is optional and defaults to 0; in that case, ⟨*B*⟩ is the apex of the cone. These are the common options plus the specific option `hollow=false`, indicating whether the cone is hollow or not.
  - **g:Pov_box(A, B, options)**, which draws a box whose faces are parallel to those of the 3D window of the graphic. The 3D points ⟨*A*⟩ and ⟨*B*⟩ define a diagonal of the box, more precisely ⟨*A*⟩ = M(xinf, yinf, zinf) and ⟨*B*⟩ = M(xsup, ysup, zsup). These are the common options.

- **CSG geometry**. These are the following methods (for each of them, the options are the common options):

  - **g:Pov_union(list, options)**, where ⟨*list*⟩ is a list of POV-Ray objects; these objects may be either the name of an already created object or a POV-Ray command given as a string. The result is treated as a single object.
  - **g:Pov_intersection(list, options)**, where ⟨*list*⟩ is a list of POV-Ray objects; these objects may be either the name of an already created object or a POV-Ray command given as a string. The result is the common part of these objects.
  - **g:Pov_merge(list, options)**, where ⟨*list*⟩ is a list of POV-Ray objects; these objects may be either the name of an already created object or a POV-Ray command given as a string. This command works like union, but removes internal surfaces (unlike union), which is useful in the case of transparency.
  - **g:Pov_difference(list, options)**, where ⟨*list*⟩ is a list of POV-Ray objects; these objects may be either the name of an already created object or a POV-Ray command given as a string. The result is the difference: ((object1 minus object2) minus object3) etc.

- **Writing directly to the source**, with the following methods:

  - **g:Pov_comment(comment)**, which writes the string ⟨*comment*⟩ to the source as a comment.
  - **g:Pov_special(code)**, which writes the string ⟨*code*⟩ verbatim to the source; it must therefore be valid POV-Ray code. Note that a 3D point written as *M(x, y, z)* in *luadraw* is written in POV-Ray code as <-*x*, *y*, *z*>; the POV-Ray coordinate system is thus not our usual one and is left-handed.

Save, execution, inclusion

- **Save and execution**. Once all objects have been created, saving and executing the file with POV-Ray is done using the method:

  g:Pov_exec([filename]).

  The argument ⟨*filename*⟩ must be a filename without path or extension, but it is optional; by default, the name of the current graphic is used. The created file will have the *pov* extension and will be saved in the *luadraw* working directory (this directory is stored in the `cachedir` variable). The command used for execution is displayed in the terminal, along with the output from the POV-Ray software, allowing you to see if it encountered an error. The image construction appears on screen, but the window closes as soon as rendering is complete[3]. The image has the same name as the source, except the extension which is *png* instead of *pov*.

  NB: POV-Ray execution requires compiling the document with the *-shell-escape* or *-enable-write18* option. Once the image is obtained, this option is no longer necessary unless the image has been modified.

- **Save only**. This is done with the method:

  g:Pov_save([filename]).

  With the same remarks as previously for the ⟨*filename*⟩ argument.

- **Image inclusion**. Once obtained, the image can be included in the graphic with the method:

  g:Pov_show([filename]).

  If the ⟨*filename*⟩ argument is not specified, it refers to the name of the current graphic; otherwise, ⟨*filename*⟩ must be a complete image filename (with extension). The inclusion is done with `\includegraphics[]{filename}`.

Examples

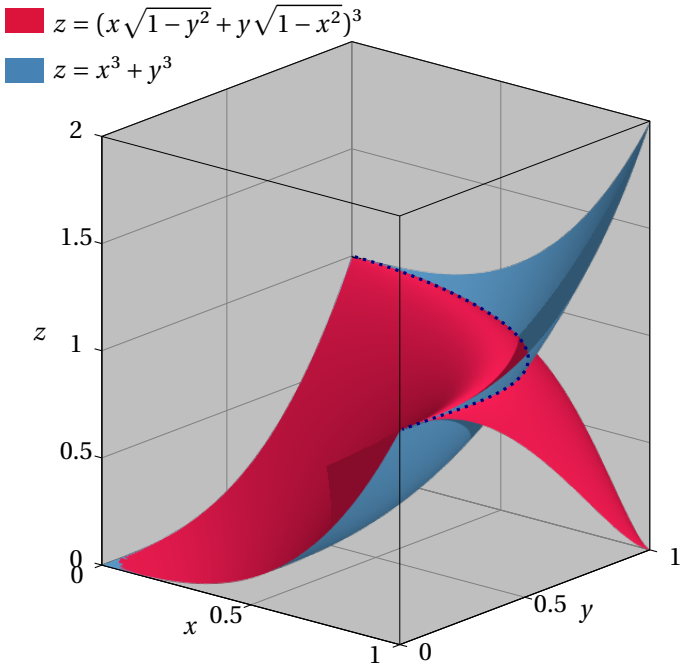
Intersection of two implicit surfaces

```
\begin{luadraw}{name=intersection_surf}
local ld = luadraw
local cpx, pt3d = ld.cpx, ld.pt3d
local Z, M = cpx.Z, pt3d.M

local g = ld.graph3d:new{window3d={0,1,0,1,0,2}, window={-0.25,1.5,-0.5,2.25}, size={10,10,0}, viewdir={-50,60}}
local sqrt = math.sqrt
require "luadraw_povray"
local f = function(x,y,z) return x^3+y^3-z end
local h = function(x,y,z) return (x*sqrt(1-y^2)+y*sqrt(1-x^2))^3-z end
local L = ld.implicit(function(x,y) return f(x,y,0)-h(x,y,0) end,0.01,1,0.01,1,{25,25})
L = ld.map(function(z) return M(z.re,z.im,z.re^3+z.im^3) end, L[1])  -- intersection curve
-- using povray
g:Pov_new()
g:Pov_implicit("pow(x,3)+pow(y,3)-z", f, {color=ld.SteelBlue})
g:Pov_implicit("z-pow(x*sqrt(1-y*y)+y*sqrt(1-x*x),3)", h, {color=ld.Crimson, containedby={M(0,0,0),M(0.999,0.999,2)}})
g:Pov_exec()
-- drawing
g:Dboxaxes3d({grid=true, gridcolor="gray",fillcolor="lightgray",xyzstep=0.5, drawbox=true})
g:Pov_show() --Pov-Ray image
g:Dpolyline3d(L, "line width=1.2pt,dotted,Navy") --we draw the curve over the image
local d = 0.1
g:Dsquare(Z(-0.25,2.25), Z(-0.25,2.25-d),1,"draw=none,fill=Crimson")
g:Dsquare(Z(-0.25,2.25-2*d), Z(-0.25,2.25-3*d),1,"draw=none,fill=SteelBlue")
g:Dlabel("$z=(x\\sqrt{1-y^2}+y\\sqrt{1-x^2})^3$", Z(-0.25+d,2.25-d/2) , {pos="E"},
    "$z=x^3+y^3$", Z(-0.25+d,2.25-5*d/2),{})
g:Show()
\end{luadraw}
```

[3]It depends on your operating system.

Figure 23: Intersection of two implicit surfaces

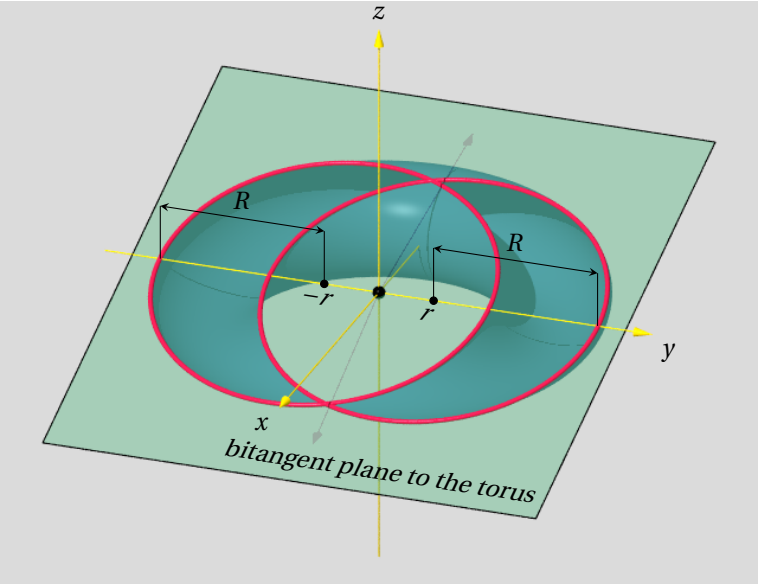


Villarceau circles

```
\begin{luadraw}{name=Villarceau_circles}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-6.5,6.5,-5,5},margin={0,0,0,0}, size={10,10}, viewdir={20,65}}
require "luadraw_povray"
local R, r = 3, 1
local N = ld.rotate3d(vecK, math.asin(r/R)*ld.rad, {Origin, vecJ})
local P = {Origin, -N}
-- using povray
g:Pov_new({bg=ld.LightGray})
g:Pov_torus(Origin, R, r, vecK, {color=ld.SteelBlue, clipplane=P})
g:Pov_plane(P,{color=ld.SeaGreen, opacity=0.4, edge=true, scale=0.9})
g:Pov_axes(Origin,{color=ld.Gold,arrows=1})
g:Pov_dots(Origin, {dotscale=1.5})
g:Pov_circle( M(0,r,0),R,N,{color=ld.Crimson, width=12})
g:Pov_circle( M(0,-r,0),R,N,{color=ld.Crimson, width=12})
g:Pov_exec()
-- drawing
g:Pov_show()
local F = g:Plane2facet(P,0.9)
g:Dpolyline3d({{-r*vecJ,-r*vecJ+vecK},{-(R+r)*vecJ,-(R+r)*vecJ+vecK}})
g:Dpolyline3d({{r*vecJ,r*vecJ+vecK},{(R+r)*vecJ,(R+r)*vecJ+vecK}})
g:Dpolyline3d({{-r*vecJ+vecK,-(R+r)*vecJ+vecK}, {r*vecJ+vecK,(R+r)*vecJ+vecK}}, "stealth-stealth")
g:Ddots3d({-r*vecJ, r*vecJ})
g:Dlabel3d("$x$",5*vecI,{pos="SW"},"$y$", 5*vecJ,{pos="SE"}, "$z$",5*vecK,{pos="N"},
  "$R$", -(R/2+r)*vecJ+vecK, {}, "$R$", (R/2+r)*vecJ+vecK, {},
  "bitangent plane to the torus", F[1], {pos="NW", dir={vecJ,pt3d.prod(N,vecJ)}},
  "$-r$",-r*vecJ,{pos="S"}, "$r$", r*vecJ,{} )
g:Show()
\end{luadraw}
```

Figure 24: Villarceau circles

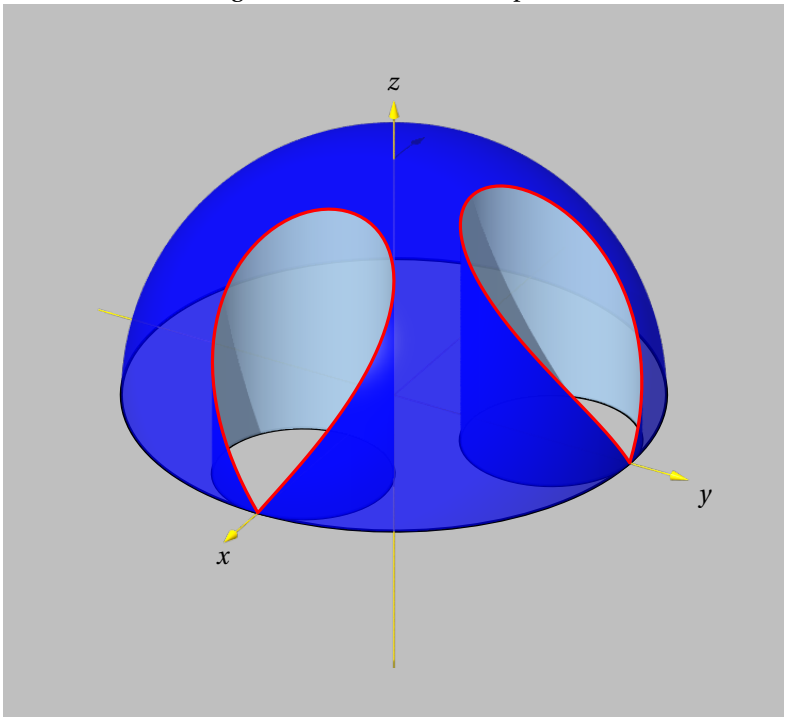


Holes in an half sphere

```
\begin{luadraw}{name=holes_in_hemisphere}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{window={-5,5,-4,5}, size={10,10}, bg="lightgray"}
require "luadraw_povray"
local O, R = Origin, 4
local A, r = 2*R/3*vecJ, R/3
local P = {Origin, vecK}
local base = ld.circle3d(A,r,vecK)[1] --circular base of the cylinder (list of 3D points)
local Cylborder = {}
for _, C in ipairs(base) do -- we project each point C of the base onto the half-sphere
    table.insert(Cylborder,C + math.sqrt(R^2-pt3d.abs2(C))*vecK)
end -- Cylborder is now the intersection between the half-sphere and the cylinder
-- using povray
g:Pov_new()
g:Pov_sphere(Origin, R, {name="sph", clipplane=P, render=false}) -- only declaration
g:Pov_cylinder(A-vecK, r, A+R*vecK, {name="cyl1", render=false })
g:Pov_union({"cyl1", "cyl1 rotate 90*z"}, {name="cylext", render=false})
g:Pov_cylinder(A-vecK, r-0.001, A+R*vecK, {name="cyl2", render=false })
-- renderings
g:Pov_difference({"sph","cylext"}, {color=ld.Blue, opacity=0.7})
g:Pov_union( {"cyl2", "cyl2 rotate 90*z"}, {color=ld.LightBlue,opacity=0.8,clipbox="sph"})
g:Pov_axes(Origin,{color=ld.Gold,arrows=1})
g:Pov_exec()
-- drawing
g:Dcircle3d(O,R,vecK,"line width=1.2pt"); g:Dcircle3d(A,r,vecK,"line width=1.2pt")
g:Dcircle3d(ld.rotate3d(A,-90,{Origin,vecK}), r,vecK,"line width=1.2pt")
g:Pov_show()
g:Dpolyline3d( {Cylborder, ld.rotate3d(Cylborder,-90,{Origin,vecK})}, "red, line width=1.2pt")
g:Dlabel3d("$x$",5*vecI,{pos="S"}, "$y$", 5*vecJ, {pos="SE"}, "$z$", 5*vecK, {pos="N"})
g:Show()
\end{luadraw}
```

Figure 25: Holes in a hemisphere



9) The *luadraw_log_axes* module

This module does not return anything; it adds new graphing methods to the *ld.graph* class, as well as a function in the namespace *luadraw*.

The *luadraw_log_axes* module allows you to display a semi-logarithmic or logarithmic grid and to place points on this grid or draw polygonal lines. Creating this grid modifies the 2D matrix of the graph; therefore, the drawing process takes place in three steps:

1. Initializing the grid with the **g:Beginlogview()** method.
2. Drawing with the **g:Dlogpolyline()**, **g:Dlogdots()**, **Dlogline()** and **g:Dloglabel()** methods.
3. Ending the grid with the **g:Endlogview()** method, which restores the original matrix and allows you to optionally add to the drawing in "normal" coordinates in the original window.

Initialization: g:Beginlogview()

The grid initialization and display are handled by the method:

g:Beginlogview(type, xmin, xmax, ymin, ymax, options).

- The argument ⟨*type*⟩ specifies the desired grid type. Possible values are the strings: `"logx"`, `"logy"`, or `"logxy"`.
- The four arguments ⟨*xmin*⟩, ⟨*xmax*⟩, ⟨*ymin*⟩, and ⟨*ymax*⟩ define the value ranges on the two axes. These ranges are completely independent of the graph's 2D window.
- The argument ⟨*options*⟩ is an array whose fields define the options. These are (with their default values):
  - `viewport=<current 2D window>`. This option is an array of the form $\{x_1, x_2, y_1, y_2\}$, indicating which area of the graph's 2D window should be used to draw the grid. By default, it's the entire current window (the one defined by the `window` option when the graph was created).
  - `clip=true`. Boolean indicating whether the grid should clip the drawings. Clipping is disabled after the execution of the **g:Endlogview()** method.
  - `grid={true,true }`. This option allows you to show or hide the vertical grid lines (corresponding to the x axis) and the horizontal grid lines (corresponding to the y axis).

- `unit={"","}`. This option specifies the increments for the graduations on the non-logarithmic axes. The default value is the step size (`xstep` on Ox, or `ystep` on Oy), EXCEPT when the `labeltext` option is not an empty string, in which case `unit` takes the value 1.
- `nbsubdiv={0 or 3, 0 or 3}`. This option specifies the number of Subdivisions between two main graduations on the axis. When the axis is non-logarithmic, the default value is zero; otherwise, the default value is 3.
- `xstep=nil or 1` and `ystep=nil or 1`. This option specifies the step size on the axes. If the axis is non-logarithmic, the default value is 1. If the axis is logarithmic, this option represents the difference between the first graduation (the one corresponding to the minimum value) and the second main graduation. If there are multiple decades, this option defaults to `nil`, and each decade is divided into 9 parts. If there is only one decade, this option defaults to one-tenth of the range of values.
- `defaultloglabels={2,3,5,10}`. This list of values is used to determine the values per decade on the logarithmic axes.
- `xdecadeloglabels=nil` and `ydecadeloglabels=nil`. This option applies to logarithmic axes. It specifies the list of values for the first decade; these values will provide the main graduations and labels for all decades. If the minimum value on the axis is *vmin*, then the first decade is the interval [vmin; 10 × vmin]. By default, this list of values is calculated as: {2 × vmin, 3 × vmin, 5 × vmin, 10 × vmin} (it is unnecessary to specify the first value *vmin*, as it is automatically added). The list {2, 3, 5, 10} is the default value for the `defaultloglabels` option
- `xexponent=0` and `yexponent=0`. This option applies to logarithmic axes. For example, when `xexponent=2`, all the labels on the x axis are divided by 10^2, and the string $(\times 10^2)$ will be added to the legend. The same principle applies to the y axis if it is logarithmic.
- `xaddloglabels={}` and `yaddloglabels={}`. This option applies to logarithmic axes. It allows you to add a list of values that will provide additional labels; these values must be between the minimum and maximum values of the axis.
- `tickpos={0.5,0.5}`. This option specifies the position of the tick marks relative to each axis. These are two numbers between 0 and 1. The default value of 0.5 means they are centered on the axis (0 and 1 represent the endpoints).
- `xyticks={0.2,0.2}`. This option specifies the length of the tick marks on the axes.
- `xylabelsep={0,0}`. This option specifies the distance between the labels and the tick marks on the axes.
- `originloc=<lower left corner>`. This option specifies the origin point of the tick marks for the non-logarithmic axis.
- `originnum={minimum,minimum}`. This option specifies the value at the origin point of the tick marks (tick number 0) for the non-logarithmic axis. The formula that defines the label at tick number n is:

  `(originnum + unit*n)"labeltext"/labelden.`
- `legend={"","}`. This option specifies a legend for the axes. By default, the x-axis legend is located below the axis and in the middle, and the y-axis legend is located to the left of the axis, in the middle, and is written vertically.
- `legendpos={0.5,0.5}`. This option specifies the position (between 0 and 1) of the legend relative to each axis.
- `legendsep={-0.5,-1}` . This option specifies the distance between the legend and the axis. The legend is on the opposite side of the axis from the graduations.
- `legendangle={0,90}`. This option specifies the angle (in degrees) that the legend should make with the axis.
- `legendstyle={"S","N"}`. Specifies the label style for the legends, with the value `"auto"` it is determined automatically, otherwise you can use the values: `"N"`, `"NW"`, `"W"`, `"SW"`, `"S"`, `"SE"`, `"E"`, `"NE"`.
- `labelpos={"bottom","left"}`. This option specifies the position of the labels relative to the axis. For the Ox axis, the possible values are: `"none"`, `"bottom"`, or `"top"`; for the Oy axis, it is: `"none"`, `"right"`, or `"left"`.
- `labelstyle={"S","W"}`. This option defines the style of the labels for each axis. The possible values are: `"auto"`, `"N"`, `"NW"`, `"W"`, `"SW"`, `"S"`, `"SE"`, `"E"`, `"NE"`.
- `labelangle={0,0}`. This option defines the angle of the labels in degrees relative to the horizontal for each axis.

- `labelcolor={"","}`. This option allows you to choose a color for the labels on each axis. The empty string represents the default color.
- `labelden={1,1}`. This option specifies the denominator of the labels (integer) for the non-logarithmic axis.
- `labeltext={"","}`. This option defines the text that will be added to the numerator of the labels for the non-logarithmic axis.
- `xynode_options=""`. String that will be passed as is to the `\node{}` instruction for all labels on both axes (but not for legends).
- `xnode_options=xynode_options`. String that will be passed as is to the `\node{}` instruction for all labels on the x-axis (but not for the legend).
- `ynode_options=xynode_options`. String that will be passed as is to the `\node{}` instruction for all labels on the y-axis (but not for the legend).
- `use_siunitx={siunitx,siunitx}`. This option specifies whether numeric values should be formatted using the *siunitx* package; the default value is that of the global variable `siunitx`, which is `false` by default.
- `mylabels=""`. This option allows you to apply custom labels to the non-logarithmic axis. When labels are required, the value passed to this option must be a list of the type: `{pos1,"text1",pos2,"text2",…}`. The number ⟨*pos1*⟩ represents an x-coordinate in the coordinate system `(A,step)` (*A* being the origin of the axis), which corresponds to the point with affix `A+pos1*step`, the step being either `xstep` or `ystep` along the axis.
- `gridstyle="solid"`. This option defines the line style for the primary grid.
- `subgridstyle="solid"`. This option defines the line style for the secondary grid. A secondary grid appears when there are subdivisions on one of the axes.
- `gridcolor="gray"`. This sets the color of the primary grid.
- `subgridcolor="lightgray"`. This sets the color of the secondary grid.
- `gridwidth=4`. Line thickness of the primary grid (which is 0.4pt).
- `subgridwidth=2`. Line thickness of the secondary grid (which is 0.2pt).

Drawing and Conversion Methods

- The **g:Dlogpolyline(L [, close, draw_options])** method draws the 2D polygonal line ⟨*L*⟩ on the grid; coordinate conversion is automatic. The ⟨*close*⟩ argument is a boolean indicating whether the line should be closed (`false` by default), and the ⟨*draw_options*⟩ argument is a string (empty by default) that will be passed to the *\draw* instruction.
- The method **g:Dlogdots(L [, mark_options])** draws the scatter plot contained in ⟨*L*⟩ (a complex number, a list of complex numbers, or a list of lists of complex numbers) on the grid. Coordinate conversion is automatic. The argument ⟨*mark_options*⟩ is a string (empty by default) that will be passed to the *\draw* instruction.
- The method **g:Dlogline(A, B [, draw_options])** draws the line passing through points ⟨*A*⟩ and ⟨*B*⟩ on the grid (coordinate conversion is automatic). The argument ⟨*draw_options*⟩ is a string (empty by default) that will be passed to the *\draw* instruction.
- Conversion function: **ld.Zlog(z)** where ⟨*z*⟩ is a complex number, returns the affix on the grid of the corresponding point.

End of grid usage: g:Endlogview()

The **g:Endlogview()** method restores the initial graph matrix and the 2D window.

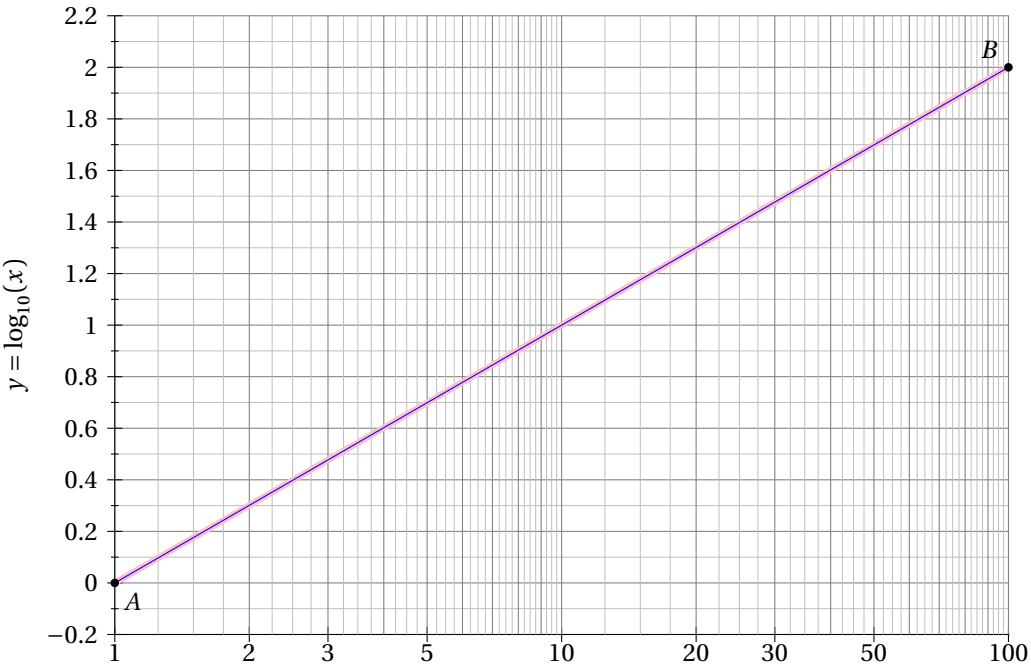
Examples

```
\begin{luadraw}{name=logx_example}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z
```

```
local log10 = math.log10

local g = ld.graph:new{ size={14,10,0} }
require 'luadraw_log_axes'
g:Beginlogview("logx", 1, 100, -0.2, 2.2, {
   nbsubdiv={3,1},
   ystep = 0.2,
   legend={"The line $(AB)$ and the curve $y=\\log_{10}(x)$ coincide.","$y=\\log_{10}(x)$"}
})
local L = ld.cartesian(function(x) return log10(x) end, 1,100)
local A, B = 1, Z(100,2)
g:Dlogline(A,B, "Pink,line width=2.4pt")
g:Dlogpolyline(L,"blue")
g:Dlogdots({A,B})
g:Dloglabel("$A$", A, {pos="SE"}, "$B$", B, {pos="NW"})
g:Endlogview()
g:Show()
\end{luadraw}
```

Figure 26: Logarithmic x-axis



The line (AB) and the curve $y = \log_{10}(x)$ coincide.

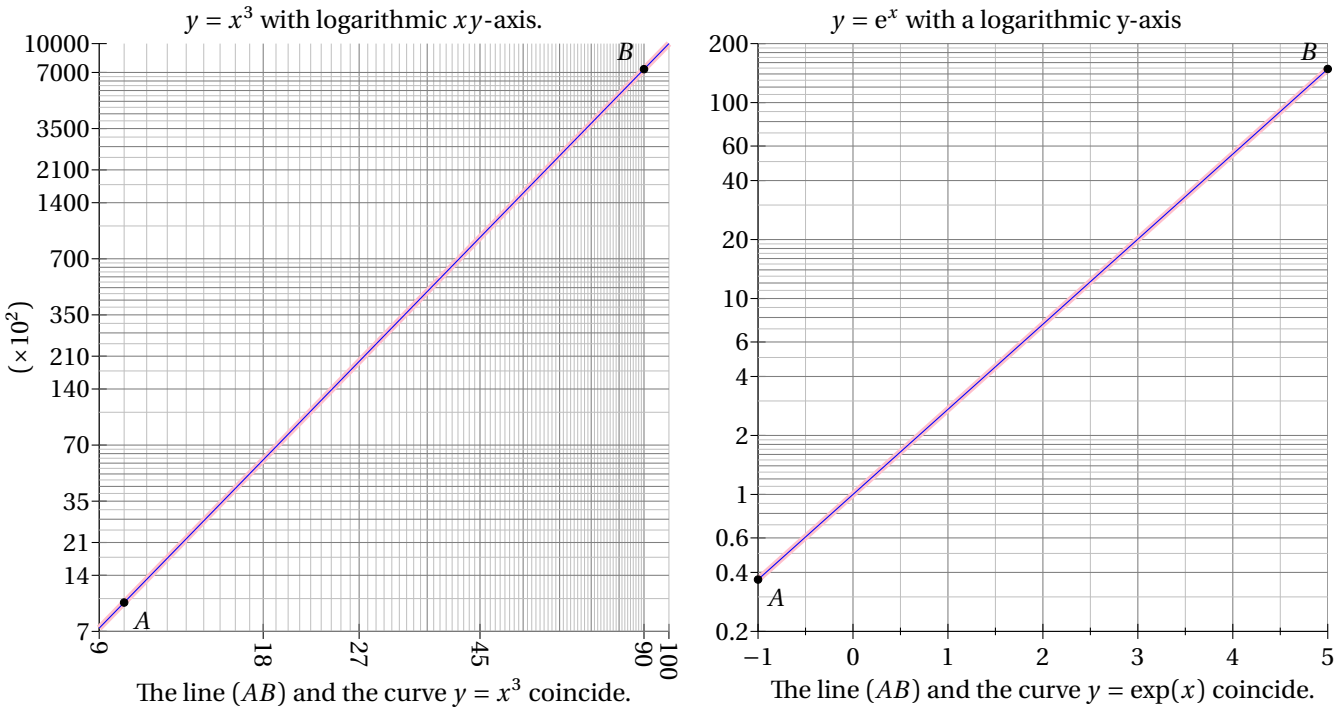
```
\begin{luadraw}{name=logy_and_logxy_example}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z
local  exp = math.exp

local g = ld.graph:new{ window={-10,10,-5,5}, size={18,10,0} }
require 'luadraw_log_axes'
g:Beginlogview("logy", -1, 5, 0.2, 200, { viewport={0.25,10,-5,4.5},
    nbsubdiv = {1,1},
    legendsep = {-0.5,-0.5},
    legend = {"The line $(AB)$ and the curve $y=\\exp(x)$ coincide.",""}
})
local L = ld.cartesian(exp, -1, 5)
local A, B = Z(-1,exp(-1)), Z(5,exp(5))
g:Dlogline(A,B, "Pink,line width=2.4pt")
g:Dlogpolyline(L,"blue")
g:Dlogdots({A,B})
g:Dloglabel("$A$", A, {pos="SE"}, "$B$", B, {pos="NW"})
```

```
g:Endlogview()

g:Beginlogview("logxy", 9, 100, 7e2, 1e6, { viewport={-10,-0.25,-5,4.5},
    nbsubdiv = {8,1},
    xaddloglabels={100},
    legendsep = {-0.5,-0.75},
    yexponent=2,
    yaddloglabels={1e6},
    labelangle={-90,0}, labelstyle={"E","W"},
    legend = {"The line $(AB)$ and the curve $y=x^3$ coincide.",""}
})
local L = ld.cartesian(function(x) return x^3 end, 9, 100)
local A, B = Z(10,1e3), Z(90,729e3)
g:Dlogline(A,B, "Pink,line width=2.4pt")
g:Dlogpolyline(L,"blue")
g:Dlogdots({A,B})
g:Dloglabel("$A$", A, {pos="SE"}, "$B$", B, {pos="NW"})
g:Endlogview()
-- drawing on the "normal" 2D window
g:Dlabel("$y=\\mathrm{e}^x$ with a logarithmic y-axis",Z(5,4.5),{pos="N"},
  "$y=x^3$ with logarithmic $xy$-axis.",Z(-5,4.5),{})
g:Show()
\end{luadraw}
```

Figure 27: Logarithmic x-axis and logarithmic xy-axis



10) The *luadraw_decorations* module

The **luadraw_decorations** module redefines certain graphics methods to add options such as adding a label, but for each of them, **the old syntax is still valid**. This module does not return anything.

2D Polygonal Lines: g:Dpolyline()

The method **g:Dpolyline(L, options)** draws the polygonal line $\langle L\rangle$ (a list of complex numbers or a list of lists of complex numbers). The argument $\langle options\rangle$ is an array whose fields define the possible options, which are (with their default values):

- `close=false`: a boolean indicating whether the line $\langle L\rangle$ should be closed.

- `clip=nil`: This option is either `nil` (default value) or a table of the form `{x1,x2,y1,y2}`. In the first case, the row is clipped by the current 2D window **after** its transformation by the 2D matrix of the graph. In the second case, the row is clipped by the window $[x_1;x_2]\times[y_1;y_2]$ **before** being transformed by the matrix of the graph.
- `draw_options=""`: String (empty by default) that will be passed as is to the *\draw* instruction.
- Options for adding a label:
  - `label=""`: label to add.
  - `anchor1d=nil`: number between 0 and 1 indicating the label's position along line $\langle L\rangle$ (0 for the beginning of the line, 1 for the end of the line).
  - `anchor=nil`: complex number representing the label's anchor point in the plane.
  - `anchor2d=cpx.Z(0.5,0.5)`: complex number representing the label's position within the bounding box $[0;1]\times[0;1]$ of line $\langle L\rangle$; therefore, by default, the label is at the center of this box.

    The order of priority is: `anchor`, `anchor1d`, `anchor2d` (if the first option is `nil`, the second is chosen; if it is also `nil`, then the third is chosen).
  - `pos="center"` indicates the label's position relative to the anchor point. It can be `"center"` (centered), `"N"` (north), `"NE"` (northeast), `"E"` (east), `"SE"` (southeast), `"S"` (south), `"SW"` (southwest), `"W"` (west), `"NW"` (northwest). By default, it is `center`, and in this case, the label is centered on the anchor point.
  - `dist=0`, the distance in cm between the label and its anchor point when `pos` is not equal to `"center"`.
  - `dir={}`, this option is a table of the form `{dirX [,dirY,dep]}` which indicates the writing direction. The three values `dirX`, `dirY`, and `dep` are three complex numbers representing three vectors. The first two indicate the writing direction, and the third indicates a displacement (translation) of the label relative to the anchor point. The vector `dep` is zero by default, and the vector `dirY`, if absent, is equal to the vector `dirX` rotated 90 degrees in the clockwise direction. By default, the `dir` option is equal to the current writing direction.
  - `node_options=""` is a string (empty by default) intended to receive options that will be directly passed to TikZ in the *node[]* instruction.
  - `showanchor=false`: with the value `true`, the anchor point is displayed along with the label's bounding box.

2D Paths: g:Dpath()

The method **g:Dpath(P, options)** draws path $\langle P\rangle$. The $\langle options\rangle$ are the same as for the **g:Dpolyline()** method (page 211) except for the options `close` and `clip`, which are ignored.

2D Lines: g:Dline()

The method **g:Dline(d, options)** draws the line $\langle d\rangle$, which is a list of the form $\{A,u\}$ where A represents a point on the line (a complex number) and u a direction vector (a non-zero complex number).

Variant: the method **g:Dline(A, B, options)** draws the line passing through the points $\langle A\rangle$ and $\langle B\rangle$ (two complex numbers).

In both cases, the $\langle options\rangle$ are the same as for the **g:Dpolyline()** method (page 211) except for the `close` and `clip` options which are not taken into account, and the following option:

- `scale=1`. The option `scale` (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.

2D Segments: g:Dseg()

The method **g:Dseg(seg, options)** where $\langle seg\rangle$=$\{A,B\}$ with A and B being two complex numbers, draws the segment $[A;B]$. The $\langle options\rangle$ are the same as for the **g:Dpolyline()** method (page 211) except for the `close` and `clip` options, which are ignored, and the following two options:

- `scale=1`. The option `scale` (which defaults to 1) is either a number (percentage) to vary the length of the displayed segment (from its midpoint), or an array of two numbers (percentages) {scaleA, scaleB} to vary the length of the displayed segment at each of its endpoints.

- `ticks="none"`: allows adding or removing tick marks (graduations) on the segment. The syntax for adding them is: `ticks={nb [, length, space, draw_options]}` where ⟨*nb*⟩ is the number of strokes to add.

2D Arc: g:Darc()

The **g:Darc(B, A, C, r, direction, options)** method draws a circular arc centered at ⟨*A*⟩ (a complex number), with radius ⟨*r*⟩, extending from ⟨*B*⟩ (a complex number) to ⟨*C*⟩ (a complex number) in the counterclockwise direction if the argument ⟨*direction*⟩ is 1, and in the opposite direction otherwise. The argument '⟨*options*⟩' is a table whose fields define the possible options. These are (with their default values):

- `arc_options=""`: string passed to the *\draw* instruction for drawing the arc.

- `sector_options=""`: string defining the fill mode for the angular sector.

- `label=""`: text to be displayed.

- `node_options=""`: string defining the options for the label.

- `pos="auto"`: specifies the label's position relative to the anchor point. Other possible values are the usual values for positioning a label: `"center"`, `"N"`, etc. By default, the anchor point is located at the intersection of the circular arc and the angle bisector.

- `dist=r`: distance between the anchor point and the center of the circle; by default, this is the radius ⟨*r*⟩ of the circle.

- `angle=0`: angle (in degrees) of rotation that the anchor point should make by default around the center of the circle.

- `rotate="none"`: indicates whether the label should be rotated around its anchor point. Other possible values are: `"auto"`, in which case the label is written along the angle bisector, or `"ortho"`, in which case the label is written perpendicular to the angle bisector.

- `ticks="none"`: allows you to add or not add markings (graduations) on the arc of a circle for an acute angle. The syntax for adding them is: `ticks={nb [, length, space, draw_options]}` where ⟨*nb*⟩ is the number of lines to add.

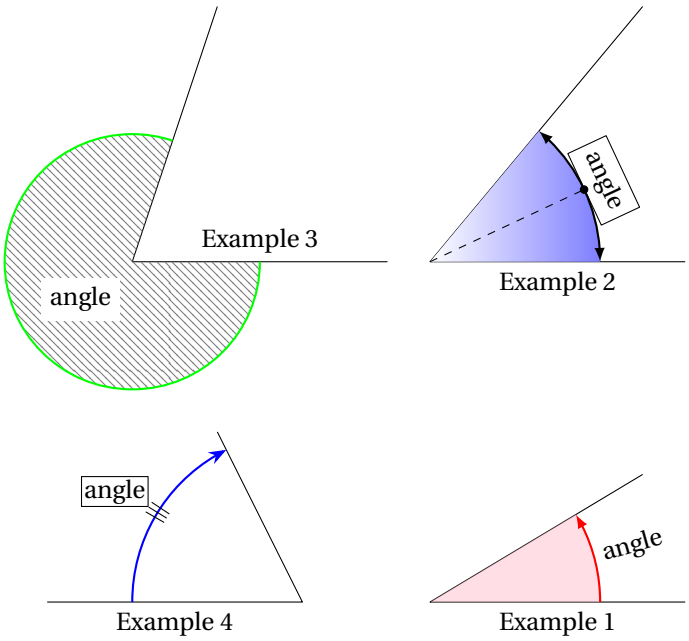
- `showanchor=false`: with the value `true`, the anchor point is drawn, as well as the bisector and the box around the label.

```
\begin{luadraw}{name=decoratedarc2D}
local ld = luadraw
local cpx = ld.cpx
local Z, i = cpx.Z, cpx.I

local g = ld.graph:new{window={-5,3,-3,5}, size={10,10}}
require 'luadraw_decorations'
g:Shift(Z(0,-2)) -- example 1
local b,a,c,r = 3, 0, 2.5+1.5*i, 2
g:Dpolyline({b,a,c})
g:Darc(b,a,c,r,1, {
    label="angle", rotate="auto",
    sector_options="fill=Pink,opacity=0.5",
    arc_options="red,line width=0.8pt,-latex"  })
g:Dlabel("Example 1",1.5,{pos="S"})
g:Shift(Z(0,4))  -- example 2
b,a,c,r = 3, 0, 2.5+3*i, 2
g:Dpolyline({b,a,c})
g:Darc(b,a,c,r,1, {
    label="angle", rotate="ortho",
    sector_options="left color=white, right color=blue!50",
```

```
    arc_options="line width=0.8pt,latex-latex",
    showanchor=true  })
g:Dlabel("Example 2",1.5,{pos="S"})
g:Shift(Z(-3.5,0))  -- example 3
b,a,c,r = 3, 0, 1+3*i, 1.5
g:Dpolyline({b,a,c})
g:Darc(b,a,c,r,-1, {
    label="angle", dist=r/2, pos="center",
    node_options="fill=white",
    sector_options="pattern=north west lines,pattern color=gray",
    arc_options="line width=0.8pt,green"   })
g:Dlabel("Example 3",1.5,{pos="N"})
g:Shift(Z(2,-4)) -- example 4
b,a,c,r = -3, 0, -1+2*i, 2
g:Dpolyline({b,a,c})
g:Darc(b,a,c,r,-1, {
    label="angle", dist=r+0.125,
    node_options="draw,inner sep=1pt",
    arc_options="line width=0.8pt,blue,-Stealth",
    ticks=3  })
g:Dlabel("Example 4",-1.5,{pos="S"})
g:Show()
\end{luadraw}
```

Figure 28: g:Darc() Method



3D Arc: g:Darc3d()

The method **g:Darc3d(B, A, C, r, direction, options)** draws a circular arc centered at ⟨*A*⟩ (3D point), with radius ⟨*r*⟩, extending from ⟨*B*⟩ (3D point) to ⟨*C*⟩ (3D point) in the clockwise direction if the argument ⟨*direction*⟩ is 1, and in the counterclockwise direction otherwise. This arc is drawn in the plane containing the three points ⟨*A*⟩, ⟨*B*⟩, and ⟨*C*⟩. When these three points are collinear, the option `normal` (non-zero 3D point) must be specified, which represents a normal vector to the plane. This plane is oriented by the cross product $\vec{AB} \wedge \vec{AC}$ or by the vector `normal` if specified. The argument ⟨*options*⟩ is an array whose fields define the possible options, which are (with their default values):

- `normal=nil`: a non-zero 3D point representing a normal vector to plane (ABC). It is optional if the three points are not collinear.
- `arc_options=""`: a string passed to the *\draw* instruction for drawing the arc.

- `sector_options=""`: a string defining the fill mode for the angular sector.
- `label=""`: the text to be displayed.
- `node_options=""`: String defining the options for the label.
- `pos="auto"`: Specifies the label's position relative to the anchor point. Other possible values are the usual values for positioning a label: `"center"`, `"N"`, `"NW"`, etc. By default, the anchor point is located at the intersection of the arc of the circle and the angle bisector.
- `dist=r`: Distance between the anchor point and the center of the circle (A). By default, this is the radius $\langle r \rangle$ of the circle.
- `angle=0`: Angle (in degrees) of rotation that the anchor point should make by default around the center of the circle (A) and in the plane of the circle.
- `rotate="none"`: indicates whether the label should be rotated around its anchor point **in the screen plane**. Other possible values are: `"auto"`, in which case the label is written along the bisector, or `"ortho"`, in which case the label is written perpendicular to the bisector.
- `rotate3d="none"`: Indicates whether the label should be rotated around its anchor point **in the plane (**(ABC)**)**. Other possible values are: `"auto"`, in which case the label is written along the angle bisector, or `"ortho"`, in which case the label is written perpendicular to the angle bisector (still in the plane (ABC)). With the value `"none"`, the label is written in the screen plane.
- `ticks="none"`: Allows you to add or not add markings (graduations) on the arc of a circle for an acute angle. The syntax for adding them is: `ticks={nb [, length, space, draw_options]}` where $\langle nb \rangle$ is the number of lines to add.
- `showanchor=false`: With the value `true`, the anchor point is drawn, along with the bisector and the box around the label.
- `bezier=true`: To draw the arc with Bézier curves, with the value `false`, the arc is a polygonal line. When TikZ adds an arrow to the end of a Bézier curve, it undergoes a slight deformation that can create an artifact when the angular sector needs to be painted; in this case, it is better to use the `bezier=false` option.

```
\begin{luadraw}{name=decorated_arcs3D}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M

local g = ld.graph3d:new{ window={-3,8,-5,8}, viewdir={"xy",0.8,60}, size={10,10} }
require 'luadraw_decorations'
local O, P = Origin, M(6.5, 6, 4)
local Px,Pz,Py,Pxz = ld.px(P), ld.pz(P), ld.py(P), ld.pxz(P)

g:Dpolyline3d({{O, 7*vecI}, {O, 7*vecJ}, {O, 6*vecK}}, "-Stealth,solid,black, line width=1pt")
g:Dlabel3d("$x$", 7*vecI, {pos="E"}, "$y$", 7*vecJ, {pos="N"}, "$z$", 6*vecK, {pos="S"})
g:Dpolyline3d({{P,Py}, {P,Pxz}, {Pxz,Px}, {O,Pxz}, {Pxz,Pz}}, "dashed,blue,thick")
g:Dseg3d({O, P}, "-latex,RosyBrown,line width=4pt")
g:Dpolyline3d({{O, 2.5*vecI}, {O, 2.5*vecJ}, {O, 2.5*vecK}}, "-latex,cyan,line width=3pt")

g:Darc3d(Px, O, P, 4, 1, {
    arc_options = "-Stealth,blue,ultra thick",
    sector_options = "fill=blue,opacity=0.2",
    label = "$\\alpha$", node_options="blue,scale=1.25",
    rotate = "ortho", pos="N" })

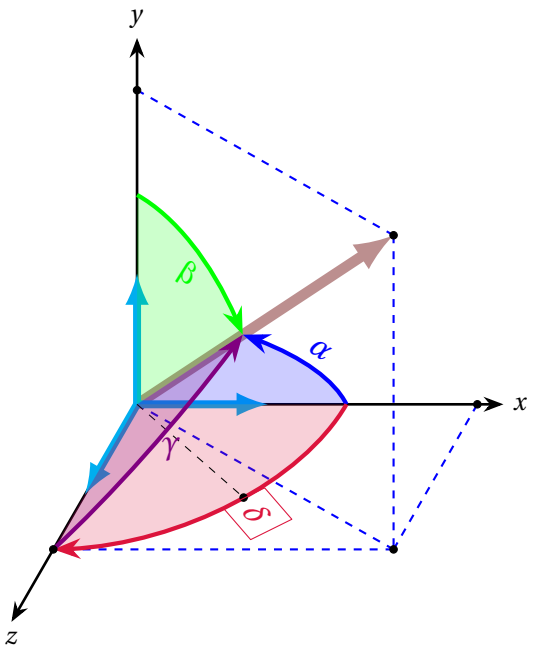
g:Darc3d(Py, O, P, 4, 1, {
    arc_options = "-Stealth,green,ultra thick",
    sector_options = "fill=green,opacity=0.2",
    label = "$\\beta$", node_options="green,scale=1.25",
```

```
    pos = "S", rotate3d = "ortho"  })

g:Darc3d(Px, O, Pz, 4, 1, {
    arc_options = "-Stealth,Crimson,ultra thick",
    sector_options = "fill=Crimson,opacity=0.2",
    label = "$\\delta$", node_options="Crimson,scale=1.25",
    rotate3d = "auto", angle=-5,
    showanchor=true  })

g:Darc3d(Pz, O, P, 4, 1, {
    arc_options = "-Stealth,violet,ultra thick",
    sector_options = "fill=violet,opacity=0.2",
    label = "$\\gamma$", node_options="violet,scale=1.25",
    pos="E"  })
g:Ddots3d({P,Pxz,Px,Py,Pz})
g:Show()
\end{luadraw}
```

Figure 29: Method *g:Darc3d()*



Angle 3D: g:Dangle3d()

The method **g:Dangle3d(B, A, C, options)** draws angle BAC with a parallelogram (only two sides are drawn). The parallelogram lies in the plane defined by points $\langle A\rangle$, $\langle B\rangle$, and $\langle C\rangle$, so these points do not need to be collinear. The argument $\langle options\rangle$ is an array whose fields define the possible options, which are (with their default values):

- `radius=0.25`: defines the length of the parallelogram's sides.
- `arc_options=""`: a string passed to the *\draw* instruction to draw the parallelogram's outline (only two sides are drawn).
- `sector_options=""`: string defining the parallelogram's fill mode.
- Adding a label: the options are the same as for the **g:Dpolyline()** method (page 211) except for the `close` option, which is ignored.

Conclusion :

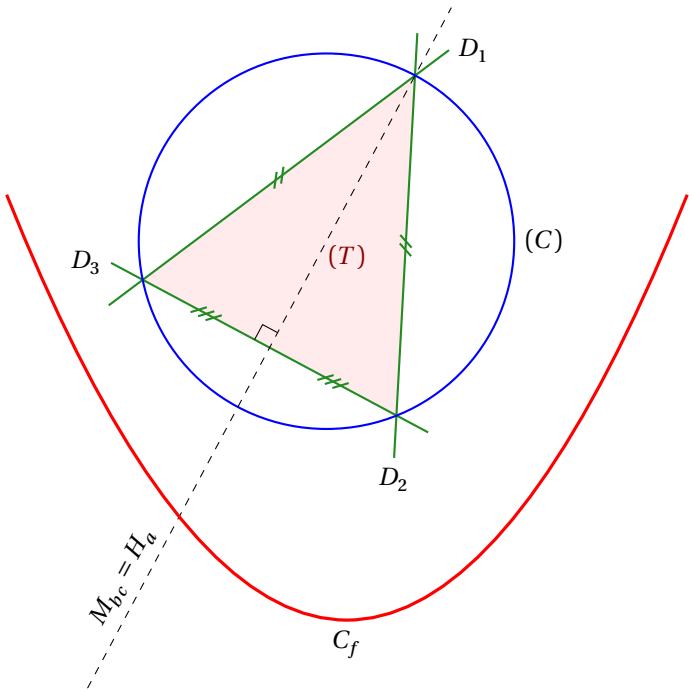
1. In all 2D or 3D line drawing methods that call one of the previous methods, for those that have an argument ⟨*draw_options*⟩, which is normally a string, this can be replaced by an array ⟨*options*⟩ as in the new method **g:Dpolyline(L, options)**.

2. In all 2D or 3D line drawing methods that already have an argument ⟨*options*⟩ (or ⟨*args*⟩), and that have one of these options called `draw_options`, which is normally a string, this can be replaced by an array ⟨*options*⟩ as in the new method **g:Dpolyline(L, options)**.

```
\begin{luadraw}{name=decorations2d}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{size={10,10}}
require 'luadraw_decorations'
local A, B = Z(1,4), Z(-3,1)
local C = ld.rotate(B,50,A)
local A1 = (B+C)/2
g:Dpolyline({A,B,C}, {close=true, draw_options="draw=none,fill=pink,fill opacity=0.3",
    label="$(T)$", anchor=(A+B+C)/3, pos="E",node_options="DarkRed"})
g:Lineoptions(nil,"ForestGreen",8)
g:Dseg({A,B}, {scale=1.25,ticks=2,label="$D_1$",anchor1d=0,pos="E"})
g:Dseg({A,C}, {scale=1.25,ticks=2,label="$D_2$",anchor1d=1,pos="S"})
g:Dseg({C,B}, {scale=1.25,label="$D_3$",anchor1d=1,pos="W"})
g:Dmarkseg(B,A1,3); g:Dmarkseg(A1,C,3); g:Dangle(B,A1,A,0.25,"black,thin")
g:Dcircle({A,B,C}, {label='$(C)$', anchor1d=0, pos="E", draw_options="blue"})
g:Dmed(B,C,{label="$M_{bc}=H_a$",anchor1d=0.15,pos="N",dir=cpx.I*(C-B), draw_options="dashed,black,thin"})
g:Dcartesian( function(x) return (x/2)^2-4 end,{x={-5,5},
    draw_options={label="$C_f$", anchor2d=Z(0.5,0), pos="S", draw_options="red,line width=1.2pt"} })
g:Show()
\end{luadraw}
```

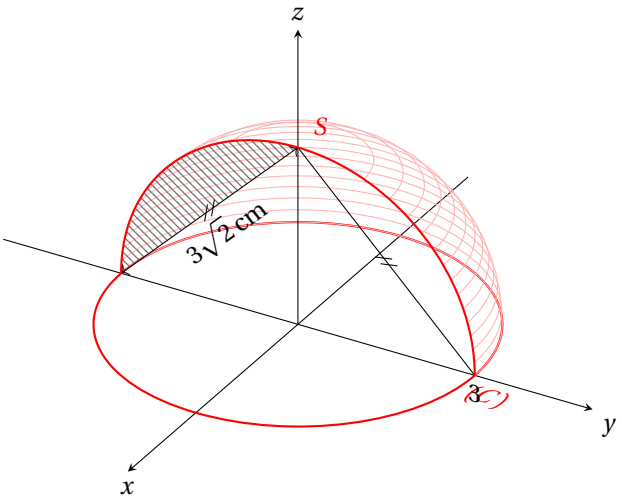
Figure 30: 2D Decorations




```
\begin{luadraw}{name=decorations3d}
local ld = luadraw
local pt3d = ld.pt3d
local Origin, vecI, vecJ, vecK, M, Z = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK, pt3d.M, ld.cpx.Z
```

```
local g = ld.graph3d:new{size={10,10}}
require 'luadraw_decorations'
g:Dline3d(-5*vecI,5*vecI, {label="$x$",anchor=5*vecI,pos="S",draw_options="-stealth"},true)
g:Dline3d(-5*vecJ,5*vecJ, {label="$y$",anchor1d=1,pos="SE",draw_options="-stealth"},true)
g:Dcircle3d(Origin, 3, vecK, {label="$(C)$",anchor1d=0,pos="SE",
    dir={vecJ,-vecI}, draw_options="red,thick", node_options="red"})
local nb = 15
local H = ld.linspace(0,3,nb)
for k = 1, nb-1 do
    local z = H[k]
    local r = math.sqrt(9-z^2)
    g:Darc3d(M(0,-r,z),z*vecK,M(0,r,z),r,-1,vecK,"thin, red!30")
end
g:Dpath3d({-3*vecJ,Origin,3*vecK,3,-1,vecI,"ca",3*vecK,"l","cl"},
    "draw=none,pattern color=black!60,pattern=north west lines")
g:Dseg3d({Origin,5*vecK},
    {label="$z$",anchor2d=Z(0,1),pos="N",draw_options="-stealth"})
g:Dseg3d({-3*vecJ,3*vecK}, {label="$3\\sqrt 2$\\,cm", pos="S", dir={M(0,1,1),M(0,-1,1)},
    ticks=2, draw_options="<->"})
g:Dseg3d({3*vecJ,3*vecK},
    {ticks=2, label="$3$", anchor=3*vecJ,pos="S",dir={vecJ,vecK}})
g:Dpath3d({-3*vecJ,Origin,3*vecJ,3,-1,vecI,"ca"},
    {label="$S$", anchor1d=0.5, pos="N",dist=0.15,draw_options="red,thick", node_options="red"})
g:Show()
\end{luadraw}
```

Figure 31: 3D Decorations



11) The *luadraw_coils_chains* module

The module *luadraw_coils_chains* allows you to draw springs (two types) and chains (two types). It adds new graphing methods to the *ld.graph* class but returns nothing.

Springs

- The method **g:Dcoil(list, radius, options)** allows you to draw a spring. The argument ⟨*radius*⟩ is the radius of the spring. The argument ⟨*list*⟩ has two possible forms:
  - Form 1: ⟨*list*⟩ = {a1, n1, a2, n2,..., aN}, where a_1, ..., a_N are points (complex numbers), n_1 is an integer,

representing the number of turns between a_1 and a_2, n_2 is the number of turns between a_2 and a_3, and so on.

– Form 2: ⟨*list*⟩ = {C, n}, where C is a polygonal line (for example, a curve) and n is an integer representing the total number of turns. The spring will be drawn along the curve C.

The ⟨*options*⟩ argument is a table whose fields are the options that will modify the aesthetic appearance of the spring. These options are (with their default values):

– `direction=1`: Direction of rotation of the coils (1 for clockwise, −1 for counterclockwise).

– `wire_dia=<2*radius/15>`: Wire diameter.

– `color="gray"`: Fill color of the coils.

– `colorB=color`: Fill color of the ”back” coils; by default, this is the same value as the `color` option.

– `border=<current color>`: Color of the coil outline.

– `border_width=<current thickness>`: Thickness of the coil outline.

– `tension=1`: The coils are slightly curved to give a 3D effect; this option allows you to increase or decrease the curvature. With a value of 0, there is no curvature.

– `start_angle=nil`, `end_angle=nil`: angles (in degrees) for the first and last half-turns. These angles are determined automatically, but they can be modified with these options.

– `wireframe=false`: with the value `true`, the turns are filled with a solid color, but with the value `false` (default), they are filled with a gradient, specifically with the following formula:

```
left color=<color>!<leftC>, right color=<color>!<rightC>, middle color=<color>!<midC>
```

with the three coefficients being options: `leftC=100`, `rightC=50`, `midC=10`.

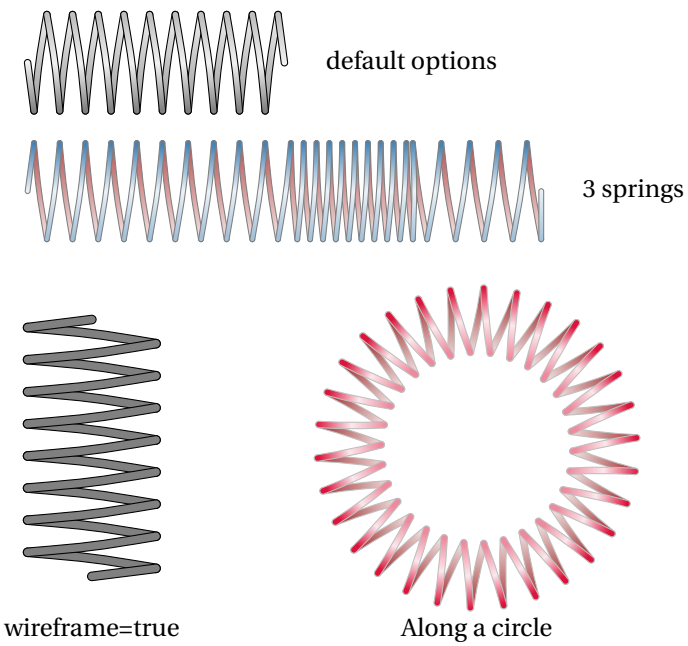
– `holes=false`: With the value `true`, a dot is drawn at the ends of the spring.

– `reverse=false`: Reverses the display order of the spring when it is made up of multiple pieces.

```
\begin{luadraw}{name=Dcoil_examples}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{window={-5.6,5,-5,5}, size={10,10}}
require 'luadraw_coils_chains'
local a, b, c, d = Z(-5,4), Z(-1,4), Z(1,4), Z(3,4)
g:Dcoil({a,10,b}, 0.75, {}); g:Dlabel("default options", b+0.5, {pos="E"})
g:Shift(Z(0,-2))
g:Dcoil({a,10,b,10,c,5,d}, 0.75, {color="SteelBlue", direction=-1, wire_dia=0.075,
    border="gray", colorB="Brown"})
g:Dlabel("3 springs", d+0.5, {pos="E"})
g:Shift(Z(0,-2))
g:Dcoil({Z(-4,4),8, Z(-4,0)}, 1, {wireframe=true}); g:Dlabel("wireframe=true",Z(-4,-0.5),{pos="S"})
local C = ld.circle(Z(2,2),2)
g:Dcoil( {C,30}, 0.5, {color="Crimson", wire_dia=0.1, border="lightgray", colorB="DarkRed"})
g:Dlabel("Along a circle",Z(2,-0.5),{pos="S"})
g:Show()
\end{luadraw}
```

Figure 32: The *g:Dcoil()* Method



- The method **g:Dcoil2(list, radius, options)** allows you to draw a wire spring. The argument ⟨*radius*⟩ is the radius of the spring. The argument ⟨*list*⟩ has two possible forms:
  - Form 1: ⟨*list*⟩ = {a1, n1, a2, n2,..., aN}, where a_1, ..., a_N are points (complex numbers), n_1 is an integer, representing the number of turns between a_1 and a_2, n_2 is the number of turns between a_2 and a_3, and so on.
  - Form 2: ⟨*list*⟩ = {C, n}, where C is a polygonal line (for example, a curve) and n is an integer representing the total number of turns. The spring will be drawn along the curve C.

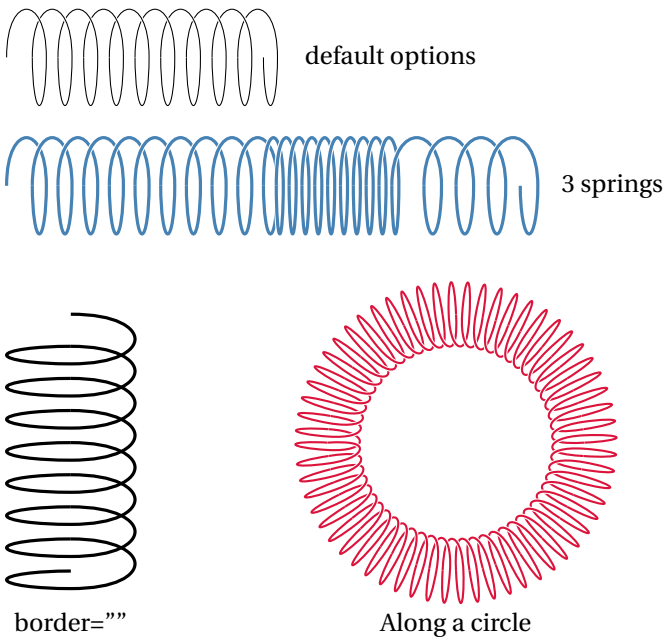
  The argument ⟨*options*⟩ is a table whose fields are the options that will modify the aesthetic appearance of the spring. These options are (with their default values):
  - `direction=1`: direction of rotation of the coils (1 for clockwise, −1 for counterclockwise).
  - `color=<current color>`: color of the coils.
  - `width=<current thickness>`: thickness of the coils (in tenths of a point).
  - `border="white"`: this option is either an empty string (`""`) or a string representing a color. In the latter case, the drawing uses the *double* option of TikZ with this color as the border, allowing the "front" coils to be seen passing in front of the "back" coils, like a 3D spring.
  - `border_width=<current thickness>`: border thickness (used when the `border` option is not equal to `nil`).

```
\begin{luadraw}{name=Dcoil2_examples}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{window={-5.6,5,-5,5}, size={10,10}}
require 'luadraw_coils_chains'
local a, b, c, d = Z(-5,4), Z(-1,4), Z(1,4), Z(3,4)
g:Dcoil2({a,10,b}, 0.75, {}); g:Dlabel("default options", b+0.5, {pos="E"})
g:Shift(Z(0,-2))
g:Dcoil2({a,10,b,10,c,4,d}, 0.75, {color="SteelBlue", direction=-1, width=12})
g:Dlabel("3 springs", d+0.5, {pos="E"})
g:Shift(Z(0,-2))
g:Dcoil2({Z(-4,4),8, Z(-4,0)}, 1, {border="", width=12}); g:Dlabel('border=""',Z(-4,-0.5),{pos="S"})
local C = ld.circle(Z(2,2),2)
```

```
g:Dcoil2( {C,60}, 0.5, {color="Crimson", width=8, border="white"})
g:Dlabel("Along a circle",Z(2,-0.5),{pos="S"})
g:Show()
\end{luadraw}
```

Figure 33: The *g:Dcoil2()* Method



Chains

- The **g:Dchain(list, link_length, options)** method allows you to draw a chain. The ⟨*list*⟩ argument is a list of at least two complex numbers; these are the points through which the chain will pass. The ⟨*link_length*⟩ argument is the length of a link in the chain, but this will be adjusted to have a whole number of links. The ⟨*options*⟩ argument is an array whose fields are the options that will modify the aesthetic appearance of the chain. These options are (with their default values):
  - `width=<(calculated length of a link)/1.618>` : width of a link, with the value `width="circle"`, the link will be circular.
  - `wire_dia=<width/4>`: wire diameter.
  - `color="gray"`: color to fill the links.
  - `colorB=color`: Second fill color for the links; by default, it's the same value as the `color` option.
  - `border=<current color>`: Color of the link outline.
  - `border_width=<current thickness>`: Thickness of the link outline.
  - `wireframe=false`: With the value `true`, the links are filled with a solid color, but with the value `false` (the default value) they are filled with a gradient, specifically with the following formula:

    `left color=<color>!<leftC>, right color=<colorB>!<rightC>, middle color=<color>!<midC>`

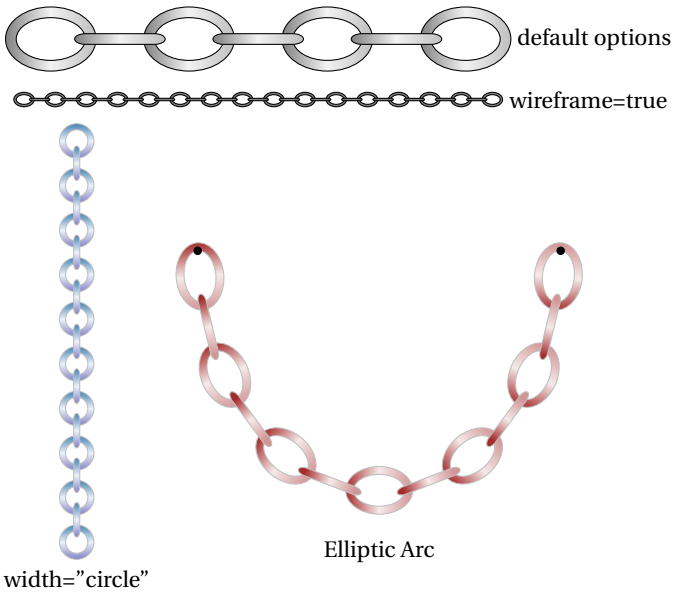
    with the three coefficients being options: `leftC=100`, `rightC=50`, `midC=10`.

```
\begin{luadraw}{name=Dchain_examples}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{window={-5.25,6,-5,5.25}, size={10,10}}
require 'luadraw_coils_chains'
g:Labelsize("small")
```

```
local a, b = Z(-5,4.5), Z(3,4.5)
g:Dchain({a,b}, 1, {}); g:Dlabel(" default options", b, {pos="E", dist=0.1})
g:Shift(Z(0,-1))
g:Dchain({a,b}, 0.25, {wireframe=true}); g:Dlabel("wireframe=true", b, {pos="E"})
g:Shift(Z(0,1))
a, b = Z(-4,3), Z(-4,-4)
g:Dchain({a,b}, 0.35, {color="SteelBlue", colorB="DarkBlue", width="circle", wire_dia=0.1, border="lightgray"})
g:Dlabel('width="circle"', b, {pos="S",dist=0.1})
a, b = Z(-2,1), Z(4,1)
local C = ld.ellipticarc(a,Z(1,1),b,3,4,1)
g:Dchain(C, 0.75, {color="Brown", border="lightgray"}); g:Ddots({a,b})
g:Dlabel("Elliptic Arc",Z(1,-3),{pos="S",dist=0.5})
g:Show()
\end{luadraw}
```

Figure 34: The *g:Dchain()* Method



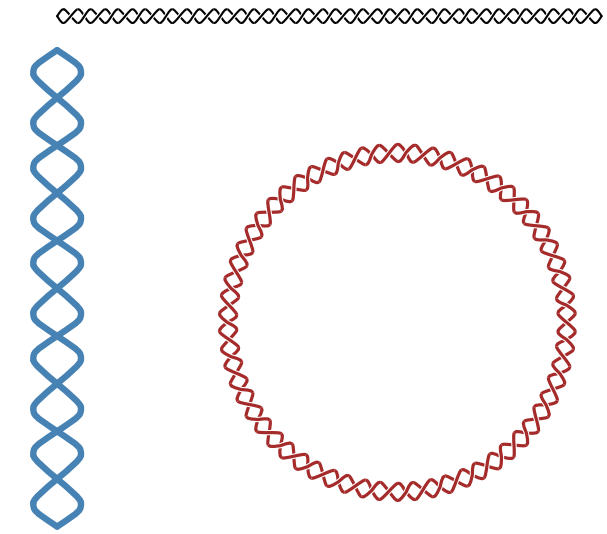
- The **g:Dchain2(list, R, options)** method draws a chain as two intersecting wavy wires. The ⟨*list*⟩ argument is a list of at least two complex numbers, representing the points through which the chain will pass. The ⟨*R*⟩ argument is the radius of the chain (the height of a wave). The ⟨*options*⟩ argument is an array whose fields are the options that will modify the aesthetic appearance of the chain.
  - `direction=1`: direction of rotation of the wires (1 for clockwise, −1 for counterclockwise).
  - `color=<current color>`: color of the chain.
  - `width=<current thickness>`: thickness of the wires (in tenths of a point).
  - `border="white"`: This option is either an empty string (`""`) or a string representing a color. In the latter case, the drawing uses TikZ's *double* option with that color as the border, thus distinguishing the wire in front from the one behind.
  - `border_width=<current thickness>`: Border thickness (used when the `border` option is not equal to `nil`).

```
\begin{luadraw}{name=Dchain2_examples}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z

local g = ld.graph:new{window={-5,5,-4.5,5}, size={10,10}}
require 'luadraw_coils_chains'
g:Labelsize("small")
local a, b = Z(-4,4.5), Z(4,4.5)
```

```
g:Dchain2({a,b}, 0.1, {width=8})
g:Shift(Z(0,1))
a, b = Z(-4,3), Z(-4,-4)
g:Dchain2({a,b}, 0.35, {color="SteelBlue", width=24, border=""})
g:Dlabel('border=""', b, {pos="S",dist=0.1})
a, b = Z(-2,1), Z(4,1)
local C = ld.circle(Z(1,-1), 2.5)
g:Dchain2(C, 0.125, {color="Brown", width=12})
g:Dlabel("Along a circle",Z(1,-4),{pos="S",dist=0.1})
g:Show()
\end{luadraw}
```

Figure 35: The *g:Dchain2()* Method



12) The *luadraw_pdfliteral* module

This module does not return a value; instead, it adds new graphical methods to the *ld.graph* and *ld.graph3d* classes.

When the volume of data TikZ needs to render becomes very large—such as a vast number of surface facets, numerous segments or paths in a tiling, or a dense scatter plot—the sheer number of macros employed by TikZ can lead to prohibitively long compilation and rendering times.

One way to circumvent this issue and significantly reduce compilation time is to generate such graphics directly in PDF format, bypassing TikZ macros. This does not mean abandoning TikZ entirely, as it remains highly efficient for tasks like style management. The approach involves remaining within a *tikzpicture* environment but—specifically when plotting a large number of segments with no style changes other than color (and notably no opacity changes) and using only solid fills (or no fill at all)—writing the drawing instructions directly in native PDF syntax rather than using the *\draw* command.

The g:Dliteralpath() method

The **g:Dliteralpath(path1, args1, path2, args2, ...)** method allows you to draw 2D paths (⟨*path1*⟩, ⟨*path2*⟩, ...) in PDF format. Each path must be followed by a table (⟨*args1*⟩, ⟨*args2*⟩, ...) containing options for that specific path. These options are:

- `fill=<table {r,g,b}>`: fill color; the default is the current color. Setting `fill="none"` disables the fill; otherwise, the color must be a table in the format $\{r, g, b\}$ (with r, g, and b between 0 and 1), and the fill is **solid** (using the current opacity)—other fill styles have no effect.
- `draw=<table {r,g,b}>`: stroke color; the default is the current color. Setting `draw="none"` disables the stroke (but

not the fill); otherwise, the color must be a table in the format $\{r, g, b\}$ (with r, g, and b between 0 and 1), and the stroke is drawn using the current style and opacity.

- `width=<thickness in tenths of a point>`: line thickness; the default is the current thickness.

NB: Options selected for a path also apply to subsequent paths unless they are modified.

The g:Dliteralpolyline() method

The **g:Dliteralpolyline(polyline1, args1, polyline2, args2, ...)** method allows for drawing the 2D polylines ⟨*polyline1*⟩, ⟨*polyline2*⟩, …in PDF. Each polyline must be followed by a table: ⟨*args1*⟩, ⟨*args2*⟩, …. These tables represent the options for each polyline; they are the same as those for the **g:Dliteralpath()** method, plus the following option:

- `close=<boolean>`: indicates whether the polyline should be closed (`false` by default).

NB: the options chosen for a polyline also apply to subsequent ones unless they are modified.

The g:Dliteraldots() method

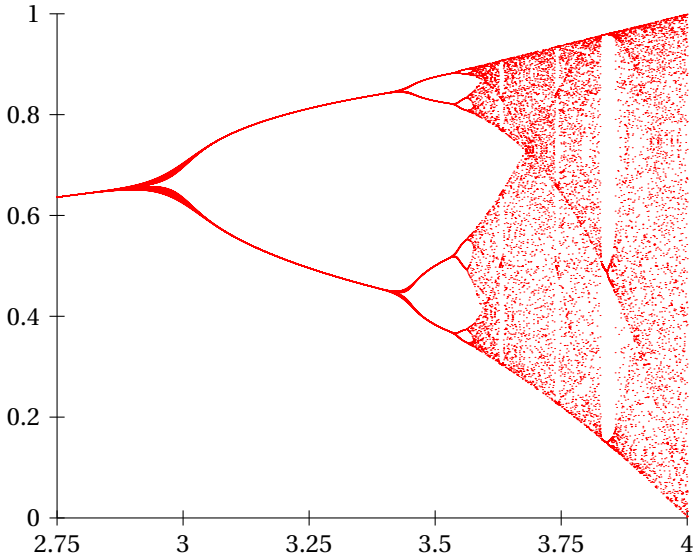
The **g:Dliteraldots(dots1, args1, dots2, args2, ...)** method allows you to draw lists of 2D points (complex numbers) ⟨*dots1*⟩, ⟨*dots2*⟩, …in PDF format. Each list of points must be followed by a table: ⟨*args1*⟩, ⟨*args2*⟩, …. These tables represent the options, which are:

- `color=<table {r,g,b}>`: color of the points; by default, this is the current color of the plot. The color must be a table in the format $\{r, g, b\}$ (with r, g, and b between 0 and 1), and the plot is drawn with a solid line and the current opacity. The points are circular.
- `width=<thickness in tenths of points>`: line thickness, this is the current default thickness. This thickness determines the diameter of the points.

NB: the options chosen for a polygonal line also apply to subsequent lines if they are not modified.

```
\begin{luadraw}{name=bifurcation}
local ld = luadraw
local cpx = ld.cpx
local Z = cpx.Z
local g = ld.graph:new{ window={2.65,4,-0.15,1}, size={10,10} }
require 'luadraw_pdfliteral'
local dots = {}
for r = 2.75, 4, 0.001 do
    local u = 0.5
    for k = 1, 25 do u = r*u*(1-u) end -- transition
    for k = 1, 25 do u = r*u*(1-u); table.insert(dots, Z(r,u)) end
end
g:Daxes({Z(2.75,0),0.25,0.2}, {limits={{2.75,4},{0,1.001}}, originpos={"center","center"} })
g:Dliteraldots(dots, {width=1, color=ld.Red}) -- 31275 points
g:Show()
\end{luadraw}
```

Figure 36: Bifurcation diagram of the sequence $u_{u+1} = r u_n(1-u_n)$



The g:Dliteralfacet() method

The **g:Dliteralfacet(F1, args1, F2, arg2, ...)** method performs the same function as the **g:Dmixfacet()** method, except that the facets are drawn using native PDF commands. The arguments ⟨*F1*⟩, ⟨*F2*⟩, ..., are lists of facets (lists of lists of 3D points), and ⟨*args1*⟩, ⟨*arg2*⟩, ..., are tables representing the options; these are:

- `mode=ld.mShaded` : defines the rendering mode. Possible values are:
  - `ld.mWireframe` : wireframe mode; only the edges are drawn. When the `usepalette` option is not `nil`, the color of each edge is calculated based on the palette (using the same method as for filled facets).
  - `ld.mFlat` or `ld.mFlatHidden` : faces are drawn in a solid color, and edges are also drawn.
  - `ld.mShaded` or `ld.mShadedHidden` : faces are drawn with shading based on their inclination, and edges are also drawn.
  - `ld.mShadedOnly` : faces are drawn with shading based on their inclination, but edges are not drawn.
- `contrast=1` : this number allows you to increase or decrease the color shading of the facets in `ld.mShaded`, `ld.mShadedHidden`, and `ld.mShadedOnly` modes.
- `edgecolor=<current color>` : string defining the color of the edges.
- `edgewidth=<current thickness>` : line thickness of the edges in tenths of a point.
- `backcull=false` : if set to `true`, facets considered invisible (normal vector not pointing towards the observer) are not displayed. This option is useful for convex objects as it reduces the number of facets to be drawn.
- `clip=false` : if set to `true`, facets are clipped by the 3D window.
- `twoside=true` : boolean indicating whether the two sides of the facets (inside and outside) are distinguished; if set to `true`, the two sides will not have exactly the same color.
- `reverse=false` : with the value `true` the orientation of the facets is reversed.
- `color=<table {r,g,b}>` : fill color for the facets; the default is `ld.White`. The color must be a table in the format $\{r, g, b\}$ (with r, g, and b between 0 and 1).
- `usepalette=nil` : this option allows you to specify a color palette for rendering the facets, as well as a calculation mode; the syntax is `usepalette={palette,mode[,minmax]}`, where ⟨*palette*⟩ refers to a table of colors—themselves tables in the format $\{r, g, b\}$, where r, g, and b are numbers between 0 and 1. The ⟨*mode*⟩ argument can be:

– one of the strings: `"x"`, `"y"`, or `"z"`. In the first case, for example, facets with the minimum x-coordinate at their center of gravity use the first color in the palette, while those with the maximum x-coordinate use the last color; for the others, the color is calculated based on the center of gravity's x-coordinate via linear interpolation.

– a function: $\langle mode \rangle : f \mapsto \text{mode}(f) \in \mathbb{R}$, where f denotes a facet (a list of 3D points). Facets with the minimum value use the first color in the palette, those with the maximum value use the last color, and the color for the others is calculated via linear interpolation.

The $\langle minmax \rangle$ argument is a table of two numbers; these two numbers are used as the minimum and maximum values for the $\langle mode \rangle$ argument. If $\langle minmax \rangle$ is omitted, these two values are calculated automatically.

Some remarks

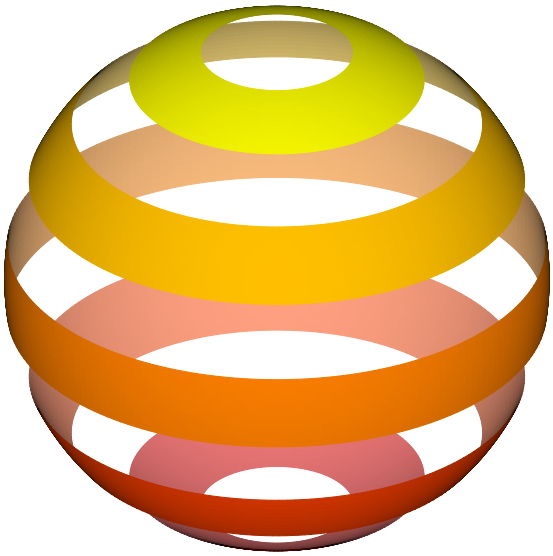
- Options selected for a list of facets also apply to subsequent lists unless modified.
- The drawing is rendered using default opacity and line style settings. These can be modified **before** execution using the methods **g:Fillopacity()** and **g:Linestyle()**.

```
\begin{luadraw}{name=Dliteralfacet}
local ld = luadraw
local M, Ms = ld.pt3d.M, ld.pt3d.Ms
local g = ld.graph3d:new{ size={10,10} }
require 'luadraw_pdfliteral'
local R, nb = 4, 5
local phi2, phi11, dphi = math.pi, nil, math.pi/(2*nb+1)

local sph_strip = function(p1,p2)
    return ld.surface( function(u,v) return Ms(R,u,v) end, -math.pi, math.pi, p1, p2,{201,11})
end

local colors = ld.getpalette( ld.palAutumn, nb,true)
local S = {}
for k = 1, nb do
    phi1 = phi2 -dphi; phi2 = phi1 - dphi
    table.insert(S, sph_strip(phi1,phi2))
    table.insert(S, {color=colors[k], mode=ld.mShadedOnly})
end
g:Dliteralfacet( table.unpack(S) ) -- 10^4 facets
g:Show()
\end{luadraw}
```

Figure 37: Spherical bands



Note: It is possible to achieve the same result without drawing the facets, but it is more technical.

13) The *luadraw_linprog* module

This module does not return a value; instead, it adds new functions and graphical methods to the *ld.graph* and *ld.graph3d* classes. Its purpose is to solve and graphically represent linear programming problems within a specified window.

This module loads the *luadraw_decorations* module.

In 2D

The goal is to represent systems of constraints of the form $ax + by < c$ or $ax + by > c$ within a given window; the solution is a list of complex numbers representing the convex solution polygon (within the specified region).

Based on this solution, one can seek to optimize a functional of the type $(x, y) \mapsto \alpha x + \beta y$ over this polygon—that is, to determine its minimum and maximum values on the polygon.

The calculation The function **ld.linprogSolve(constraints, x1, x2, y1, y2, objectives)** performs the calculations[4] and returns a sequence consisting of two items:

1. the solution polygon (a list of complex numbers),
2. the list of solutions for each objective function.

The ⟨*constraints*⟩ argument is the list of constraints; each constraint is a string in the form `"a*x+b*y>c"` or `"a*x+b*y<c"`, where a, b, and c are numerical values, and x and y are two letters (the multiplication symbol is mandatory).

The arguments ⟨*x1*⟩, ⟨*x2*⟩, ⟨*y1*⟩, and ⟨*y2*⟩ define the solution window: $[x_1; x_2] \times [y_1; y_2]$.

The ⟨*objectives*⟩ argument is the list of objective functions to be optimized (it may be empty); each objective function is a string in the form `"u*x+v*y"`, where u and v are numerical values. The solution for an objective function is a table with the following fields:

- `expr`: contains the expression of the objective function,
- `min`: contains the numerical value of the minimum,
- `mindot`: contains the point where the minimum is reached (complex number),

[4] This is a geometric solution method. The chosen window is a polygon that is cut out with the different lines corresponding to the constraints.

- `minline`: contains the line passing through the point where the minimum is reached,
- `max`: contains the numerical value of the maximum,
- `maxdot`: contains the point where the maximum is reached (complex number),
- `maxline`: contains the line passing through the point where the maximum is reached,

Graphical representation Depending on convention, some people draw and shade the solution polygon and then add the optimization lines for each objective function (if any); others prefer to shade the half-plane containing the points that do not satisfy each constraint, leaving the solution polygon as the unshaded area.

- The method **g:DlinprogRegion(constraints [, options, objectives, objectives_options])** solves the system of constraints contained in the list ⟨*constraints*⟩ and draws the solution polygon, as well as any optimizations. The ⟨*options*⟩ argument is a table of possible options for this representation; these are (with default values):
  - `view=<default 2D window>`, a list in the form $\{x_1, x_2, y_1, y_2\}$ representing the viewing window: $[x_1; x_2] \times [y_1; y_2]$.
  - `draw_options=""`, a string containing drawing options for the solution polygon (string passed to *\draw*).
  - `lines=false`; if set to `true`, the line corresponding to each constraint (with equation $ax + by = c$) is drawn with a label in the middle of the line, oriented along the line's direction.
  - `color=<current color>`, the color used to draw the lines and their labels (if `lines=true`).
  - `width=<current thickness>`, the line thickness in tenths of a point for the lines (if `lines=true`).
  - `style=<current style>`, the line style (if `lines=true`).
  - `dots=false`; if set to `true`, the vertices of the solution polygon are drawn.
  - `mark_options=""`; a string containing drawing options for the vertices when `dots=true` (this string is passed to *\draw*).
  - `outside=false`; if set to `true`, the area outside the solution polygon is filled instead of the interior.
  - `out=nil`; if a list-type variable (table) is assigned to this option, then after the method executes, the table will contain the list of vertices of the solution polygon, followed by the list of solutions for each objective function.
  - In addition to these options, the options from the *luadraw_decorations* module for polygonal lines are also available.

  The ⟨*objectives*⟩ argument is the list of functionals to optimize and plot (lines achieving the minimum and/or maximum).

  The ⟨*objectives_options*⟩ argument is the table of available options for this representation; these are (with default values):

  - `color=<current color>`, color for plotting the line and its label.
  - `width=<current thickness>`, line thickness in tenths of a point.
  - `style=<current style>`, line style.
  - `sense="minmax"`, other possible values: "min" or "max". This option allows you to select which solution line to plot (both by default).
  - `dots=false`; if set to `true`, the solution point(s) are plotted (where the optimization occurs).
  - `mark_options=""`, string containing drawing options for the solution point(s) when `dots=true` (string passed to *\draw*).
  - The following options come from the *luadraw_decorations* module for lines:
    * `label=<auto>`, a default label is displayed with the line (or lines); it consists of the functional's expression along with the min or max value, as applicable.
    * `node_options=<auto>`, string containing options for the label; by default, it matches the line color and has a transparent white background.

    * anchor=<auto>, label anchor point; by default, this is the point where the extremum occurs (one can also use anchor1d).
    * dir=<auto>, label writing direction; by default, this follows the direction of the line.
    * pos=<auto>, position of the label relative to its anchor point; by default, the label is placed on the opposite side of the solution polygon.

- The method **g:DlinprogHalfPlanes(constraint1, options1, constraint2, options2, ...)** allows you to draw the non-solution half-plane for each constraint. Each constraint is a string in the form `"a*x+b*y>c"` or `"a*x+b*y<c"`, followed by its list of options; these are:
  - color=<current color>, the color used for drawing the line, its label, and the half-plane fill.
  - width=<current thickness>, the line thickness in tenths of a point.
  - style=<current style>, the line style.
  - pattern="none", the fill pattern for the half-plane (none by default). Use pattern="fill" for a solid fill, or any other fill pattern recognized by TikZ.

    The values of these four options for a given constraint also apply to subsequent constraints unless modified. However, the following options are specific to each constraint; these are options derived from the *luadraw_decorations* module for lines:
    * label=<auto>, a default label is displayed with the line.
    * node_options=<auto>, a string containing options for the label; by default, it matches the line color and has a transparent white background.
    * anchor1d=0.5, the label's anchor point; by default, this is the midpoint of the line (you can also use anchor).
    * dir=<auto>, the label's writing direction; by default, this follows the direction of the line.
    * pos=<auto>, position of the label relative to its anchor point; by default, the label is placed on the side of the solution polygon opposite the anchor.

- The method **g:DlinprogObjectiveLine(objective_sol,objective_options)** draws an optimization line. The ⟨*objective*⟩ argument represents the solution to the optimization of a functional—specifically, a table with the following fields:
  - expr: contains the expression of the functional,
  - min: contains the numerical value of the minimum,
  - mindot: contains the point where the minimum is reached (complex number),
  - minline: contains the line passing through the point where the minimum is reached,
  - max: contains the numerical value of the maximum,
  - maxdot: contains the point where the maximum is reached (complex number),
  - maxline: contains the line passing through the point where the maximum is reached,

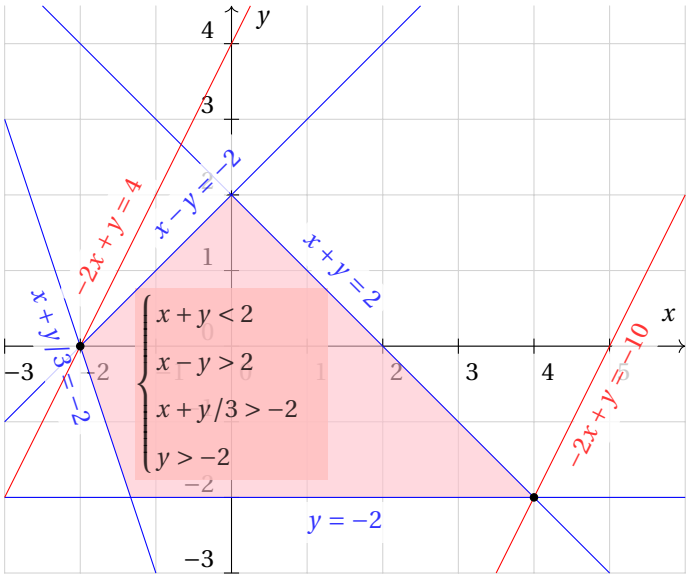
  The ⟨*objective_options*⟩ argument is a table of possible options for this representation; these were previously described in the section regarding the **g:DlinprogRegion()** method.

Examples

```
\begin{luadraw}{name=linear_prog1}
local ld = luadraw
local cpx = ld.cpx
local Z = ld.cpx.Z
local g = ld.graph:new{window={-3,6,-3,4.5}, size={10,10}}
require 'luadraw_linprog'
local system = \luastringO{$\begin{cases}x+y<2\\ x-y>2\\ x+y/3>-2\\ y>-2\end{cases}$}
g:Daxes({0,1,1},{grid=true,gridcolor="LightGray",arrows="->",legend={"$x$","$y$"}})
g:DlinprogRegion(
    {'x+y<2','x-y>-2','x+y/3>-2','y>-2'}, -- contraints
    {draw_options="draw=none,fill=Pink, fill opacity=0.6",lines=true, color="blue", label=system, anchor=Z(0,-0.5),
    node_options="inner sep=0,draw=none,fill=pink, fill opacity=0.8" }, -- options
```

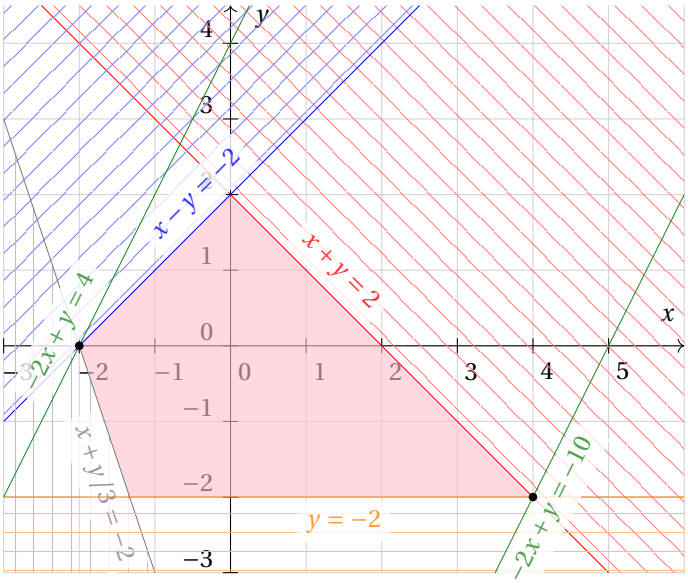
```
    {"-2*x+y"}, {color="red",dots=true, anchor1d=0.5} -- an objective and its options
    )
g:Show()
\end{luadraw}
```

Figure 38: The *g:DlinprogRegion()* method




```
\begin{luadraw}{name=linear_prog2}
local ld = luadraw
local cpx = ld.cpx
local Z = ld.cpx.Z
local g = ld.graph:new{window={-3,6,-3,4.5}, size={10,10}}
require 'luadraw_linprog'
local C = {'x+y<2','x-y>-2','x+y/3>-2','y>-2'} -- contraints
local function mypattern(angle, distance)
    angle = angle or 0
    distance = distance or 0.25 -- distance in centimeter
    return "{Lines[angle="..angle..",distance="..distance.."cm]}"
end
local polygon, optimization = ld.linprogSolve(C,-5,5,-5,5,{"-2*x+y"})
g:Daxes({0,1,1},{grid=true,gridcolor="LightGray",arrows="->",legend={"$x$","$y$"}})
g:Dpolyline(polygon, true, "draw=none,fill=Pink, fill opacity=0.6")
g:DlinprogHalfPlanes(
    C[1], {color="red", pattern=mypattern(-45)},
    C[2], {color="blue", pattern=mypattern(45)},
    C[3], {color="gray", pattern=mypattern(90), anchor1d=0.2},
    C[4], {color="orange", pattern=mypattern(0)}
    )
for _, obj in ipairs(optimization) do
    g:DlinprogObjectiveLine(obj, {color="ForestGreen", dots=true})
end
g:Show()
\end{luadraw}
```

Figure 39: The *g:DlinprogHalfPlanes()* method



In 3D

This involves representing systems of constraints of the form $ax+by+cz<d$ or $ax+by+cz>d$ within a given 3D window; the solution is a convex polyhedron.

Based on this solution, one can seek to optimize a functional of the type $(x,y,z)\mapsto \alpha x+\beta y+\gamma z$ over this polyhedron—that is, to determine its minimum and maximum values on the polyhedron.

The function **ld.linprogSolve3d(constraints, x1, x2, y1, y2, z1, z2, objectives)** performs the calculations[5] and returns a sequence consisting of two items:

1. the solution polyhedron (a table with two fields: *vertices* and *facets*),
2. the list of solutions for each functional.

The ⟨*constraints*⟩ argument is the list of constraints; each constraint is a string of the form `"a*x+b*y+c*z>d"` or `"a*x+b*y+c*z<d"`, where a, b, c, and d are numerical values, and x, y, and z are letters (the multiplication symbol is mandatory).

The arguments ⟨*x1*⟩, ⟨*x2*⟩, ⟨*y1*⟩, ⟨*y2*⟩, ⟨*z1*⟩, and ⟨*z2*⟩ define the solution window: $[x_1;x_2]\times[y_1;y_2]\times[z_1;z_2]$. The ⟨*objectives*⟩ argument is the list of objective functions to optimize (it may be empty); each function is a string in the format `"u*x+v*y+w*z"`, where u, v, and w are numerical values. The solution for an objective function is a table with the following fields:

- `expr`: contains the expression of the objective function,
- `min`: contains the numerical value of the minimum,
- `mindot`: contains the point where the minimum is reached (complex number),
- `minplane`: contains the plane passing through the point where the minimum is reached,
- `max`: contains the numerical value of the maximum,
- `maxdot`: contains the point where the maximum is reached (complex number),
- `maxplane`: contains the plane passing through the point where the maximum is reached,

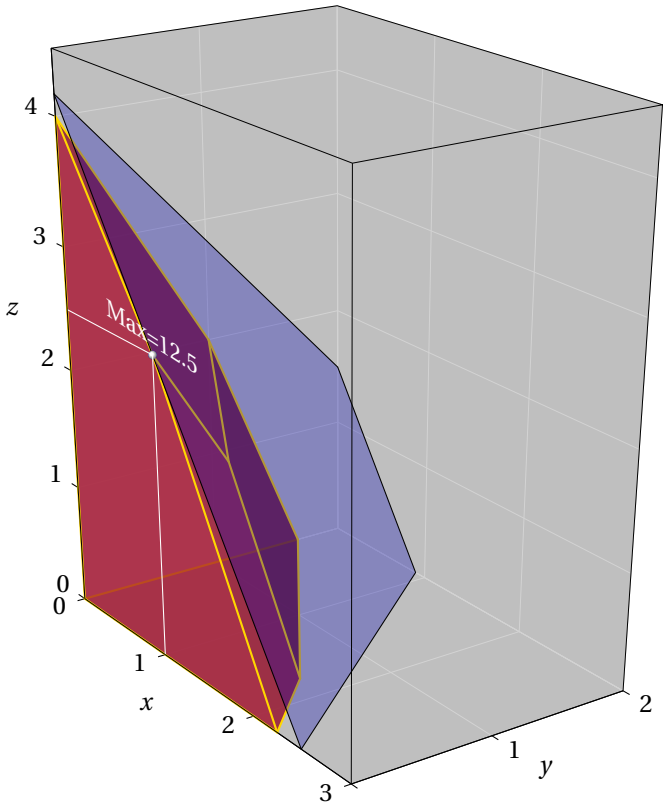
The module does not provide a specific 3D graphical method. Here is an example of how to use the **ld.linprogSolve3d()** function.

[5] This is a geometric solution method. The chosen window is a polyhedron that is subdivided by the various planes corresponding to the constraints.

Example: Maximize $5x+4y+3z$ subject to: $\begin{cases} 2x+3y+z<5 \\ 4x+4y+2z<9 \\ 3x+4y+2z<8 \\ x>0,\ y>0,\ z>0 \end{cases}$

```
\begin{luadraw}{name=linear_prog3}
local ld = luadraw
local cpx = ld.cpx
local pt3d = ld.pt3d
local Z, M = cpx.Z, pt3d.M
local O, I, J, K = pt3d.Origin, pt3d.vecI, pt3d.vecJ, pt3d.vecK
local g = ld.graph3d:new{ window={-2,2,-3,3}, size={10,12,0},window3d={0,3,0,2,0,4.5},
    viewdir={"central",-35,65,15,M(1.5,1,2.25)}}
require 'luadraw_linprog'
local C = {'2*x+3*y+z<5', '4*x+4*y+2*z<9', '3*x+4*y+2*z<8'}
local poly, sol = ld.linprogSolve3d(C,0,5,0,5,0,5,{"5*x+4*y+3*z"})
local P = sol[1].maxplane
local A = sol[1].maxdot
local max = sol[1].max
g:Dboxaxes3d({grid=true,gridcolor="LightGray",fillcolor="lightgray", drawbox=true})
g:Dscene3d(
    g:addPlane(P, {color="blue", edge=true, opacity=0.3, scale=1}),
    g:addPoly(poly, {color="Crimson",opacity=0.7, edge=true, edgecolor="Gold", edgewidth=8}),
    g:addDots(A, {scale=0.75, color="white"})
    )
g:Dlabel3d("Max=$"..max.."$", A, {pos="N", dir={I,K}, node_options="text=white"})
g:Dpolyline3d({ld.pz(A), A, ld.px(A)}, "white")
g:Show()
\end{luadraw}
```

Figure 40: The *ld.linprogSolve3d()* function



II History

1) Version 3.7

- Added the function **ld.poly2graph()** which allows converting a polyhedron into a graph.
- Added the **g:Dpolyline2()** method; it functions the same way as the **g:Dpolyline()** method, except that the list of points—in addition to complex numbers—may also contain TikZ nodes expressed as strings, for example: `"(A.south)"`.
- For the methods **g:Cylinder_outline()**, **g:Cone_outline()**, and **g:Frustum_outline()**: the returned result contains two additional fields: the *hiddenside* field and the *hiddensection* field.
- In the module *luadraw_decorations*: added the method **g:Dangle3d()** which draws a right angle in space with options to paint the parallelogram and to add a label.
- Added the method **g:Dmarkseg3d()** which allows marking a segment of space with small segments.
- Added new 3D projection modes: `"yx"`, `"zx"`, `"zy"`, `"osi"`, `"yOx"`, `"zOx"`, and `"zOy"`, which are the "symmetrical" versions of the modes `"xy"`, `"xz"`, `"yz"`, `"iso"`, `"xOy"`, `"xOz"`, and `"yOz"`.
- Addition of the function **ld.cutfacet2()**, which allows for the cutting of a list of facets (or a polyhedron)—not by a plane (which is the purpose of the **ld.cutfacet()** function), but by an implicit surface defined by the equation $f(x,y,z)=0$, subject to certain conditions on f.
- In the *luadraw_spherical* module, three methods have been added: **g:Dhalf_sphere()**, **g:Dquater_sphere()**, and **g:Dheighth_sphere()**.
- For the **g:Dcut_sphere** method, the following options have been added: `section=""`, which controls whether or not the circular section is drawn, and `outline=false`, which controls whether or not the outline of the spherical part is drawn.
- Bug fixes...

2) Version 3.6

- In the *luadraw_shadedforms* module: addition of the `bilinear=true/false` option for the **g:Dshadedrectangle()** method, allowing the use (or not) of bilinear shading on each grid cell (`true` by default).
- Addition of the function **ld.pg_gonreg()**, which returns a list of lists of complex numbers representing a regular star polygon.
- In the *luadraw_spherical* module:
  - Most of the module's global functions can now be applied to a sphere other than the current one.
  - Added the **g:Split_facet()** method, which calculates and returns—as 3D paths—the outline of the visible part of the spherical facet and the outline of the hidden part.
  - Added the **g:Split_region()** method, which calculates and returns—as 3D paths—the outline of the visible part of the spherical region and the outline of the hidden part.
  - Two options—`front` and `back`—have been added to the **g:DSfacet()** and **g:DSregion()** methods; these allow the user to retrieve the path corresponding to the outline of the visible part (for the first option) and the path corresponding to the outline of the hidden part (for the second option).
  - Most graphical elements in this module now have options to add a label.
- For the **ld.section2tube()** function, the `opensection=true/false` option has been added; with `true`, the section will not be closed—meaning there will be no facets between the last and first points of the section, and the tube will have a lateral opening.

- For the **ld.line2tube()** function, the `opensection=true/false` option (as described above) and the `angle={angle1,angle2}` option have been added; the latter allows the start and end angles (in degrees) to be modified so that the section forms only part of a circle.
- Added three options for the **g:BeginOnPlane()** method: the `border=true/false` option and the `border_options` option, which allow drawing a frame around the chosen 2D window in the plane, as well as the `tikzmatrix=true/false` option which allows using or not the TikZ transformation matrix for this method.
- Added the function **ld.parabola(A, B, C)**, which returns a Bézier curve representing a parabolic arc from point ⟨*A*⟩ to point ⟨*C*⟩, passing through point ⟨*B*⟩ (three complex numbers).
- Added the function **ld.param_path(P)**, which returns a function representing a parametrization of the 2D path ⟨*P*⟩.
- Addition of the **ld.splitpath()** function, which allows a path to be split into two paths at a given point.
- Added the function **ld.common_tangent()** and the corresponding drawing method **g:Dcommon_tangent()**; this allows for drawing the common tangents to two parametric curves.
- Added the function **ld.parallel_tangent()** and the corresponding drawing method **g:Dparallel_tangent()**; this allows for drawing tangents to a parametric curve that are parallel to the tangent at a point with a given parameter, or parallel to a given direction.
- The optional argument of the **ld.solve()** function can now take an additional value: `"bisection"`, in which case the method used will be bisection (this can be useful if the Newton method fails).
- Added the function **ld.getmatrix(T1, T2)**, which calculates and returns the matrix transforming triangle ⟨*T1*⟩ (a list of three non-collinear 2D points) into triangle ⟨*T2*⟩ (a list of three 2D points).
- Added the function **ld.getmatrix3d(T1, T2)**, which calculates and returns the matrix transforming tetrahedron ⟨*T1*⟩ (a list of four non-coplanar 3D points) into tetrahedron ⟨*T2*⟩ (a list of four 3D points).
- Bug fixes...

3) Version 3.5

- In the *luadraw_spherical* module:
  - The **g:Define_sphere()** method has three additional options: `back=true/false`, `inside=true/false`, and `front=true/false`. These allow for toggling the display of elements located behind, inside, and in front of the sphere.
  - Added the **g:DSaxes()** method, which draws the three axes.
  - Most graphics methods now include additional options for adding a label.
  - Added the functions **ld.smidpoint()** and **ld.sbarycenter()**.
- In the *luadraw_povray* module:
  - Added the option `clip=<boolean>` for the **g:Pov_polyline()** method to clip the polygonal line to the current 3D viewport.
  - Added the option `csg=true/false` for the **g:Pov_facet()** method. With the value `true`, the defined object can be used in CSG operations (with some limitations).
- For 3D paths: addition of the instructions *"e"* for ellipse and *"ea"* for elliptic arc.
- New 3D functions for ellipses and elliptic arcs: **ld.ellipse3db()**, **ld.ellipse3d()**, **ld.ellipticarc3db()**, **ld.ellipticarc3d()**, as well as the associated drawing methods: **g:Dellipse3d()** and **g:Dellipticarc3d()**.
- Added the method **g:Dcut_sphere()** which allows drawing a sphere after it has been cut by a plane.
- Added the `apex=true/false` option for the **g:Dcone()** method; with the value `true`, the apex is marked by a point when the cone is viewed from above.

- The **g:Saveattr()** method now accepts an argument: **g:Saveattr(scope_options)**; this is a string (empty by default) that will be passed to the *scope* instruction: `\begin{scope}[<scope_options>]`.
- Added the `lowlevel=true/false` option for the **g:BeginOnPlane()** method; with the value `true`, the transformation matrix is applied at the lowest level, so all graphical elements—arrows, text, etc.—are drawn in the desired plane.
- Addition of the *luadraw_linprog* module, which enables the solving and graphical representation of solutions to 2D or 3D linear programming problems.
- Added the option `gradient=true/false` for the methods **g:Dcylinder()**, **g:Dcone()**, and **g:Dfrustum()**. This specifies whether the fill should be done with a gradient or not (`true` by default).
- Bug fixes...

4) Version 3.4

- For the **g:Dfrustum()** method, the old syntax **g:Dfrustum(C,R,r,V,options)** has been changed to **g:Dfrustum(C,R,r,A,options)**; the difference is that the center of the second circular base is point $\langle A\rangle$, whereas in the old syntax it was point $C+V$. Similarly, for the **ld.frustum()** function, the old syntax **ld.frustum(C,R,r,V,nb,open)** becomes **ld.frustum(C,R,r,A,nb,open)**, with the same difference. This change ensures a consistent syntax for cylinders, cones, and truncated cones.
- For the method **g:Beginclip(path, inverse)**, the ⟨*path*⟩ can be a 2D or 3D path.
- In the *luadraw_spherical* module: addition of the **g:DSregion()** method, which allows painting a region of the sphere bounded by a simple closed spherical curve.
- Optional arguments have been added to the **g:Endclip([draw_path, draw_options])** method: ⟨*draw_path*⟩ is a boolean (`false` by default); when set to `true`, the path used for clipping is drawn after the group is closed, and in this case, the optional argument ⟨*draw_options*⟩, which must be a character string (empty by default), is passed to the *\draw* command.
- Addition of functions specific to 2D or 3D paths: **ld.path()**, **ld.path3d()**, **ld.convpath()**, **ld.polyline2path()**, **ld.polyline2path3d(** and the methods, **g:Convpath3d()**, **g:Path3d2path2d()**.
- Added the methods **g:Cylinder_outline()**, **g:Cone_outline()**, **g:Frustum_outline()**, and **g:Sphere_outline()**, which return a table containing the characteristic elements (outline, visible edges, hidden edges, etc.).
- Added the methods **g:Cylinder_tangency()**, **g:Cone_tangency()**, **g:Frustum_tangency()**, and **g:Sphere_tangency()**, which calculate and return the list of tangency points for cylinders, cones, truncated cones, and circles drawn on a sphere.
- For the function **ld.read_table3d()**, added the option `mode` allowing you to choose a type of 3D table (types `"xyz"`, or `"y/x"` or `"x/y"`.
- The `usepalette()` option (for drawing facets) now accepts a third (optional) argument:

  `usepalette(palette, mode, minmax)`,

  where ⟨*minmax*⟩ is a table of two numbers; these two numbers are used as the minimum and maximum values for the ⟨*mode*⟩ argument. If ⟨*minmax*⟩ is omitted, these two values are calculated automatically.
- Bug fixes...

5) Version 3.3

- Added the function **ld.read_table3d()** to read and use an array (list of lists) containing the values of one (or more) function(s) of two variables on a tile $[x_1;x_2]\times[y_1;y_2]$.
- Added the option `reverse=true/false` to the facet drawing methods (**g:Dfacet()**, **g:Dmixfacet()**, **g:Dpoly()**, **g:addFacet()**, ...). With the value `true`, the facet orientation is reversed.

- The *luadraw_pdfliteral* module has been added; it allows for direct drawing within the PDF stream—including 2D paths, 2D polylines, sets of 2D points (circular), or 3D facets—without using the *\draw* command. This significantly reduces compilation time when dealing with large datasets, subject to certain limitations: no opacity changes, and only two fill styles (none or solid).
- Three new methods have been added to the *luadraw_spherical* module: **g:DSaddback()**, **g:DSaddinside()**, and **g:DSaddfront()**, which allow for the addition of graphical elements (paths or polylines) behind, inside, or in front of the sphere.
- In the **g:Dfacet()** and **g:Dmixfacet()** methods, when in *ld.mWireframe* mode (drawing edges only), if the `usepalette` option is not `nil`, the color of each edge is calculated from the palette using the same method as for facets when they are rendered.
- Adding the option `out=<variable>` to the method **g:BeginOnPlane()**, it allows retrieving a 2D matrix corresponding to the new axes.
- Added the `graphics_options` option for the **g:Dmapimage()** method. Added the `node_options` option for the **g:Dmapimage()** and **g:Dimage()** methods.
- For the **g:addPolyline()** method, there is a new (experimental) option called `double={border color, border width}`, which adds a border to each side of the polygonal line. With the default value (`nil`), there is no border.
- The syntax of the functions **ld.lineEq()** and **ld.planeEq()** has been extended to take into account a possible inequality.
- Bug fixes...

6) Version 3.2

- Added the methods **g:BeginOnplane()** and **g:EndOnPlane()**, which allow drawing on a plane in space using 2D graphics methods.
- In the *luadraw_fields* module, vector fields tangent to a surface have been added[6]. The **ld.surfacefield()** function calculates and returns the vector field, the **g:Dsurfacefield()** method allows drawing the vector field with (or without) the surface.
- The **ld.linspace()** function has second possible syntax: **ld.linspace(a1, b1, n1, b2, n2, ..., bp, np)** which returns a list of ⟨*n1*⟩ evenly distributed numbers from ⟨*a1*⟩ to ⟨*b1*⟩, followed by ⟨*n2*⟩ evenly distributed numbers from ⟨*b1*⟩ to ⟨*b2*⟩ (but ⟨*b1*⟩ is not repeated), etc.
- In the *luadraw_spherical* module, these functions have been added:
  - **ld.interGreatC(C1,C2)** which returns, as a sequence, the two points of intersection of the two great circles ⟨*C1*⟩ and ⟨*C2*⟩ belonging to the sphere.
  - **ld.interSphericalC(P1, P2)** which returns, as a sequence, the points of intersection (if they exist) between two circles belonging to the sphere (not necessarily great circles).
  - **ld.projstereo_Scircle(P, N, h)** which returns, as a path, the stereographic projection of a circle drawn on the sphere.
  - **ld.projstereo_Sfacet(L, N, h [, close])** which returns, as a path, the stereographic projection of a spherical facet.
- Adding three options to the **g:Dboxaxes3d()** method: `xlabels={x1,…,xn}`, `ylabels={y1,…,yn}`, `zlabels={z1,…,zn}`. These options allow you to apply labels to the axes. By default, these options have the value `nil`, in which case the default labels (one per tick mark) are displayed.
- In the *luadraw_povray* module, there is a new option in the default settings: `arrowscale={1,1}`, which is a table of two numbers. The first number is a scale factor for the radius of the arrow bases (the arrows are cones), and the second is a scale factor for the arrow heights.

[6] On an initiative of Explorer.

In the **g:Pov_polyline()** and **g:addPolyline()** methods, the option of the same name can now be either a table of two numbers or a single number (in which case the two numbers are considered equal).

- For 3D, there is a new global variable `ld.Hiddenlinescale` which defaults to `2/3`. This means that the thickness of hidden lines will be equal to that of visible lines, multiplied by this number, when using the **g:Dscene3d()** method.
- Bug fixes...

7) Version 3.1

- Added the function **ld.implicit_inequality()** which returns a **path** representing the outline of the part of the plane located within a certain block and satisfying a condition of the type $f(x,y) >= 0$ or $f(x,y) <= 0$.
- Added the method **g:Dimplicit_inequalities()** which can fill the set of points satisfying a constraint system of the type $f_i(x,y) > 0$ or $f_i(x,y) < 0$.
- Adding the function **plane2rectangle(P, V, L1, L2)** which returns a rectangle (a list of 3D points) to represent this plane. This is the same rectangle drawn by the method **g:Dplane(P, V, L1, L2)**. It can be drawn using the method **g:Dpolyline3d()** or as a facet.
- For the methods **g:addPlane()** and **g:addPlaneEq()** (which are used in **g:Dscene3d()**), the `rectangle` option has been added. When `rectangle=nil` (the default value), the plane is cut by the 3D window and the resulting facet is drawn. When `rectangle={V,L1,L2}`, the plane is drawn as a rectangular facet; this is the same rectangle drawn by the method **g:Dplane(P, V, L1, L2)** (where P denotes the plane).
- When creating a 3D graph, the `window3d` option now takes into account the scales on the three axes:

  `window3d={x1,x2,y1,y2,z1,z2 [,xscale,yscale,zscale]}`.

  These three scales are optional and are set to 1 by default; they determine the initial 3D matrix of the graph (which is therefore no longer the identity if one of the scales is different from 1).
- When creating a graph, the `margin` option can now be reduced to a single number when all four margins are equal, for example `margin=0`.
- Added the option `useclip=<boolean>` for the method **g:Dinequalities()** which allows you to choose the technique to use, either contour calculation (with the value `false`, default value), or with a series of clips (with the value `true`).
- The **rectangle()** function now has a second possible syntax: **rectangle(a, b)**, in which case the rectangle has its sides parallel to the axes and has opposite vertices $\langle a \rangle$ and $\langle b \rangle$. The same applies to the corresponding graphical method **g:Drectangle()**.
- For the **line2strip()** function, the argument $\langle ends \rangle$, which was previously a boolean, can now take the values `"none"` (equivalent to `false`), `"butt"` (equivalent to `true`), or `"round"`. Boolean values are still accepted.
- The function **line2strip()** now has an additional argument called `mode`, which can be `"center"` (default value), or `"left"`, or `"right"`, in the first case the strip is centered on the polygonal line, in the second case it is on the left of the line, and in the third case it is on the right of the line.
- In the function **read_csv_file()**, add the option `comment=<char>`, this indicates the characters that begin a line of comments.
- All methods for drawing lines or half-lines (**g:Dline()**, **g:Dperp()**,...) now have an `scale` option, like the **g:Dseg()** method. This option (which defaults to 1) can be a number (percentage) or a table of two numbers (percentages). The second case allows you to vary the two endpoints separately.
- Bug fixes...

8) Version 3.0

This version introduces a major change: all data related to the *luadraw* package is now stored in the namespace (table) named *luadraw*. This necessitates the use of dot notation, for example, `luadraw.graph` instead of `graph`. However, it is possible to create shortcuts; for instance, the instruction `local ld = luadraw` will allow you to use *ld* instead of *luadraw* in dot notation. Refer to the very beginning of this document for more details.

This change results in incompatibility with previous versions; however, the changes required to adapt older code are minimal, especially since there are no changes to the graphics methods (they were already encapsulated in two classes).

This change also brings some (minor) modifications to extensions; refer to the documentation for more details.

- Added a second possible syntax for the functions **ld.surface()** and **ld.obj_surface()**: **ld.surface(f, mesh)** and **ld.obj_surface(f, mesh)** where $\langle mesh\rangle = \{\{u_1,\dots,u_n\},\{v_1,\dots,v_m\}\}$ (increasing list of values of parameter u and increasing list of values of parameter v).
- In the *luadraw_decorations* module: the 2D and 3D line drawing methods have been overridden so that the $\langle draw_options\rangle$ argument, which is normally a string passed to the *\draw* instruction, can be replaced by a table whose fields represent possible options (such as adding a label). The method names remain unchanged, and the old syntax is still valid.
- In the functions **ld.curve2cone()**, **ld.curve2cylinder()**, **ld.line2tube()**, **ld.section2tube()**, **ld.rotcurve()** and **ld.rotline()**, add the option `obj=true/false`; with the value by `false` (default) the functions return a list of facets, with the value `true` they return a table `{vertices={3D points}, facets={{index1,…},…}, normals={3D vectors}}`. If the **g:Dpoly()** method does not take into account the *normals* field, the **g:Pov_facet()** method of the *luadraw_povray* module, on the other hand, uses this field when it is present.
- Added the function **ld.cutpolyline2(P,f,sg,close)** where $\langle P\rangle$ is a polygon (list of complex numbers), $\langle f\rangle$ is a function ($x\mapsto f(x)\in\mathbf{R}$), and $\langle sg\rangle$ is a string equal to `">"` or `"<"`. This function returns the outline of the part of the polygon satisfying the constraint $y>f(x)$ or $y<f(x)$ depending on the value of $\langle sg\rangle$.
- Added the method **g:Dinequalities(constraints, options)** which draws the set of points satisfying a constraint system of the type $y>f_i(x)$ or $y<f_i(x)$.
- In the *luadraw_povray* module: added the option `usepalette={palette, mode, minmax}`.
- Bug fixes...

9) Version 2.8

Non-exhaustive list:

- Added functions: **nth_root(n,x)** (nth root of a real x, defined on $\mathbb{R}$ when n is odd); **cpx.cosh()**, **cpx.sinh()** (complex hyperbolic cosine and sine) and **cpx.pow(z,a)** (for calculating z^a with z a complex number and a a real number).
- Added the *luadraw_coils_chains* extension which allows drawing springs and chains.
- Added the *luadraw_log_axes* extension which allows you to create and draw on a logarithmic grid in x, or in y, or in x and y.
- Added the `use_siunitx` option for the **g:Daxes()**, **g:DaxeX()**, **g:DaxeY()**, **g:Dgradbox()**, **g:Dgradline()** methods. This allows you to locally use or not use the formatting of numeric values by the *siunitx* package.
- Added the `showlines` option for the **g:Dgrid()** method which allows you to show or hide horizontal and/or vertical lines.
- Added the *luadraw_decorations* extension, which enhances certain drawing methods by adding options. Currently, there is **g:Ddecoratedarc()** for 2D arcs and **g:Ddecoratedarc3d()** for 3D arcs.
- For the methods **g:Dpoly()**, **g:Dfacet()**, **g:Dmixfacet()**, and **g:addFacet()**, in the option `usepalette={palette, mode}`, the second argument can now be a function, $\langle mode\rangle: f\mapsto \text{mode}(f)\in\mathbb{R}$, where f denotes a facet (a list of 3D points). Facets with the smallest value have the first color of the palette, those with the smallest value have the last color of the palette, and for the others, the color is calculated by linear interpolation. Previously, the argument $\langle mode\rangle$ could only be `"x"`, `"y"`, or `"z"`.

- Added the function **obj_surface(f,u1,u2,v1,v2 [, grid])** which returns the surface parameterized by $\langle f\rangle\colon (u,v)\mapsto f(u,v)\in\mathbf{R}^3$ in *obj* format, that is, a table with three fields {vertices={...}, facets={{...},...}, normals={...}}. The first two fields are identical to the case of polyhedra, and the third field contains the unit vectors normal to the surface at each vertex. The facets are triangular.

- In the *luadraw_povray* module: a second syntax has been added for drawing smooth parametric surfaces:

  g:Pov_surface(f,u1,u2,v1,v2,options)

  where $\langle f\rangle\colon (u,v)\mapsto f(u,v)\in\mathbf{R}^3$ is the parameterization. This method is faster than the previous one.

- In the *luadraw_spherical* module: add the global variable `Hiddendelayed` = `false`. With the value `false`, hidden parts are drawn at the end of the **g:Dspherical()** instruction; with the value `true`, they are drawn at the very end of the current graph, which can be useful if you have added elements after the sphere that hide part of it.

- For the **g:Daxes()** method: the `originloc` option is still the point used as the origin for the graduations, but it is no longer automatically the point of intersection of the two axes.

- For the method **g:Daxes()**: addition of the options `xynode_options = ""`, `xnode_options = xynode_options` and `ynode_options = xynode_options` which allow passing options to the instruction `\node{}` for all labels (except legends).

- For the **g:addAxes()** method, add the option `labels={"$x$","$y$","$z$"}` to manage the labels displayed at the end of each axis.

- Bug fixes...

10) Version 2.7

Non-exhaustive list:

- The basic solid drawing methods: **g:Dcylinder()**, **g:Dcone()** and **g:Dfrustum()** now have two additional options: `edgestyle` and `edgewidth` (as for the **g:Dsphere()** method).

- In the module *luadraw_compile_tex*, for the methods:

  g:Dcompiled_tex(L, anchor, options) and **g:Compiled_tex2path3d(L, options)**,

  add the option `pos` identical to the labels.

- In the *luadraw_compile_tex* module, three global variables are added to manage access to the *pdflatex*, *pdf2ps* and *pstoedit* programs. These variables are `pdflatexcmd`, `pdf2pscmd` and `pstoeditcmd`, they allow you to optionally add a path to the program, for example: `pstoeditcmd = "/usr/bin/pstoedit"`.

- New syntax for the function **circle(data, nbdots)**, where $\langle data\rangle$ is a list (center and radius, or three points on the circle) and $\langle nbdots\rangle$ is the desired number of points. The old syntaxes remain valid.

- New syntax for the function **ellipse(data, nbdots)**, where $\langle data\rangle$ is a list: {center, rx, ry, incline}, and $\langle nbdots\rangle$ is the desired number of points. The old syntaxes remain valid.

- Added the function **mixpalette(pal, percent, color)** which returns a new palette after mixing each color of the palette $\langle pal\rangle$ with $\langle color\rangle$.

- Correction of numerous typos in the documentation.

- Bug fixes...

11) Version 2.6

Non-exhaustive list:

- Added the *luadraw_povray* extension which allows you to create an image with Pov-Ray and include it in the current graphic to draw over it.
- Added the *luadraw_fields* extension which allows drawing vector fields or gradient fields.
- Added the *luadraw_shadedforms* extension, which allows drawing 2D polygonal lines or filling a shape with a color gradient based on the chosen calculation method and palette.
- The **g:Dshadedpolyline()** method is now part of the *luadraw_shadedforms* extension.
- Added the methods **g:Dimage()** and **g:Dmapimage()**. The first allows you to include an image in the graph, and the second allows you to map an image onto a parallelogram.
- Added the function **parallelogram()**, which returns the vertices of a parallelogram constructed from a vertex and two vectors. Also added is the corresponding drawing method **g:Dparallelogram()**.
- Added the options `gradside` and `gradsection` which allow modification of gradient parameters in the drawing of cylinders, cones and truncated cones (methods **g:Dcylinder()**, **g:Dcone()** and **g:Dfrustum()**).
- Extension of the method **g:Newcolor(name, color)**, the second argument can now be either a table of three RGB components, or a string representing a color.
- Bug fixes...

12) Version 2.5

Non-exhaustive list:

- Added the function **read_csv_file()**[7] which allows reading a *csv* file with different options.
- The *luadraw_palettes* extension has been updated to version 1.3.0 of the `@prism` project of Christophe BAL.
- Added the method **g:Dshadedpolyline()** which allows drawing a 2D polygonal line with a color gradient depending on the calculation method and palette chosen.
- Added the **g:Dpolynames()** method which allows displaying a polyhedron with the number of faces and/or those of vertices.
- Added the *luadraw_cvx_polyhedra_nets* extension, which allows you to determine a net for convex polyhedra.
- Bug fixes...

13) Version 2.4

Non-exhaustive list:

- Added the central projection.
- Added the `legendstyle` option for axes, to impose a label style ("auto", "N", "E", ...) for legends when there are any (until now, the style was necessarily "auto").
- Added the **g:Labeldir()** method which allows global management of the writing direction.
- Added the functions **interCS()** (intersection between a circle in space and a sphere), and the function **interSSS()** (intersection between 3 spheres).
- Added the function **voronoi()** as a complement to Delaunay triangulation, it allows you to make Voronoi diagrams.

[7] Based on an idea by Christophe BAL.

- Added the function **parallel_polyline()** which returns a parallel polygonal line.
- Added the function **tangent_from()** and the method **g:Dtangent_from()** which allows drawing the tangents to a given curve from a given point.
- Bug fixes...

14) Version 2.3

Non-exhaustive list:

- Added cavalier perspective projections: on yz, on xz or on xy, as well as the isometric projection.
- Added the function **section2tube()**.
- Added the *luadraw_compile_tex* module.
- Added the **g:Proj3dV()** method for calculating the projection of space vectors onto the screen plane.
- Added the functions **circumcircle()** and **incircle()** in 2D, they return a sequence: center and radius.
- Added the function **line2strip()** which returns a path representing a "strip" centered on a given polygonal line.
- Added the function **delaunay()** which performs a Delaunay triangulation on a list of points and returns the list of triangles obtained.
- Added the function **cpx.normalize(z)** which returns the complex number z divided by its modulus (or `nil` if it is zero).
- Added the instruction **whatis(variable, msg)** which displays the status of a ⟨*variable*⟩ (along with the message ⟨*msg*⟩) and its contents in the terminal.
- Bug fixes...

15) Version 2.2

Non-exhaustive list:

- Added the `clip` option for the methods: **g:Dfacet()**, *Dmixfacet()*, **g:addFacet()**, **g:addPoly()** and **g:addPolyline()**, as well as for point cloud drawing methods, and line drawing methods such as **g:Dpolyline3d()**, **g:Dparametric3d()**, **g:Dpath3d()**, etc.
- Added the `xyzstep` option for the **g:Dboxaxes3d()** method. This option defines a common step for all three axes (1 by default).
- Added the **g:DSdots()**, **g:DSstars()**, **g:DSinvstereo_curve()**, and **g:DSinvstereo_polyline()** methods to the *luadraw_spherical* module.
- Added the *luadraw_palettes* module.
- Added the **interDC()** function (intersection between a line and a circle in 2D) and the **interCC()** function (intersection between two circles in 2D).
- Added the **curvilinear_param()** and **curvilinear_param3d()** functions, which allow you to parameterize a list of points (one in 2D and the other in 3D) with a function of a variable t between 0 and 1.
- Added the function **cvx_hull2d()**, which returns the convex hull (polygonal line) of a list of points in 2D, and the function **cvx_hull3d()**, which returns the convex hull (list of facets) of a list of points in 3D.
- Added the methods **g:Beginclip(path)** and **g:Endclip()**, which make it easier to set up clipping using TikZ.
- Added the functions **normal()**, **normalC()**, and **normalI()**, which return the normal to a 2D curve at a given point. The corresponding graphics methods have also been added.

- Added the function **isobar()**, which returns the isobarycenter of a list of complexes.
- Added the `usepalette={palette,mode}` option for the **g:Dpoly()**, **g:Dfacet()**, **g:Dmixfacet()**, and **g:addFacet()** methods.
- Added the **clipplane()** function, which allows you to clip a plane with a convex polyhedron. The function returns the section, if it exists, as a facet.
- Added the **cartesian3d()** and **cylindrical_surface()** functions, which calculate and return surfaces, with the option to add dividing walls for the **g:Dscene3d()** method.
- Added the function **evalf(f, ...)** which allows a protected evaluation of $f(\ldots)$. It returns the result of the evaluation if there is no runtime error from Lua, otherwise it returns `nil` but without causing the script execution to terminate.
- Added the function **split_points_by_visibility()** (3d) to separate a curve into two parts: visible part, hidden part.
- In the methods **g:Dfacet()**, **g:Dmixfacet()**, **g:Dpoly()**, **g:Dedges()**, **g:addFacet()**, **g:addPolyline()**, **g:addPoly()**, the default values for the line drawing options (thickness, color, and style) are the current values.
- Bug fixes...

16) Version 2.1

Non-exhaustive list:

- By default, TikZ files are saved in a subfolder called *_luadraw*. The new package option `cachedir` allows you to change this.
- The `line join=round` option is automatically added to the *tikzpicture* environment.
- Two additional options for the *luadraw* environment: `bbox` and `pictureoptions`.
- A number of additional 2D and 3D geometric construction functions.
- Graduated axes (2D, 3D) use the *siunitx* package to format labels when the global variable `siunitx` is set to `true`.
- Added upright and slanted truncated cones (**frustum()** and **g:Dfrustum()**).
- Added regular pyramids (**regular_pyramid()** and truncated pyramids **truncated_pyramid()**).
- Cylinders and cones are no longer necessarily upright; they can now be slanted.
- Added the **cutpolyline(L, D, close)** function.
- (Elementary) drawing of sets (**set()** function) and operations on sets (**cap()**, **cup()**, **setminus()**).
- Modification of the `mode` option of the **g:Dplane()** method.
- Addition of the `close` option for the **g:addPolyline()** method.
- Bug fixes...

17) Version 2.0

- Introduction of the *luadraw_graph3d.lua* module for 3D drawings.
- Introduction of the `dir` option for the **g:Dlabel()** method.
- Minor changes in color management.

18) Version 1.0

First version.